FUNDAMENTAL SOURCES OF EFFICIENCY

BY

FLETCHER DURELL, Ph.D.
HEAD OF THE MATHEMATICAL DEPARTMENT OF THE LAWRENCEVILLE SCHOOL;
AUTHOR OF A SERIES OF MATHEMATICAL TEXTBOOKS

PHILADELPHIA AND LONDON
J. B. LIPPINCOTT COMPANY
1914

PRINTED IN THE UNITED STATES OF AMERICA

To
My Wife

PREFACE

The present work is an attempt to analyze the various forms and sources of efficiency into a few elemental principles. It is felt that the study of such primal elements will not only aid in the mastery of efficiency in a given field, but will prevent this important idea from assuming a narrow meaning and thus leading to limited or even, in some respects, harmful results.

The above treatment of the subject also facilitates, on the one hand, the use of efficiency principles in all the various details of work and experience and, on the other, their application in the formation of a general philosophy of life.

While the book has been cast in a form adapted to general reading, groups of exercises have been inserted which, it is hoped, will add to its value if it is used as a textbook in any institution where the principles of efficiency are taught either generally or in any specific field.

It is also believed that such a presentation of the principles of efficiency may throw light on the problem of vocational and cultural studies, and perhaps suggest how the study of the principles of efficiency, as such, may be made a central study in educational systems, helping to give both vocational and cultural values to other more conventional studies.

The leading ideas contained in this book were suggested to the author when reading the philosophy of Herbert Spencer more than twenty years ago. The importance of investigating the elements which constitute fitness, that is, efficiency, becomes evident as soon as the principle of the survival of the fittest is recognized. While all the principal ideas in the book were thus arrived at before the present energetic movement toward efficiency in more or less concrete fields arose, new light and many important details have been obtained from a reading of the publications of the Efficiency Society, and the works of writers like F. W. Taylor, Harrington Emerson, and Hugo Münsterburg.

The writer wishes to express his indebtedness to Ida Kruse McFarlane and Charles William Cuno of the Department of Efficiency, University of Denver, who have kindly read the manuscript and made important suggestions concerning the same.

CONTENTS

CHAPTER I

PRELIMINARY SURVEY. DEFINITIONS

Recent Advances in Efficiency

In the year 1900 a competent authority estimated that the cost of unskilled labor, such as digging dirt and carrying burdens, when done by man unaided, was one thousand times as great as the cost of the same labor when done by the aid of the best steam engines. But even since that statement was made, there has been remarkable progress. For instance the Diesel oil and tar engine, which is three, or under some circumstances four times as efficient as the steam engine, has been invented and come into use.

When he was a young man Harrington Emerson, on one occasion, had an opportunity to watch the work of digging the Suez Canal. He thus describes the crude way in which part of the digging was being done: "Many of the workers were girls digging up the sand with their hands, throwing it into the rush basket each had woven for herself, lifting the baskets to their heads, and carrying the load of 20 or 30 pounds a hundred feet up the bank and dumping it." He calls attention to the contrast of this method of excavating and removing earth with that used in digging the Panama Canal. At the latter place the digging has been done almost entirely by steam scoops, each lifting two or more cubic yards of dirt at a single stroke. The earth has, in some cases, been removed for twenty miles by trains of cars, and the tracks on which these cars ran, when a change in their position became necessary, were moved in large sections at a single operation. Yet in the highly efficient process of digging the Panama Canal, many improvements were made from time to time. It was estimated that in this work in the year 1912 a ton of dynamite was twice as effective as in the year 1908.

Nor has the advance in the efficiency of skilled labor been less noteworthy than that in such crude and fundamental work as digging dirt. For instance in the weaving of cotton fabrics, the use of power machinery has increased the efficiency of a workman at least eightfold.

Primitive methods of agriculture, as compared with hunting and fishing, increase the productivity of the land 200 times. Here again, the use of machinery and other improved methods have greatly increased the efficiency of man's labor. Thus "between the years 1830 and 1896 the time of human labor required for the production of a bushel of wheat was reduced from three hours to ten minutes, and the cost of the human labor to produce this bushel declined from 17¾ cents to 3½ cents".

In the year 1866 the death rate of New York City was 34 per thousand; in the year 1912 this rate had been reduced to 14.1 per thousand. This means that twelve years have been added to the length of the average human life in that city. Similar illustrations might be given of increased efficiency in the various other departments of human activity.

Opportunities for Further Increases in Efficiency

In spite of the large advances which have been made, even a slight examination shows that opportunities exist for large extensions of the efficiencies which have been mentioned. Even in the Diesel engine nearly two thirds of the fuel energy is wasted. Owing to losses in mining coal, and in converting coal energy into electric light, only a little over one half of 1 per cent of the energy latent in coal as it lies in the ground is utilized by man in using coal as a source of light.

In the fundamental industry of agriculture, an efficiency expert like Harrington Emerson estimates the farms in the United States as about 30 per cent efficient. At the Conservation Congress in the year 1911, the statement was made that one-third of the barnyard manure in the United States is wasted, and that this means an annual loss of $800,000,000. In the World's Work in the year 1906, it was stated that "all authorities are now agreed that we might double the yield of our principal crops". In his address to the Pennsylvania Pilgrims, May 1913, President Van Hise of the University of Wisconsin said: "We know enough about agriculture so that the agricultural production of the country could be doubled if the knowledge were applied. We know enough about disease so that if the knowledge were utilized infectious and contagious disease would be substantially destroyed in the United States—and that within a score of years. We know enough about eugenics so that if the knowledge were applied the defective classes would disap-

pear within a generation. Similarly in other fields our knowledge has expanded far beyond utilization."

Growing Need of Increased Efficiency

Not only do opportunities for increased efficiency thus exist, but there is also an increasing need for the development and application of such methods. On February 14, 1912, President Taft signed the bill for the admission of Arizona as the forty-eighth and last state in the Union. Henceforth the people of the United States must look forward primarily, not to new territory, but to the best possible use of the resources now in their possession. The same principles apply essentially to the world as a whole. The leading nations have explored and taken possession of all the productive parts of the earth's surface. By the use of steel and electricity this process has recently been brought to a conclusion with great suddenness. Henceforth the law of the survival of the fittest will have a new and more crushing power.

Increasing Value of Efficiency

On the other hand improved methods of efficiency will now have an ever increasing value for those who know how to master and apply them. Of the 110,000 species of flowering plants known to exist in the world, less than 1 per cent have been studied in any adequate way and reduced to man's service. The non-flowering plants and the animal life of the world has been studied and utilized in an even smaller degree. Quite as great is the wealth which man can make by his own exertions. Thus in the field of chemistry, between the years 1883 and 1900 the number of carbon compounds increased from 16,000 to 67,000. As yet but few of these products have been fully studied and employed. Humus, because it improves the texture of the soil, conserves moisture, supplies food to the growing plant, and is the home of the nitrogen fixating bacteria, is one of the most important elements in agriculture. Yet notwithstanding the vast and manifold investigations of science, the Encyclopedia Brittanica says concerning humus, "little is known with regard to its chemical constitution". Hence man's knowledge of the most efficient way to use humus is correspondingly limited, and an important field here lies open by which to obtain new power.

Improved methods of travel and communication bring each of the

world's inhabitants into ever closer touch with its artistic treasures, its literature, scenery, and history. Without proper methods for its mastery, the vast mass of culture wealth thus made accessible, may become a burden or even a danger. With adequately developed methods of assimilation, it becomes a near and instant wealth to every person.

Hence supreme need and supreme opportunity alike spur on mankind to a study of the science, art, and philosophy of achieving results. It is desirable to analyze the concept of efficiency until primal units of it are obtained which can be used in many ways each day by the individual person, can be taught in the schools, and developed into elaborate systems.

DEFINITIONS AND FIRST PRINCIPLES

Definition of Efficiency

An efficient process is one in which the available results exceed the expenditure. Thus if a farmer expends $1000 in raising a crop and receives $1500 for the crop, his work may be said to be efficient.

Efficiency is the property of being efficient: that is, of producing results in excess of expenditures.

The degree of efficiency of a process is the ratio of the available result of the process to the expenditure. Thus in the above illustration the degree of efficiency is $1500 ÷ $1000, or 3/2. Often "degree of efficiency" is shortened to "efficiency", as when we say that the efficiency of a process is 2, or twofold.

In case the degree of efficiency is less than one or unity, the given process is one of loss or waste. Thus if a farmer spends $1000 in raising a crop of wheat and receives only $800 for the crop, the efficiency of the process is the fraction 4/5, serving to indicate the degree of loss involved.

Principal Kinds of Efficiency

Relative efficiency is the degree of efficiency in a process in comparison with the efficiency in a like process taken as standard. Thus if land which should produce 32 bushels of wheat per acre is being made to yield only 24 bushels per acre, the efficiency of the process of agriculture involved is 75 per cent. Or if one farmer raises 20 bushels of wheat per acre while another farmer raises 24 bushels

per acre on the same kind of land the efficiency of the second farmer as compared with the first is 24 ÷ 20, or 6/5.

In comparison with relative efficiency the first definition of efficiency given is a definition of absolute or independent efficiency.

A case of *homogeneous efficiency* is one in which the material expended and the results obtained are of the same kind. Illustrations are the expenditure of money to obtain more money; or the planting of grain to procure more grain of the same kind; or the sacrifice of present happiness to obtain more happiness in the future.

A case of *heterogeneous efficiency* is one in which the output obtained is different in kind from the material expended. Particular instances are the burning of coal to produce light, or the taking of strychnine in small doses to improve one's health.

It is important to note that the degree of a case of heterogeneous efficiency cannot be expressed or measured directly. For the quantities compared in any ratio must be of the same kind. Thus bushels cannot be divided by dollars, nor light by coal, nor health by arsenic. Two ways, however, exist by one or both of which the degree of a case of heterogeneous efficiency may usually be expressed. The first of these methods consists in transforming the given case into one of homogeneous efficiency and then treating the result directly. Thus if money is expended to produce wheat which is consumed on the farm and thereby results in health and happiness the process is heterogeneous; but if the wheat crop, instead of being consumed on the farm, is converted into money, the process becomes homogeneous and its efficiency can be expressed as the ratio of the sum of money received to that expended. Similarly if the money received from the sale of light be compared with that expended in producing the light, the efficiency involved is determined numerically.

The second method of measuring heterogeneous efficiency is that of expressing the efficiency in the relative form; that is, comparing the output in a given case with that in another case where the expenditure is the same, the latter case being taken as a standard. Thus if the burning of a ton of coal will produce a given number of units of light in one case, but will produce twice as many units in another case, in the latter case the fuel efficiency is twice what it is in the former. Or, to look at the matter from a somewhat different point of view, if a given current of electricity on passing through tungsten filaments will produce 2½ times as much light as when passing through carbon filaments, the efficiency of the tungsten filament is said to be 2½ times that of the carbon. Similarly if a given area of land will support two persons who live by hunting and fishing, but when used agriculturally will support 300 persons, the efficiency of the agricultural use of the land as compared with its use for hunting is 150.

Qualitative Efficiency

In some cases the results of a process are so abstract or complex that it is not possible to reduce them to a numerical basis. Instances are the expenditure of money to procure health or to save life. This abstractness or complexity may characterize either the expenditure or the results or both of these. As illustrations we have the sacrifice of home associations in order to make money in a foreign land, or of health in order to obtain an education and power of social service. Hence we may say in general that *qualitative* (or mass) *efficiency* is efficiency whose existence is evident but which cannot be expressed in an exact numerical form on account of the abstractness or complexity of some of the elements involved.

Efficiency Relative to an Ideal

Certain important cases of relative efficiency call for special mention.

The first is *efficiency relative to an ideal or perfect standard.* Thus a perfect steam engine would be one which converts all the energy latent in the coal consumed by it, into mechanical power. No steam engine, however, can convert more than 18 per cent of its fuel into power. Hence the fuel efficiency of a steam engine cannot exceed 18 per cent. That of a small steam engine is frequently as low as 1 per cent. There are indications that the amount of sugar in a sugar beet cannot be made to exceed about 17.5 per cent. A further increase seems to cause the vegetative processes of the beet to degenerate and the whole organism to decay. If this view of the matter is correct, with respect to the ideal beet the degree of efficiency in a beet containing 10.5 per cent of sugar is $10.5 \div 17.5$, or 60 per cent. Similarly if a pupil solves 8 problems out of 10 on an examination paper, his efficiency is .80 with respect to the ideal which consists of solving all of the problems.

Bogey and Competitive Efficiency

The second form of relative efficiency is that with respect to some degree of efficiency taken as normal, or "bogey", under given conditions. Thus if the thoroughly trained and equipped but average workman is able to lay 2800 bricks per day, a workman who lays 2240 bricks has an efficiency of 80 per cent; while one who lays 3220 bricks has an efficiency of 115 per cent. This kind of efficiency may be termed *bogey efficiency.*

In the third place we have what may be termed *competitive efficiency*. By this is meant the degree of one's efficiency in comparison with that of one's immediate competitors. Thus if with a given outlay of time and money one farmer produces only 1000 quarts of milk while his neighbors, with the same outlay, produce 1500, the competitive efficiency of the first farmer is 66⅔ per cent. It is to be noted that the competitors involved in a given case may fall below, equal, or exceed bogey efficiency. The importance of the matter lies in the fact that, when a man's competitive efficiency is small, he may be undersold, driven from business, or ruined commercially. Efficiency of this kind which does not reach a certain standard, is therefore often called inefficiency or absolute failure. Competitive efficiency is recognized in such statements as "He has made good", or "This is worth while".

Economy

Cases of efficiency may be relative in other ways as with respect to the emphasis laid on expenditure or on output. For instance, in a given case, it may be desirable to strive for that form of efficiency which consists of diminishing the cost instead of that which is characterized by an increase in the output. This form of efficiency is usually termed economy. Or, in general,

Economy is a form of efficiency in which the output compared with that in a standard case is unchanged but the expenditure is diminished. Thus if a farmer who has been raising 1000 bushels of wheat at a cost of $470 is enabled in a later year to produce this same amount of wheat at a cost of $360, the process is primarily one of economy. Hence, in general, efficiency means primarily an increase in results; economy, a diminution in expenses. Economy is thus the inverse of efficiency. The proverb "A dollar saved is a dollar made" illustrates the fact that economy and efficiency are two aspects of the same thing.

It is frequently convenient to treat cases of relative efficiency as cases of relative economy; that is, to estimate degrees of efficiency by a comparison of costs instead of outputs. This is illustrated by comparing the cost of raising a bushel of wheat on one farm with that on another. For example if one farmer raises a bushel of wheat at a cost of 48 cents and his neighbor at a cost of 40 cents, the efficiency of the first as compared with the second is $\frac{40}{48}$, or $\frac{5}{6}$, or 83⅓ per cent. It is to be carefully noted that in forming this ratio, the two costs are compared in the inverse order from that used in determining the degree of efficiency by a comparison of two outputs.

Infinite Efficiency

Important cases of efficiency are those in which the degree of efficiency is immeasurably large (or infinite). As examples we have all those cases where what is valueless when treated in one way becomes useful when treated in some other way. For instance some forms of lignite when burned directly are valueless for power generating purposes, but become valuable when converted into gas and used in a gas engine. Other and still more important cases are those of the conversion of loss, waste, or defeat into results having a positive value. An instance is that of saving life in a case where death has hitherto resulted uniformly, as in certain kinds of surgical operations. Another instance of infinite efficiency is that of using two points to represent a straight line, or of the drawings called a plan and elevation to represent a building.

Terms related to Efficiency

Usefulness means efficiency regarded as a broad qualitative fact and with but slight emphasis on the quantitative elements involved.

Utility is the same as usefulness except that in utility results are usually more concrete, or more definitely realized.

Efficacy (or *effectiveness*) is a form of efficiency in which emphasis is laid on the complete attainment of a specified end, comparatively small attention being paid to the cost involved.

It is convenient at times to denote cases of waste or loss by the term negative efficiency.

Knowledge of Equivalence an Aid to Efficiency

Before concluding the discussion of cases of relative efficiency, a word should be said as to the relation of efficiency processes to the equivalence or invariant processes with which they are associated. In certain respects the results of a process must be equivalent to the data involved. Thus by the laws of the conservation of energy and of matter, the amount of force and of matter in the entire results of a process cannot differ from those in the data. Nevertheless in a given case the amount of those results, which are usable or available for a given purpose, may differ from the amount available in the data. Thus when a given amount of food is fed to one kind of cow, more milk is often obtained than when the same amount of food is fed to another variety of cow. In the present investigation we take this principle of profitable non-equivalence between

datum and result in certain processes as a fact or axiom and try to take the utmost advantage of it. In the course of our investigation we shall find that equivalence laws and facts are useful in obtaining efficiency results. An illustration is the fact that when coal is burned under the boiler of a steam engine only 15 per cent of the power latent in the coal is converted into mechanical energy, the rest being lost as friction in parts of the engine, or as heat which passes up the chimney, or radiates from the parts of the engine. If, however, the coal is first converted into gas and then used in a gas engine, the amount of useful energy obtained is often increased threefold. A knowledge of the equivalences concerned has been an important aid in obtaining this more efficient result.

COMPLEX CASES OF EFFICIENCY

Efficiency Factors Acting in Succession

An efficiency process may often be separated into a number of parts acting in succession, each part acting upon the results of a preceding part. The resultant efficiency of all the parts is then obtained by taking the product of the respective efficiencies of the different parts. By way of illustration we may consider a metal cutting machine which has been run for 5 hours per day at a speed of 10 revolutions per minute and which does its cutting by means of a fine diamond point. If the machine is afterwards run 8 hours per day with a speed of 45 revolutions per minute and a round nose tool with a cutting end 10 times as wide as the diamond point is used for the cutting, the efficiency elements of time, speed, and width of cut are increased 8/5, 9/2, and tenfold respectively. Hence the efficiency of the new process, as compared with that of the old, is $\frac{8}{5} \times \frac{9}{2} \times 10$, or 72. When elements of efficiency are combined in this way, they may be regarded as acting tandem and the resultant obtained called *tandem* or multiplicative *efficiency*.

Parallel Efficiency Factors

In other cases of resultant efficiency the elements concerned act *parallel* to each other, and the degree of *efficiency* in the process as a whole is equal to the average degree of efficiency of the components. Thus if a farmer is raising four crops at the same time, all of equal importance, and their respective efficiencies are .80, .90, .86, .92, his efficiency for the year is the average of these rates, or .87. In the same way the efficiency of a student is sometimes determined

by his average grade in all the subjects pursued by him. If however the elements involved in a given case are of unequal importance, to obtain the resultant efficiency it is necessary to multiply each elemental efficiency by a number which indicates its weight or importance, to take the sum of all the products and divide the sum obtained by the sum of all the weights. Thus if the respective efficiencies of four crops raised by a farmer are .60, .90, .80, .72, and the second and third crops are each twice, and the last three times as important as the first, his resultant efficiency is $\frac{.60+2\times.90+2\times.80+3\times.72}{1+2+2+3}$ or .77.

Tandem and Parallel Efficiencies Combined

In is evident that mixed cases of efficiency also occur, in the sense that some of the elements involved act in tandem or multiplicative fashion in whole or part, and others are parallel or additive in relation. For instance while the chief crops of a farmer during a given year may be developed in parallel fashion, the operations involved in raising one or more of the crops may act in multiplicative succession. Thus the manner of preparing the seed bed affects the development of the seed, care and efficiency in selecting the seed influence all later processes, and so on. Also if the crops are rotated from year to year the efficiency of any one crop influences the efficiency of the crops which follow in succeeding years. This may also result in certain multiplicative cross relations during any one year. Thus large hay and corn crops will supply feed for many cattle which in turn will supply a larger amount of fertilizer for several later crops.

Another Method of Analyzing Efficiency Processes

Emphasis should be laid in this connection on what is for our purpose a still more important method of analyzing an efficiency process into constituent elements. To illustrate this method we take the case of a particular crop raised by a farmer, as a crop of wheat. The agriculturist obtains efficiency in preparing the seed bed of the crop by several distinct means one of which, for instance, is the group principle in the form of the gang plow and also of the extra wide harrow. The group as a source of efficiency is also found in the drill used in sowing the seed, and in the reaper used in harvesting the grain. In some form, often in several forms, it is an aid in the efficient marketing of the crop. Other general primal efficiency principles are at work in the various concrete divisions of

the process which have been enumerated. Even the division of the process into such parts as fertilizing the ground, preparing the seed bed, planting the seed, etc., for separate study, is an application of one of these primary principles, viz.: that of the use of units, or of diversity. Owing to their fundamental and inclusive nature, a thorough grasp of these primal efficiency agencies will mean increased efficiency in the various departments of life. These primary efficiency principles will be taken up one by one in chapters which follow and each investigated as closely as possible. Afterward they will be considered to some extent in combination.

EXERCISE 1

Determine the degree of efficiency in each of the following cases:

1. A farmer expended $2000 in raising a crop of corn and received $2800 for the crop.
2. A farmer spends $1200 in raising a crop of wheat and receives but $1000 for the crop.
3. Land which should yield 80 bushels of corn per acre is producing but 56 bushels per acre.
4. By an expenditure of $800, 2,200 bushels of potatoes which sold for 80 cents a bushel were produced.
5. A machine capable of 400 revolutions per minute is making 250 revolutions per minute.
6. It has been estimated that ten square miles of land used for hunting and fishing will support 2 persons, while the same land when used agriculturally will support 1000 persons.
7. In a certain town the light given by a 16 candle power carbon bulb costs ¾ cents per hour, while that given by a 32 candle power tungsten bulb costs ½ cent per hour.
8. On one field an expenditure of $800 produced a yield of 2000 bushels of potatoes, and on another field $1200 produced 4200 bushels.
9. A field which, by following the best available methods, could have been planted in 8 days, was actually planted in 10 days.
10. It costs $4000 per annum to run a certain poultry establishment, and the annual receipts average $4800.
11. A man who should work 8 hours per day, on account of ill health works on the average 2 hours and 15 minutes per day.
12. A gold mine which cost $400,000 at first paid 8 per cent dividends, but later when operated by improved methods paid dividends of 20 per cent.
13. Between the years 1840 and 1900 the per cent of sugar in the sugar beet was increased from 5½ to 13.
14. When coke is burned directly, 45 per cent of its energy is utilized. If it is converted into water gas before it is consumed, 82 per cent of its energy is utilized.

15. At one time 61 hours of labor by one man were necessary to cultivate one acre of wheat. At the present time this number of hours has been reduced to 3.

16. When the Americans acquired the Philippine Islands, they found the natives transporting mahogany logs at a certain place from the interior to the seashore by such crude methods that it took 16 buffalos and 20 men to transport one log a distance of one mile per day. An American taught the Filipinos to suspend one end of a log from the axle of two large wheels, and thus made it possible for 2 buffalos and 2 men to transport a log 20 miles in one day. If a day's work by a buffalo is taken as worth 50 cents, and that by a man as worth the same, compute the efficiency of the second process.

17. State which of the preceding instances are cases of relative efficiency. Also which are cases of independent or absolute efficiency.

18. State which are cases of economy, and which of direct efficiency.

19. State which are cases of homogeneous and which of heterogeneous efficiency.

Give an example of each of the following kinds of efficiency and determine the degree of efficiency in each illustration given:

20. Economy
21. Direct Efficiency
22. Relative Efficiency
23. Homogeneous Efficiency
24. Heterogeneous Efficiency
25. Bogey Efficiency
26. Competitive Efficiency

27. Give an example of the transformation of a case of heterogeneous into homogeneous efficiency.

28. At one time 40 hours of labor produced 2 tons of hay. Later 40 hours of labor produced 20 tons of hay. Transform these statements so that they shall express an economy instead of a direct efficiency.

29. At one time 100 hours of labor produced 48 bushels of oats. Later by the use of improved methods 40 hours of labor were made to produce 180 bushels of oats. Transform these statements so that the expenditure of labor in the two cases is the same. Also so that the crop obtained is the same.

30. Give an illustration of efficiency and transform it into an economy.

31. Give an illustration of economy and transform it into an efficiency.

Find the resultant degree of efficiency

32. In a process in which the efficiencies of the four constituent (tandem) elements are 3, 2, 5, $\frac{1}{4}$ respectively.

33. Are .60, .80, .90, .75 respectively.

34. Of a process where the five constituent (parallel) elements are $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{2}$, 5-4, 5-2.

35. Where the three (parallel) elements are .60, .84, .72 respectively, and the second and third elements each count twice as much as the first.

36. A metal cutting machine has been running $3\frac{1}{2}$ hours per day, and making 18 revolutions per minute. If it should run 10 hours per day, make 80 revolutions per minute, and use a cutting point 6 times wider than that in use, how many fold would the efficiency of the machine be increased?

37. A farmer raises four crops with efficiencies of .60, .80, .70, .50. If the second and third crops are each twice, and the last three times as valuable as the first, find his average efficiency for the year.

38. Give a numerical illustration of resultant efficiency composed of tandem elements.

39. Also of one of parallel elements.

40. Also of one containing both tandem and parallel elements.

41. Ascertain the methods of judging milk cattle, horses, and a corn stalk with ears, and determine whether each of these is a case of tandem or of parallel efficiency.

42. By using an auxiliary instance in which the result equals the cost, show that each case of absolute or independent efficiency may be regarded as a case of relative efficiency.

43. Ascertain the original meaning of the word economy. Is this word ever used in its original or some similar sense?

44. Give an example of infinite negative efficiency.

45. Give an illustration of a process which is efficient when judged by one standard but is waste or loss when judged by another.

46. Give an illustration of reciprocal efficiency.

47. On a certain field 80 days labor by one man will produce a yield of 300 bushels of corn, while 90 days labor will produce a yield of 400 bushels. Which method of cultivation is the more efficient and how much?

48. Which is the more fundamental test of efficiency in raising a given crop in a given country, the yield per acre, or that per workman? Give reasons for your answer.

49. Explain the meaning of the proverb "Strength saved is strength made." Also of "Economy is wealth."

State the kind or kinds of efficiency involved in each of the following instances; also compute the degree of efficiency wherever possible:

50. The office expenses of a given business were $2750 annually. After the introduction of the card and loose leaf ledger system of keeping accounts, the annual office expenses were reduced to $1600.

51. In Watt's steam engine 1/25 of the energy in coal was utilized. In some modern steam engines 1/6 of this energy is utilized.

52. The employment in a productive way of capital which has been lying idle.

53. The efficiency of a given process should be 3/2 but is actually only 5/4.

54. By changing the seed used a farmer increased the yield of wheat per acre on a given farm from 20 bushels to 24 bushels.

55. A series of separate pictures on a moving picture film when thrown on the screen in succession gives the impression of a continuous whole.

56. By an expenditure of $20,000, gold worth $800 was obtained from sea water.

57. A given amount of grain and fodder when fed to beef cattle will produce 4000 lbs. of beef, but when fed to milch cows will produce milk which is equivalent in food value to 18,000 lbs. of beef.

58. Reciprocating steam engines on the average consume 4½ lbs. of coal per horse power per hour; turbine engines, 1¾ lbs.; gas engines, 1⅛ lbs.

59. In a certain city the expenditure of $1,500,000 for a water filtration plant reduced the annual death rate from 22 per thousand to 18 per thousand.

60. In fattening a beef steer it was found that the first 100 pounds of increased weight in the animal were produced by 730 pounds of grain used as food, and the last 100 pounds of weight required 1000 pounds of grain food.

61. The efficiency of a given business firm in buying is .80, in manufacturing is .95, and in selling is .90.

62. The use of an improved textbook in a given school caused 30 per cent more of the pupils to pass in the subject taught and saved the teacher much time and effort.

63. On a certain railroad when operated by steam 90 lbs. of coal on the average were burned to send a train of cars a distance of one mile. After the railroad was electrified 50 lbs. of coal did this work.

64. In a certain year 9,000,000 agricultural workers in the United States produced 600,000,000 bushels of wheat, while in Europe 60,000,000 farm laborers produced 1,200,000,000 bushels of wheat.

65. In a certain neighborhood one farmer by spending $2,000 produces crops worth $4,000. Another farmer by spending $2,000 produces $3,000. Compute the absolute efficiency for each farmer. Also compute the relative efficiency of the second farmer as compared with the first.

CHAPTER II

REUSE

Illustrations of Reuse

A cook who has succeeded in inventing a new recipe frequently saves herself much labor by preserving this recipe and using it repeatedly. Often also the cook does mankind a service by giving the recipe to other cooks to use. Similarly a man who has made a successful speech may at times save himself much work by writing out the speech and using it in whole or in part on other occasions.

The automobile is possible because the gas engine, the pneumatic tire, and ball bearings, all invented in some other connection, have been reused in combined form to produce the motor carriage. The commission form of city government originated in Galveston after the city had been devastated by a severe storm, and has since been adopted by hundreds of other cities.

If we go back far enough in human history we find a primitive manlike creature who had been in the habit of picking up from time to time any club that chanced in his way, and of using it as an aid in killing wild animals for food. To this progenitor of man it occurred at some time to keep a good club which he had found that he might use it again and thus save himself the labor of searching for other clubs. This act of his lifted his life to a new level of efficiency and indeed constituted a distinct epoch in human progress.

For in this act and in the other instances given above we have illustrations of a primary and fundamental principle of efficiency which we may term reuse or repeated use. This is the first of the specific efficiency principles which we shall study.

CLASSIFICATION

Cases of repeated use may be classified according to the character of the objects which are used over and over again.

Classification according to Materials Reused

Material inorganic objects may be reused. Instances are the repeated use of a spade, a gun, or a silver coin; of ammonia when

used over and over again in an ice-making machine; or of sea water, which by being reoxygenated is used repeatedly in a vivarium, the expense of obtaining a new supply of sea water each day being thereby avoided. The basis of civilization is the soil which has been cleared so that it can be reused in raising crops in successive years and which, in advanced methods of cultivation, will even produce six or seven crops in one season. Similarly all the material objects which enter into the formation of tools and machinery, including large objects like buildings, are reused. A noticeable case is the reuse of a mold or pattern in manufacturing metallic objects.

Living organic objects are frequently reused in whole or part. The amoeba improvises an arm or mouth out of its substance when opportunity or need arises for the use of such a member. But higher animals have organs and members which are grown once for all and then reused. This repeated use often includes not merely reuse for one given purpose but in many different ways. Thus the human hand is used repeatedly not merely for the purpose of grasping a specified object many times in succession, but as an instrument by which to strike, push, pull, write, swim, gesture, give, and to perform a variety of other acts.

The advantage in using domesticated animals instead of living by the chase is largely the gain which comes from reuse. Thus if a wild animal is killed and eaten it is used but once. Whereas if it is kept and milked, or used as a beast of burden, or in breeding other animals, it is used many times in succession. A similar advantage led to the change from torturing and killing men captured in war, to keeping them as slaves.

In fact a high degree of social organization means that the members of society are using and reusing each other in many ways daily.

Reuse of more Abstract Data

Mental data are also capable of reuse. Thus by the aid of memory and imagination, we reuse perceptions, feelings, and concepts. We are thereby saved the labor of acquiring these data anew, and, what is often of even greater importance, acquire a surplus mental energy which we can use for higher purposes. An adequate analysis would also show that we reuse mental data by means of the reasoning faculty and by unconscious habit. One great source of gain in doing work that is consecutive rather than broken and fragmentary work is that in the former the initial act of the will

which starts a piece of consecutive work does not need to be repeated but is, so to speak, used repeatedly and continuously till the work is ended. Also in the process termed apperception, new phenomena and experiences are stated in terms of the concepts or units of knowledge which are already in the mind; that is, the latter units are reused and the process of learning is thereby economized.

Similarly the perceptions, feelings, thoughts, and ideas of other persons may be reused by an individual. The volitions of the teacher are reused by a class; those of a general by an army; and those of any pace maker or leader by his followers.

Also larger, more objective and *complex mental products* may be and are reused with fruitages correspondingly large and high in the scale of values. Thus experience, education, theories, hypotheses, and systems of thought of all kinds may be reused by different individuals in a great variety of ways. For instance the alphabet has been used by many nations and generations besides its inventors; and for many purposes which its inventors did not intend, as in lettering diagrams in mathematics, or in expressing systems of classification in science and logic.

In certain aspects *work* or *energy* can be reused. The savage in using the same club repeatedly, reuses not only the object itself but also the work done in searching for the club and in shaping it. This principle applies in the reuse of all tools, machinery, and systems of thought; to the imitation of other persons and the copying of their work. In setting timepieces we reuse the work of the astronomer who has accurately determined the time of day. In riding on a railroad we reuse the work of the engineers and laborers who constructed the road. Civil law is based on the reuse of precedents.

The Fuegians observed by Darwin kept the time of day, not by a clock, but by having that old man in their midst in whom the natural sense of time measurement was most accurate, strike a bell at what he judged to be equal intervals. In this way the Fuegians reused the work of nature in endowing this man with peculiar faculties and his own work in developing these faculties. Similarly in reusing various inventions we reuse the toil, or genius, or both, which have made these discoveries and inventions possible. The data thus reused become very extensive and complicated when traced back in all their ramifications.

Limitations in Reuse of Objects

It should be noted in this connection that *not all objects can be reused,* nor can every object be reused in all ways. If a material object has been once used and afterward has become inaccessible in time or space, it cannot then be reused. Certain objects may be too large, or too small, or otherwise unfit for reuse of a given kind. Also an object which has been worn out, lost, or destroyed cannot be used again. A ton of coal cannot be burned a second time. A system may be beyond the comprehension of the person who seeks to reuse it. In other words, the repeated use of things is often prevented by subjective limitations in the user as well as by objective limitations in the things themselves. This matter will be discussed more fully in Chapter XV (p. 265).

Numerical Forms of Reuse

Instances of reuse may also be classified according to individual forms in which they occur.

Thus certain cases of reuse may be characterized by a numerical element prominent in them. For instance if a single object is reused, the case may be described as unitary. Similarly if two, three, or any number of data (denoted as n data) are reused in a given case, the process involved may be termed *dual, triple,* or *n-fold* reuse. Also, if a given object is reused one, two, or n times, the corresponding processes may be denominated by a similar set of numerical terms.

An important special case under this head may be termed dual *reciprocal reuse*. As an illustration we have the fact that a hunter reuses the power of smell in his dog, while the dog in effect reuses the superior powers of vision and reasoning in the hunter. Each blade of a pair of scissors reuses the other. Each of two friends or two partners in any enterprise repeatedly uses the other.

If this method of reuse be extended to any number of agents (that is, to n agents) it becomes what may be termed n-reciprocal reuse. Thus a set of salesmen may all reuse the special devices invented by any one of the set. So in any good organization, system, society, or corporation, all the parts and members reuse the good or virtue in any one part or member.

Spatial Forms of Reuse

Cases of reuse may also be characterized by some spatial or quantitative element prominent in them. For example different kinds of reuse may be local or comprehensive. A process or datum may be reused in a near field as when the spectroscope is applied to a new chemical substance in the laboratory; or in a remote field, as when the spectroscope is used in studying the stars. So a case of reuse may be characterized by the transfer of the reused material from a large domain to a small one, as when we use our knowledge of the solar system in explaining the constitution of a molecule; or vice versa.

Cases of reuse may also be described as *one, two,* or *three dimensional.* For instance, a travelling crane moves back and forth in a line (that is, along a beam), and is usable in its various positions. Hence this may be described as a case of linear reuse. Similarly we have cases of circular and spiral reuse, or reuse expanding from a point in a fanshaped, conical, or spherical way.

Marginal Reuse

Certain forms of the efficiency process under discussion may be termed partial, differential, or *marginal reuse.* Thus if an engineer should want to measure the distance from Washington to Albany, and he already knew the distance from Washington to New York, he could save himself labor by reusing his knowledge of the distance from Washington to New York, and measuring only the distance from New York to Albany. Similarly the astronomer diminishes the work of determining the position of many stars and other bodies on the celestial sphere by first measuring once for all with great accuracy the position of certain fundamental stars, and then measuring the distance of other objects from these primary stars. In doing a large amount of work in this way the astronomer obviously reuses many times the work done on the fundamental stars.

Similarly in the mastery and reuse of certain representative types of organisms in biology, or of symbolisms in art, or of minds in psychology, we are applying that principle of efficiency which we have termed marginal or variant reuse. We have an even more important case of it in the mastery and use of fundamental principles in ethics, religion, logic, and philosophy, and then applying with slight variations this knowledge in various special fields.

In forming the plural of a word by annexing the letter *s* (see

apples as formed from apple) instead of forming an entirely new word, we save labor by means of marginal reuse. The same economic principle is at work in the use of the various suffixes, prefixes, and other modifiers of words employed in language.

An application of this principle in another field is found in the use of the so-called universal machine; that is, a machine which by slight modifications of its fundamental framework or of its details, may be made to do the work of a variety of distinct machines. A simple instance of like nature is the use of one brace and a number of bits instead of several augers. A similar case is the use of one tug boat and a number of barges instead of several steamboats. In a steamboat the engine must be idle while the boat is being loaded or unloaded; but in a tug boat the engine may be kept in almost constant use in moving some barges while others are loading or unloading. Hence one engine in a tug boat takes the place of several engines in steamboats, or is relatively reused. If all the bits mentioned above as used with a brace are replaced by a single adjustable bit, the case becomes one of marginal reuse of the second order. An important special case of marginal reuse is what may be termed repair reuse. An instance of this is replacing a broken mainspring in a watch by a new mainspring instead of buying a new watch; or having a pair of shoes half-soled instead of buying a new pair.

Temporal Forms of Reuse

With respect to the category of time cases of reuse may be characterized in a number of ways, as for instance by the adjectives temporary or permanent. The most important special case in this connection is what may be termed *prophetic* or anticipative *reuse*. Thus if a man sees a small leak in a dam and stops it at once by wheeling a small barrow of dirt to it, his work is reused in the sense that it is a substitute for the much greater labor of rebuilding the dam after it has been swept away. Similarly we have the proverb that a stitch in time saves nine. What is called preventive medicine is an instance of the same sort of efficiency. In fact a large part of the virtue in farsightedness, forehandedness, and all wise planning is of the same kind. So also care in laying a good foundation, or in making a good start, partakes of the nature of prophetic reuse.

Other Special Forms of Reuse

Space and time together give rise to the idea of *motion* and order, and to corresponding classes of reuse. Thus in the case of a travelling library, the motion of the books is the means of their repeated use, and the reuse involved may be described as motional.

Reuse obtained by means of the idea of *order* is illustrated by the repeated use of a road obtained by travelling over the road in opposite directions, or by the temporary conversion of a dynamo into an electric motor by simply reversing its action.

When a given object or idea is propagated, spreads, or grows, the element of force enters into the case in a special way. Cases of this kind may perhaps be denominated as growth reuse.

With respect to the category of quality, instances of reuse may be characterized, for example, as *homogeneous* or *heterogeneous*. Thus a certain method of bracing the framework of bridges having been invented, so long as it was reused in this kind of work only, it constituted a case of homogeneous reuse. When it was later used in making the framework of freight cars, it became a case of heterogeneous reuse. Almost all schoolhouses were originally built as places to be used exclusively for the training of children. Recently many such buildings have come to be used by adults also for various social and cultural purposes, this latter application being a case of heterogeneous reuse. Mr. Westinghouse having invented a system of economical transmission of gas by high pressure mains, reused this principle in devising an economical method of transmitting electricity by means of high voltages on the principal wires. This also is evidently a case of heterogeneous reuse.

Heterogeneous reuse may also be regarded as a kind of qualitative marginal reuse.

Negative Reuse

In this connection should be mentioned what may be termed negative reuse. An instance of this is learning from the results of a mistake not to repeat the error. Another form of negative reuse is that in which, having learned an exclusive property of an object or group of objects, we infer the opposite or negative of this property for other objects. This form of reuse is useful in some varieties of proof, in detective work, and in certain kinds of scientific investigation.

Scales of Reuse

It is also important to observe that species of reuse may be arranged according to a certain scale of value. Thus the reuse of one's powers of invention and discovery is a higher kind of efficiency than copying or imitating the work of another person directly. When fire was first discovered, it was an economy for man to reuse it by keeping it from going out and by transferring it from place to place. But later, noticeably after the invention of the lucifer match, man substituted for the reuse of fire once made, the higher reuse of work done by the inventor of the match and by other scientific workers. A still more general case of highly important repeated use is reuse of reuse. Thus when a farmer after reusing the methods of a government expert in raising corn and finding them profitable, later reuses the methods of other experts in raising cotton, wheat, clover, etc., he practises reuse of reuse, or reuse of the second order. In general, from our point of view, repeated use of the primal agencies of efficiency now under investigation is reuse of the highest order.

Obscure Cases

Certain cases of more or less obscure reuse should be mentioned. These are illustrated by the action of enzymes and catalysts in chemistry. For example the presence of a plate of platinum greatly accelerates the combination of oxygen and hydrogen to form water. This action takes place without any noticeable change in the platinum plate. Hence the plate can be reused indefinitely for the purpose mentioned. The fact is plain but the nature of the process involved is obscure. Other cases of like nature are regarded as less obscure only because they are more familiar. In this connection may be mentioned the repetition in nature of like forms such as atoms, molecules, cells, crystals, and organisms, and of the qualities of parents in their offspring, often after a number of generations in which the given qualities have not been present.

Another case of partially obscure reuse is that which is often called influence. Examples of this are the effect of a large and efficient army in preserving peace, of a reserve of capital in preventing a panic, or many of the changes wrought in a community by the mere presence in it of a strong personality.

Complex and other Forms of Reuse

Other species of reuse might be characterized as conscious or unconscious, swift or slow, feeble, automatic, or irresistible, but it does not seem necessary to examine these and like cases in detail. In concluding the topic of classification of cases of reuse, special mention should be made of compound and complex forms. An illustration is the case of a soldier discharging his gun. In doing so he reuses the metal in the gun, the wisdom of his general, the work of scouts and officers, the work of drill, and the labor of men who invented the use of metals and gunpowder, and much more implied by these items and also outside of them. An even more complex case is the reuse of western civilization, for instance, by the Japanese. Looked at from another point of view, a single act of reuse may involve not only the repeated use of many different objects and kinds of work, but also a variety of forms of reuse, as those which have been named plural, prophetic, fanshaped, heterogeneous, and unconscious.

ADVANTAGES IN REUSE

Economics due to Reuse

We shall next consider the advantages in reuse as a source of efficiency. Some of these benefits are so obvious as to need no extended discussion. Among these are economy of material, and hence of money and other resources. Thus if a manufacturer instead of building an additional factory uses his old factory by night as well as by day, he saves the cost of the new building, of ground on which to place it, of taxes, repairs, watchmen, and insurance.

In like manner repeated use means a saving of time and strength, and thus, in effect, gives new strength, and lengthens and enlarges life.

Added Power

Other advantages, not always so obvious as the above, often appear in connection with reuse. Thus economy of resources sets free certain powers, often higher powers, in the reuser, to be employed in more efficient ways than reuse itself. These methods will be indicated more definitely in later chapters.

Infinite Value in Reuse

Special mention should be made of the absolute, or relatively infinite gain which sometimes results from reuse, as when it enables a person to overcome an otherwise insuperable difficulty. Thus if a man wishes to carry to the top of a hill a rock which is too large for him to manage as a whole, he may often accomplish his purpose by breaking the rock to pieces and carrying it up the hill piecemeal by reuse of his powers. Similarly, the lever, pulley, inclined plane, and like mechanical devices often enable us to achieve otherwise impossible results. A realization of the general value of this principle of efficiency is expressed in the proverb "Divide and conquer".

Other Values

In some cases reuse gives beneficial results which are obvious but which are difficult fully to analyze, either on account of their complexity or for other reasons. Thus reuse of a scaffolding in a sense produces the building. The inertia or persistence reuse of each of a set of pictures on the retina or in the brain gives the result called the moving picture.

As a more abstract illustration of the species of efficiency function under consideration we have the method of proof called mathematical induction. In this form of proof, a unit step of reasoning is first demonstrated, and then reused an unlimited number of times, and a final result thus obtained, often apparently not demonstrable in any other way.

Reuse Greater in Higher Organisms

It may be well also to look at the advantages of reuse in a collective, historical, and, in a sense, pictorial way.

If we examine the forms of animal life, as we proceed from lower to higher forms, we find a progressive increase in the amount of reuse and in the elevation of the character of reuse. In fact the extent and quality of repeated use practised by an organism may be said to form a measure or test of the place of the organism in the scale of life.

Thus if we first consider subjective reuse, that is, the reuse of parts or members of itself by an object, we find that a lower organism such as the amoeba, while it of course reuses its own substance, often does not reuse forms or structures of this substance.

On the contrary, as has been already pointed out, it improvises a new arm or mouth whenever the need for such arises. On the other hand, it is the rule for a higher organism to reuse its limbs and organs. In man this reuse of the physique or body is surrounded by and included in a still higher class of reuse, viz.: that of certain mental powers.

If we next consider objective reuse, that is, the reuse by an organism of objects outside of itself, we find that the same law holds; that is, the higher the organism in the scale of life, the more extended in both time and space is the reuse practised by it, and the higher is the order of this reuse. Thus the spider reuses its web for but a few days. Man reuses his houses and railroads for many years; and he reuses systems and theories which are entirely above and beyond the world of the spider and of lower organisms in general.

High Degree of Reuse in Great Men

We find also that the greatest men and the greatest nations have been the greatest reusers. Newton said he had been able to accomplish what he did because he stood on the shoulders of giants. Shakespeare freely reused the forms of ancient art, plots and stories, preceding philosophies as summed up by Montaigne, Christianity, and all the current thought and discoveries of his own age. In fact it has been laid down as a law that "men of the largest original power make the most use of the results obtained by other men".

What we regard as the sudden and miraculous manifestations of genius are often, and perhaps always in some respect, the unobserved accumulations of acts of reuse. Thus the marvellous products of Greek sculpture were in large part due to many generations of Greek workmen and artists, each generation using what the preceding ones had learned, and each adding its own contribution to artistic tradition.

Departments of Knowledge Based on Reuse

If we look at the matter from another point of view, each department of human activity or thought is found to have at its foundation certain elements or units which are reused in certain characteristic and systematic ways. The fruitage of the entire body of reuse in each case is so great as to make it an advantage to mark

off each aggregate and its possible future developments, as a separate domain of knowledge or life.

For instance, the existence of trigonometry as a distinct branch of mathematics is due to the fact that certain men once calculated certain numerical tables, and that other men in using these tables have reused and got for nothing the work done by these computers. Also the tables having been computed, men using them found it an advantage to give characteristic names to the functions represented in the tables, later to denote these functions by specific symbols, and by the aid of these symbols to build up an extensive system of relations among the concepts thus treated. In a word, a simple system of reuse developed into an extensive yet definite domain of economic and efficient relations.

In this connection it should be noted that every great invention or idea, almost without exception, at first occurred to its originator in some special or individual form, and was afterward developed by various kinds of reuse into a more general form.

Functional Foremen

At present in many lines of business the realization of the value of reuse is increasing. Thus in F. W. Taylor's system of conducting business, called scientific management, it is proposed to observe or work out the best method of performing every item of work, and to have so-called functional foremen appointed so that by them each workman can be thoroughly instructed in the methods thus worked out.

Loss due to Absence of Reuse

Another method by which to grasp the importance of the principle of reuse in human life is to try to realize what existence would become if this method of obtaining results were eliminated. Thus, in the individual life, actions like walking, breathing, and the use of memory would cease. In the relations of individuals, a large part of what goes by the name of co-operation, as well as all use of the work done by one's predecessors, expanding as it does into a remote past, would be cancelled.

As we study other primal efficiency principles in succeeding chapters, we shall find that reuse has additional important uses in connection with these principles.

Definition of Reuse

It may be well at this point to define the term reuse as clearly as possible.

To *use* an object means to combine the given object with other objects so as to produce results of value (a surplusage or positive non-equivalence: see Chapter IX, p. 152).

To *reuse* an object means to repeat the process of use in whole or part.

METHODS OF APPLICATION

We shall now consider the most efficient methods of applying the principle of reuse.

Vast Number of Species of Reuse

In order to reap the largest possible benefit from the employment of this principle of efficiency it is important to consider it first with reference to the category of number; that is, to realize

(1) The vast number of objects and entities which may be reused;
(2) The vast variety of forms which each species of reuse may assume;
(3) The great number of elements which may enter into a seemingly simple act of reuse.

A mastery of the category of space also aids in the extended and improved use of the principle of efficiency under consideration. For instance such mastery aids in learning to reuse data in widely separated domains; in realizing the various spatial ways in which reuse may spread; and in determining which of these forms will be most effective in a given case.

Possible Extent of Reuse

It is also important in this connection to realize the extent to which reuse may be carried. Thus, in the department of scientific work alone, it is estimated that 10,000 men of substantial ability are at work making original researches, and that each volume of a scientific journal contains the results of 100,000 brain-power hours of work. All of these results are available for each and every worker as material of reuse.

Reuse increased by Auxiliary Objects

Of high importance also is the study of the use of auxiliary objects in connection with reuse (see Chapter VII, p. 111). Thus paper, ink, and language, as used in making permanent records of otherwise transitory material, are important aids in the reuse of ideas and experience. Buildings in which books can be stored are a farther aid to the extended reuse of the facts and ideas recorded in books. If the books are occasionally sent out in travelling libraries, railroads and the forces which operate railroads may also be regarded as agencies auxiliary to repeated use. Paint on a house preserves the house and thus leads to the continued use of the house long after it would otherwise have perished. A servant by means of a letter is enabled to reuse or get advantage from work done in a previous situation. Photographs are aids to reuse in many evident ways.

A more abstract form of auxiliary entity is an extended view of things as an aid in selecting the best material or devising the most efficient units and methods of reuse in a given case. A wide outlook gives a greater variety of material to select from, and thus opens the way to the utilization of more efficient data and methods of reuse.

By the use of auxiliaries certain limitations in reuse may often be overcome. Thus a ton of coal cannot be burned a second time, but the heat generated in burning the coal may be used to make a machine which may be reused many times, or to sustain a thinking person whose thought product may be reused and expanded indefinitely.

Reuse of the Best Materials

This leads us to the question as to what are the most efficient kinds of reuse. Many elements enter into a solution of this problem. We can here mention only a few of the more important. Among the best data for reuse purposes are those which are most accurate, have been most thoroughly tested, or are most finished or perfected in any way. In other words, whatever we do or make it pays to do in such a way that the results obtained shall be reusable. Hence the proverb that whatever is worth doing at all is worth doing well, here receives a new illumination. Thus the best plays, like Shakespeare's, are reused most extensively, especially if we consider the higher quality of the hearers of such plays and the large amount of indirect

reuse that follows from this fact. The best books live and continue to be reused; so of the best scientific work, political work, and work of any kind.

If a work of art be of a high order, it will be reused not only frequently but in a great variety of ways as by copies, photographs, and variations. The same law holds for any unusually meritorious product. Such work will also be reused with great speed and ease. The Arabic system of Arithmetic was introduced into Europe about the year 1200 A.D. It was so superior to the Roman system that, notwithstanding the primitive state of culture of that time, it spread rapidly and practically without opposition from land to land. So smooth and rapid was the transition that it is almost impossible to trace its stages.

To give a negative illustration of the principle under discussion, the Chinese alphabet is so crude and the characters composing it capable of so little reuse and therefore so numerous that it is estimated that a Chinese compositor walks 20 miles a day in setting up type, while a compositor in the English language may remain in a practically stationary position.

Highest Forms of Reuse

The most efficient cases of reuse imply not only material but also forms of a high order. Thus imitation is usually a low form of repeated use and often detrimental in many of its results, while prophetic reuse is almost invariably highly valuable. Reuse of reuse is also one of the highest forms of this source of efficiency. For example, if a particular instance of reuse of any kind has been observed, it is frequently a highly efficient process to reuse this reuse in other domains. Thus after it had been observed that germs of certain diseases travelled by the reuse of the motion of one insect, as the mosquito, it might at once have been inferred that other germs travel by other insects as ticks, fleas, and flies, and investigations made accordingly.

Often it is profitable to start with a low order of reuse and develop it step by step into a higher order. Thus in teaching geography it is usually best to have the pupil begin with the study of his local surroundings and to reuse these till he forms general conceptions of the earth as a whole which he can then reuse deductively. So in general it is often well to begin with psychological or inductive reuse and develop it into the more rapid and powerful logical and deductive reuse.

Limitations in Reuse

Of highest importance in the practical application of reuse is the study of the limitations connected with it. Every source of efficiency is hedged in by characteristic restrictions. Thus in the case in hand the materials capable of reuse are limited in certain ways some of which have been indicated on a previous page. Similarly the forms of reuse are limited. More specifically, a given material is capable of only certain special forms of reuse. Also each form or kind of reuse brings its own limitations. Repeated use is also hedged in by certain deficiencies in the persons or agencies which attempt to carry out the process involved.

Adaptation in the Application of Reuse

Hence in many cases much study is necessary in order to determine that particular species of reuse which it is most advantageous to employ in a given case, or whether indeed it may not be better to employ some other prime agent of efficiency. Thus imitation is one of the simplest and easiest forms of reuse, but it often tends to an atrophy of reuse processes of a higher kind, as of powers of invention and discovery. Just as the tape worm, living directly on another organism, has neither mouth nor digestive tract so the mere imitator loses the powers of invention and initiative.

Herbert Spencer reused the principles of biology in the field of psychology with good results, but when he applied the same principles in sociology, the results were misleading rather than illuminating.

Similarly different kinds of reuse wax in value or lapse into danger with changing circumstances. Marginal reuse is particularly variable in this respect. In a multitude of cases where its application is possible, it is necessary to determine whether it will pay best to repair an object or to procure a new one; that is, whether to mend or end the object.

One of the best ways in which to avoid the limitations and evils of reuse of a low order is to use such forms of reuse in combination with reuse processes of higher order, or indeed with other of the primal sources of efficiency. For example in undertaking any specific investigation it is of the utmost importance to learn at the outset what other workers in the same line have accomplished. These results can be reused with the utmost freedom and without

any ill effect (rather with multiplied benefit) if at the same time we reuse our higher powers of invention and adaptation.

If we surround direct concrete reuse with other reuse of higher orders, the more of the former the better. As a result of this free combination of different orders of the principle in hand, many of the processes involved will in time become automatic and instinctive (and at the same time in a sense hyper-rational).

Ideal Species of Reuse

In conclusion it may be well to state that the final and inclusive method of applying this principle is to reuse the most efficient materials in the most efficient way for the most efficient ends. In order to reduce this ideal to definite form under any given circumstances a knowledge of other primal principles as given in the chapters which follow will be useful and, indeed, necessary.

Summary of Chapter

The substance of the preceding chapter may be summed up as follows: The most primitive and, in a sense, fundamental source of efficiency is the act of reuse or repetition. Reuse takes many special forms, each of which has a wide variety of applications, both in the daily detailed life of the individual, and in elaborate systems. Among the most important individual forms of reuse are those which have been termed marginal, prophetic, and reciprocal.

Reuse means not only economy of material and effort, but also new and aggressive power. Often this new power is infinite or unlimited in nature, in that it produces an advantageous result not otherwise obtainable. The value of reuse is also shown by the fact that the higher the organism, or the greater the man, the greater is the power of reuse. So human progress in general is marked by an increasing amount and by higher forms of reuse.

In order to obtain the utmost results in the application of reuse, it is important to realize the vast number of possible species of reuse, to select or devise the best adapted form in each application, and to perfect every product of one's work so that it shall be capable of the utmost reuse. The highest form of reuse is reuse of the fundamental agencies of efficiency.

EXERCISE 2

Give an example* of reuse of

1. An inorganic object 2. An idea 3. An act of volition.
4. Name an object which cannot be reused in a specified way.
5. Give an example of dual reuse.
6. Of reuse indefinite with respect to the number of repetitions.

Give an illustration of each of the following species of reuse:

7. One dimensional 10. Reciprocal 13. Negative
8. Two dimensional 11. Prophetic 14. Obtained by motion
9. Marginal 12. Growth
15. Give two related cases of reuse one of which is of low and the other of higher order.

Name the form of reuse in each of the following:

16. Killing weeds before they come up.
17. Use of a portable garage.
18. Of a bedroom bureau convertible into a writing desk.
19. Of the word apples as the plural of apple.
20. Of the results obtained at a government agriculture experiment station.
21. Symbiosis (or commensalism) in biology.
22. The law observed in old China that "if any official thinks that a custom generally observed in his province might with advantage be used in the while country" he is to send an account of it to the central government so that it can be utilized by other provinces.
23. A given Morris chair is adjustable at four different angles. Hence it will take the place of how many different chairs so far as the resting of a single person is concerned?

What is its degree of efficiency in this respect?

What form of reuse is involved in this instance?

24. A given pair of gloves when mended at a cost of 10 cents will last half as long as a new pair which will cost $1.10. Determine the degree of efficiency in mending and using the old pair. What form of reuse is here involved?

Give an illustration where reuse results in

25. Economy of material.
26. A saving of time and strength.
27. Attainment of a result otherwise impossible.
28. How is an envelope with a transparent front a means of reuse to a business man?
29. State some of the advantages in a son's learning and following his father's business. Some of the disadvantages.
30. Specify the different items and species of reuse involved in the use of the graphophone.

---

* By "example", as thus called for, is meant one that is different from those already cited in the text.

31. State the advantages in the use of a universal machine. The disadvantages.

32. Name the different kinds of reuse possible in a large corporation. How do these differ from the forms of reuse possible in a private firm?

33. Give an example where reuse otherwise impossible is obtained by means of an auxiliary object.

34. Give an instance where reuse produces loss of some kind.

35. A universal machine which occupies 40 square feet of floor space takes the place of five other machines which would occupy 25, 30, 20, 18, and 32 square feet respectively. Compute the degree of space efficiency in using the universal machine.

36. How many items of reuse occur in one application of the principle "things equal to the same thing are equal to each other"?

37. Show in what way logical deduction from general facts is marginal reuse. Illustrate.

38. Define imitation as a special form of reuse.

39. The solar calendar originated in Egypt. Trace the reuse of this calendar.

40. Discuss the meaning and functions of analogy as a kind of reuse.

CHAPTER III

THE UNIT AND ITS MULTIPLIER

Illustrations of Manifold Use

Efficiency is sometimes obtained by using an object many times simultaneously instead of many times in succession. This process differs from reuse in being manifold in space instead of manifold in time. Thus if a book is taken from a library by several persons in succession and read by each of them, the book is said to be reused. But if the book is taken out but once and read aloud to a company of people, we have a case of simultaneous manifold use, or, more briefly, of manifold use. Other illustrations of simultaneous manifold use are the publication of an article in several newspapers at the same time by a press syndicate; the cutting of several coats or other garments simultaneously by the use of a machine; duplex, or sextuple telegraphy whereby a number of messages are sent over a wire at the same time; the substitution of "James and John swam" for "James swam and John swam", the word swam in the former expression being put to a double use.

Illustrations of Multuse

By combining the principle of successive use in time with that of manifold use in space we arrive at a principle more general than either of these efficiency processes, and including each of them as a special case. Thus if a book is taken from a library repeatedly and each time is read to a different audience, the use of the book is manifold in both time and space, and we shall have what may be termed multuse. Other illustrations of multuse are a speech repeated to several successive audiences; a painting exhibited several times and on each occasion viewed by a different and often changing group of people; a school building used not merely for the instruction of children but for various social and cultural purposes on successive occasions by the community as a whole; the imitation of a model farm or of a piece of model road by different sets of men; the various uses of the alphabet after it had once been invented. Evidently the list of such illustrations may be enlarged without limit.

Manifold use and multuse are so similar to reuse that they do not seem to need any special treatment apart from that given incidentally in the remainder of this chapter.

Illustration of a Unit and its Multiplier

A still more comprehensive principle of efficiency than any that has yet been mentioned is that of the unit and its multiplier. This source of efficiency includes reuse, manifold use, and multuse, as special cases, and also has characteristic efficiencies of its own. For instance if the book mentioned above as taken from the library, contains a certain important idea this idea may be regarded as a unit. Then the number of uses of this idea by the reader of the book, or which follow directly or indirectly from such reading, may be regarded as the multiplier of the specified unit. This mode of viewing the matter has the advantage of enabling us to drop for the time being all the processes of transmission between the datum and the result and freely to use the datum and its result in ways to be pointed out later.

Other Advantages in the Unit and Multiplier

The unit used with a multiplier as a source of efficiency also has certain other characteristic advantages. Thus both reuse and multuse include or imply the element of personality in two ways. (1) They imply a user or actor. (2) They imply an aim or end, and a person enjoying this aim or end when attained. In the idea of the unit and multiplier we set aside, in a measure, this idea of personality, and thus free ourselves from the caprices, limitations, and various entanglements inherent in it. We are thus enabled, for instance, to put in the forefront objective and quantitative elements, and to shape and adjust these to meet given aims and needs. Also we are set free to expand the application of a given case without limit in many respects. In a word in the unit and multiplier we have the category of multuse in a more general, objective, and manifold form.

Looked at from another standpoint, in reuse the normal method is to take the data material of reuse as we find them and to let the methods and results of the process depend upon or follow from the natural qualities of the material. But in the use of the unit and multiplier we have much greater freedom. Because of the somewhat abstract nature of both the unit and multiplier we may often

create more or less artificial units which will meet given ends or needs in a more effective way than will natural data. Thus in a primitive stage of human development, the length of the human foot, or some other part of the human body as the arm or a finger, was utilized in measurement reuse. In later and better systems units such as the meter have been devised. These units are more uniform than the former anatomical ones, and are better adapted in many other ways to meet fundamental and far-reaching conditions of life. Similarly the atom has been devised to give a certain kind of control of the material universe, and electrical units to perfect the use of electricity. In a word, in the processes of reuse, the unit creates the multiplier and result. On the other hand, in the unit and multiplier, the multiplier and result may create the unit.

Other advantages in the use of the unit and multiplier will be evident after we have catalogued and illustrated the different principal forms which these instruments of efficiency may assume.

CLASSIFICATION OF UNITS

Units Classified According to their Materials

Units, like the data of reuse, may be classified either according to the character of the materials of which they are composed, or according to the forms which they assume.

With reference to the materials composing them, units are classified in much the same way as are the data of reuse (see pp. 15–18). Thus we may have an *inorganic unit* such as an atom, molecule, or crystal, or any object composed of these. Similarly we may have as a unit of work the horse-power, or the energy required to lift 33,000 pounds one foot in one minute. So also we may have *biologic units* of various kinds, as cells, tissues, organs, or entire organisms such as different plants or animals. Similarly we have *psychic units,* examples being perceptions, feelings, ideas, or acts. Social and *political units* are also in use, as cities, counties, and states. So each department of thought and action contains its own characteristic units. Various examples of these departmental units will be given in the course of the discussions which follow.

It may be well to state at this place that, owing to the more abstract nature of units, many of the limitations mentioned on p. 18 with respect to materials proposed for reuse do not apply to the same extent to objects treated as units in a multiplicative process.

Units Classified According to Form

We next consider the classification of units according to their forms or certain other abstract characteristics.

Considered with reference to the categories of space and quantity, units may be grouped according to their *size, shape,* or *position.*

With respect to number, units may be *single, dual, triadic,* or *n-fold,* according to the number of elements composing them. Thus the ton-mile and acre-foot are each dual in nature. The unit of work mentioned above (the horse-power) is triadic. The numerical characteristic of units is frequently expressed as the number of dimensions in the unit.

As to *time* units may be temporary or lasting.

As to *force* they may be active or passive.

Natural and Artificial Units

Since more or less artificial units are the source of special superior results which are characteristic of the unit and multiplier principle, the distinction between natural and artificial units is of the first importance for our purpose.

A *natural unit* is any individual object as we find it in the world about us, as a man, tree, grain of wheat, or a day.

A *semi-artificial unit* is a natural object which has been partially modified so as to facilitate its use as a unit. An example is the mean or average of a number of objects, as the mean solar day, or the fractional part of a physically undivided object, as ¼ of the area of the United States. Another instance is a room in a house taken as a measure of the size of the house, as when we speak of a six-room, or an eight-room house. A like case is the description of a cotton farm in the South as a one-mule or a two-mule farm.

The value of slightly artificial units is illustrated by the operation of that form of universal suffrage where one man has one vote, regardless of differences among men, and questions are decided by the principle of majorities. Among the advantages which arise from this somewhat artificial idea of uniformity among voters are the ease and speed with which decisions are thus obtained as compared with decisions left to popular plaudits, riots, or revolutions, and often to a series of violent actions and reactions. It is to be noted that in this use of the average voter, the inequalities among men are not wholly ignored but are left to adjust themselves by

indirect methods as by the various influences which more able men exert upon those less able.

An *artificial unit* is a unit devised by man. Instances of such units are an hour, dollar, or pound; a lesson, dose, or experiment, a letter, word, sentence, paragraph, page. Other more extreme illustrations are the various legal fictions, such as the one which regards a corporation as a person, or the hypothetic units employed in science and in philosophy.

A set of units often forms a *series* increasing regularly in size or in importance in some other way. The inch, foot, yard, and mile form an example of such a scale of units. Similar instances are furnished by the different tables of compound numbers given in Arithmetic. Each developed field of work is usually supplied with such a set of units in some shape. Such scales of units will be treated more fully in Chapter VI (p. 95).

Many other special forms of units, each of more or less value, will appear as we proceed.

It is also to be observed that units of the various kinds mentioned above may be combined to form a vast variety of complex units. An instance of a complex unit is a library, or a civilization.

CLASSIFICATION OF MULTIPLIERS

A multiplier is primarily a number but this numerical essence in many cases is embodied in certain materials or entities, which, while disguising the nature of the multipliers in some ways, in others furnish the immediate instrumentalism by which the multiplier is handled or applied (see pp. 49-50 for analysis and definition of multiplier).

Classification of Multipliers According to Materials

It is convenient to classify many multipliers according to the materials in which they are thus more or less embodied.

Many kinds of what may be regarded as material or *concrete multipliers* are in use. For instance, from one point of view, the lever is a multiplier of man's available energy. In like manner every mechanical appliance which is a modification of the lever, as the wheel, pulley, inclined plane, screw, and every machine composed of these, is a multiplier belonging to the same general class.

Many other instruments used by man are multiplicative agencies. Thus the lens manifolds man's power to see both small objects and

distant objects. In the same class of multipliers may be mentioned the sounding board of a piano, the needle and mirror of a galvanometer, the relay circuit of a telegraph instrument, and the vibrating membrane of a telephone receiver. In fact, from one point of view, all the machinery which man uses, taken as a whole, may be regarded as a gigantic multiplier of his power.

An instance of a different kind of material multiplier is the use of capital to enlarge the application of the business power in a man.

Similarly various forces in nature, when properly applied, become in effect multipliers of man's power to achieve results. The hole drilled in a rock (the unit) is vastly increased by the explosion of a charge of dynamite (the multiplier). In a much larger sense, coal and water-power are manifolders of man's power. Acting in conjunction with mechanical multipliers these forces produce noteworthy results. Thus it is estimated that the steam and gas engines in use in the world do the physical work of 2,000,000,000 of men and thus increase the manual power of the human race at least sevenfold.

It is to be noted in this connection that *motion* from certain points of view may be regarded as a multiplier. Thus it is possible for a man of small resources to manifold them by keeping them in rapid action. Also we have the case of a point generating a line by motion, and similar geometric illustrations. The physical and chemical properties of matter under certain circumstances are important multipliers of man's powers. Thus a granite monument, or the chemical and physical properties of granite which make such a monument lasting, multiply an inscription carved on the monument by means of the duration imparted to the inscription. So ink and paper impart a certain permanence to a fact or idea recorded by their use and give a manifold extension to such records.

The *biological* field is full of illustrations of the principle in hand. The oölites and certain bones in the ear are magnifiers of the vibrations in the membrane of the ear made by sound waves. Other nerves of sensation have multipliers of different kinds, as hairs, papillae, etc. Many of the bones of the body act as levers in other ways. Reproduction itself is a process of multiplication.

More Abstract Multipliers

In the realm of *mind,* psychic units have different multipliers as memory, imagination, reason, fear, surprise, joy, interest, faith, and

courage. In connection with an act of wrong doing, deliberate intention accompanying an act gives it a peculiar gravity. So in ethics, the spirit in which a gift or sacrifice is made is a multiplier of its value.

Special mention should be made of certain imperfectly understood subjective multipliers, such as that psychic action which enlarges perceptions of discrete phenomena into a continuity; or that complex of different multiplicative powers which expands a hint or suggestion into a complete whole; or causes that which is partly hidden to be most fully expressed.

Many *social agencies* are multipliers of a high order of power. Among these are language, the printing press, railroads, the post-office, publicity, political systems, and civilization as a whole.

In a certain sense the *quality* of a unit is its multiplier. Thus the power of an act is multiplied if this act be applied to the apexal (or strategic) point in a domain of material. Thus the dog jumps for the throat of its victim; and the bee when attacking a person tries to sting the eye. Hence comes the importance of acting at a psychological moment, or of seizing a strategic point, or of controlling routes of transportation in business and in war. Hence also the value of being able to influence processes of education, and of being able to control the fountain sources or underlying causes of any movement. Reform a man, it has been said, and you reform one man; reform a boy and you reform a whole multiplication table of men.

Multipliers Classified by Forms

Multipliers may also be characterized and classified according to certain formal elements prominent in them. Since material often in a measure determines form, in many of the species to be mentioned in this relation form and material are closely connected.

A multiplier, like a unit, may be characterized *numerically*. For instance, when we kill two birds with one stone, the multiplier is dual in power and effect. In like manner it is evident that it may be any other number large or small. Also a multiplier may be dual (or larger) numerically in that it is a combination of two (or more) multiplicative agencies. Thus a dual multiplier may be formed by the combination of a subjective and objective multiplier, as when conscience and public opinion unite in the condemnation of a given act.

A multiplier may also be characterized by some *spatial* principle prominent in it. For instance a multiplier may be linear in form. Thus when a cannon ball is fired not across but down a line of soldiers its effect is multiplied in a linear way. Similarly we have manifolders that are fan-shaped, conically expanding, radiating from the center of a circle or sphere, spreading like the trunk and branches of a tree, or of other geometric forms. Similarly a multiplier may be one, two, three, or *n*-dimensional in character.

An important spatial case, though one often difficult to analyze completely, is that where the multiplying power is due to relative position. Thus in the number 876508 the left hand 8, as compared with the right hand 8, is multiplied one hundred thousand times in value by its relative position. Similarly position is a multiplier of one's power in many social and other relations.

A margin of empty space also, by virtue of its power to give freedom and room for expansion, often constitutes an important, though sometimes subtle enlarger of utilities.

With respect to time a multiplier may be temporary or permanent. Often also time itself is spoken of as a multiplier though time in this case is of course but a symbol for the causes or multipliers acting in time. With respect to time we also have *predictive multiplication* as when a gardener multiplies the efficiency of his work by killing young weeds before they have broken the surface of the ground. In fact many of our mental powers such as fear, conscience, interest, or even reason, have predictive multiplication as one of their chief functions.

By *reciprocal multiplication* is meant the action of two objects each of which acts as both unit and multiplier with respect to the other. Illustrations are the hunter and his dog, the general and his army, theory and practice, matter and form, labor and capital, the plan and elevation of a building, humor and pathos, two friends, two elements in a good dish, as pork and beans, two flowers which fertilize each other. Also any number of elements may enter into a reciprocal multiplicative relation. Thus when a number of friends or business men coöperate, the ideas of one member of the circle are multiplied (often many times) by the use made of them by other members of the circle.

Multipliers may be of different orders of power. This matter also will be discussed more fully later (pp. 51-52).

Indirect and Obscure Multipliers

Some multipliers are *negative* in character. An instance is the removal of an obstacle in order that a direct multiplier may act. Thus, instead of fanning oneself on a hot day, one may often get the same result with far less labor by opening a window and letting a breeze blow through the room. In a vastly higher field results of the first importance are often achieved by merely setting free intellectual or social forces. Most important for our purpose is the disentanglement of the prime efficiency agents or principles in a given domain and allowing them to act without hindrance.

Another important kind of negative multiplication is that of division of error or loss. Thus if we assay a specimen of the ore in a poor mine before working the mine extensively, we may divide by thousands the loss which would have resulted from attempting to operate the mine on a large scale. In general, every experiment or testing on a small scale of a proposed plan or method, constitutes a unit which, if successful, will later have a positive multiplier, but which if it fails to give good results has a negative multiplier in that it prevents extensive loss.

Multipliers are often *vague* or *obscure* in whole or part. Instances are that action in an organism by which small doses of a vaccine or serum lead to immunity from a specified disease. Other instances are various kinds of influence, growth, and propagation.

Complex Multipliers

The different species of multipliers enumerated above may be compounded and complexed together in various ways. Thus, to consider an extreme case, various objective multipliers compounded together give us our whole system of civilization as a multiplier of any good (or evil) unit. Similarly subjective multipliers when combined give us the entire mind and its experiences as an enlarger of any impression.

Subjective and objective multiplicative agencies give us, for example, the system of credit and banking which multiplies the usefulness of the small stock of gold possessed by the world; or the process which has expanded the observation of the attractive power of a piece of rubbed amber into the science and art of electricity; or that which has enlarged the observation of the smudge made by sunlight on a piece of silver into the science and art of photography.

An important special case of the complex multiplier is that which often creates enormously large rewards for the winner of a race by a margin however small. Similar are the multipliers which act in the process of evolution and cause those organisms to prevail and survive which excel by however small a margin of fitness.

An important special case of abstract and complex multiplier is the process called generalization.

For our purpose the most comprehensive and powerful multipliers will be found to be the principles of efficiency which are being investigated in this book.

ADVANTAGES IN THE USE OF UNITS AND MULTIPLIERS

Many of the statements that were made in Chapter II (pp. 23-26) concerning the efficiency functions of reuse apply also to the unit and multiplier. But the fact that the unit and multiplier principle is more abstract and general (less restricted as to the categories of personality and time) gives it in some respects a deeper and broader meaning and usefulness. Just as the unit and multiplier process is more thoroughgoing and manageable so also are its fruits.

Other Superiorities to Reuse

For instance, the application of a pulley to lift a weight may be viewed either as a case of reuse or as a case of the multiplication of one's powers. The advantage of expressing the process in the unit and multiplier form is that it enables us to adapt our pulley machinery accurately to the size of the weight to be lifted, and thus at times to avoid costly experiments and much waste of energy. Similarly in many other cases by putting a given efficiency principle in the unit and multiplier form, we obtain increased economy or efficiency in its application.

The superior efficiency of highly developed forms of the unit and multiplier principle over more or less haphazard reuse is illustrated by a comparison of the English language with the Chinese. The printing of English has been reduced to the use of a few well selected units with large multipliers, while the Chinese characters still consist of a multitude of ancient ideographs. As a result a font of English type weighs 50 pounds and costs $5 while a font of Chinese type weighs 1000 pounds and costs $100. Furthermore the typewriter and linotype greatly facilitate the use of the English language, but cannot be employed in the use of Chinese because of

the multitude of its primary symbols. The comparative inefficiency of the Chinese language is also illustrated by the fact mentioned on a previous page that the Chinese compositor walks 20 miles a day while the compositor of English type stands still.

Increase of Power obtained by Multiplication of Units

Machines also, by the reuse of which they are capable, increase human power. But this increase is greatly magnified by standardizing machines, that is, by reducing machines as far as possible to the unit and multiplier form.

Similarly the reciprocal reuse of their work by scientific men is much enlarged by putting the results of their work in a manifold unit and multiplier form as in journals and periodicals, and by expressing these results as far as possible as principles, laws, and formulas stated in terms of more or less conventional units.

Systems due to Units and their Multipliers

This suggests a still more general and far-reaching advantage which characterizes the use of the unit and multiplier methods viz.: the fact that the development of adapted species of unit and multiplier often leads to the combination of these into extended systems with a corresponding development of efficiency results.

Duality of Unit and Multiplier a Source of Power

Another specific and prime source of efficiency in the use of the unit and multiplier is found in the duality which characterizes the unit and multiplier when considered in comparison with the single concept prominent in the principle of reuse. As a result each of the two instruments involved, the unit and the multiplier, can be developed in independent aggressive fashion. Thus the unit can be adapted to any given purpose and the multiplier, owing to its explicit numerical form and abstract nature, can in like manner be freely extended. Also the limitations of the dual concepts may be more readily studied and remedied, and auxiliary objects, such as symbolisms, may be more readily applied to them.

Transcendent Results

The result of a multiplication often transcends in some special way the data employed. Thus a drop of water when multiplied be-

comes the ocean. But the ocean is much more than a drop of water with a numerical multiplier. So rock is more than a grain of sand or an atom manifolded.

Taking up a different form of transcendence, the microphone by multiplying the force of a vibration makes the vibration audible and the microscope by multiplying the light which falls on the retina of the eye causes the invisible to be seen. A disease germ when multiplied may produce first sickness and then death. A variation in the anatomy of a reptile when given a large multiplier has produced the bird. A point when multiplied by motion produces a line. Trifles when multiplied together in a work of art beget what is called perfection. According to the law of evolution a multiplicative increase in the number of individuals belonging to a species, leads on the one hand to the death of many individuals in the species, and on the other hand to the development of other individuals into a higher kind of organism. In many different domains a multiplicative accumulation of minute changes often produces a conversion or a revolution. These transformational fruits may all be hastened and increased by the choice or creation of adapted units and multipliers, as contrasted with more accidental reuse.

Progress obtained in Various Departments

We may realize the importance of the unit and multiplier principle in a more pictorial way by noting that the history of human progress, in one aspect, has been largely a history of improving units and multipliers. Thus in the field of inorganic science, progress has been made by the use of the atoms devised by the ancient world, and still further progress by replacing atoms in part by ions and electrons. So immense advances have been made in biology by recognizing the cell as the fundamental unit, and in political life by regarding the individual citizen, not the tribe or the city, as the primal unit. The same source of increasing efficiency is prominent in other departments of life or thought.

Historical Illustrations of Advantages

In like manner human progress has been marked by increasing and improving multipliers, and every period of unusual achievement has been one where large intense multipliers have been at work. Thus the epochs of Greek and Italian art, the cathedral building age, the

various religious eras, each came and flourished when they did, from one point of view, because of the large popular multipliers of the new item of efficiency or value connected with each of these periods. More recently the printing press, the railroad, steamboat, telegraph, and similar agencies have produced new multipliers which have given new values to every unit of progress and greatly accelerated many forward movements.

Also in the history of progress new units and new multipliers have acted and reacted on each other in various ways. This is illustrated by the fact that the new social and political units in the United States have given rise to vast multipliers of every new invention and improvement. The new multipliers in turn have led to the development of new units, particularly of small units which could not have established themselves but for the large multipliers ready to be applied to them. These new units are illustrated in the numerous small improvements made in various machines and tools.

Advantages Illustrated by Great Men

Similarly every great man as a part of his work has created at least one new efficiency unit, or given some old unit a new and notable multiplier. Thus Christ brought into the world in a vitally new form the principles of the fatherhood of God and the brotherhood of man. Paul by his knowledge of the Greek philosophy and the Roman political system gave a large multiplier to the units which Christ supplied.

Departments Determined by Units and Multipliers

In a fuller sense also we see that every department of knowledge or activity is based on (or means) a few fundamental units reused and multiplied. This principle holds true even in apparently informal and unsystematic fields of activity. Thus we have the elementary units of dancing, a few fundamental strokes in penmanship, and similar primary units in boxing and music. Likewise, though often in a far higher sense, each branch of science has its alphabet, grammar, and logic, and because of the great and distinct body of economy and efficiency thus generated its existence as a distinct department is justified.

Absence of Units and Multipliers Means Loss

Another collective way in which to realize the value and importance of the unit and multiplier principle is to try to picture what life would be without this instrument of efficiency. In this case mankind would need to dispense with the dollar, mile, inch, quart, day, year; with the science of number and measurement; and with the more informal sets and systems of units and multipliers which pervade all life.

Limitations in the Utility of Units and Multipliers

Before closing this presentation of the efficiency functions of the unit and multiplier, some statement should be made of the limitations connected with this source of efficiency. Prominent among these are certain limitations in the nature of things (to be studied more fully later, see p. 265) which on the one hand often prevent the formation of units and on the other check the action of multipliers.

Also a unit may be evil or destructive in its nature; hence its multiplier may operate to produce injurious results. Illustrations are the Colorado potato beetle, the gypsy moth, and mistakes and sins of all kinds.

Also the unit and multiplier principle has certain drawbacks due to the abstract and impersonal nature of units and of number. Hence arises the need of other efficiency principles to be used in connection with the one under consideration.

Complex Fruits

It is evident that the various fruitages of the unit and multiplier may be compounded and complexed in many ways. In this connection the operation of the principle of dialectic (see Chapter XIV, p. 249) is important. Thus a single observation made by Prof. Roentgen has developed into the whole science of X-rays with its world of surgical and other more general applications.

The most important use of the unit and multiplier principle, as will later be made more evident, is the power of this agent to set free and develop other efficiency principles both objectively and subjectively.

ANALYSIS OF THE UNIT AND MULTIPLIER

A general statement of the improved sources of efficiency in the unit and multiplier principle was made on pp. 35-36. In making a closer examination of this matter it will be an advantage to make use of the principle of marginal reuse as presented in Chapter II (p. 19). Thus by showing in what respects the unit and multiplier principle differs from reuse, we utilize the work which has already been done in analyzing and defining reuse.

Tabular Comparison

The corresponding qualities and elements which enter into the two principles under discussion are shown in the following table:

Reuse	*Unit and Multiplier*
Personal	Impersonal
Time is essential	Time is secondary and quantity is primary
Emphasis on datum and result	Emphasis on process
Concrete	Abstract
Fruitage usually direct and immediate	Fruitage often indirect and remote
Qualitative	Quantitative
Voluntary and intentional	Multipliers often act independently of or even contrary to human will
Synthetic	Analytic

Summing up the matter in a single sentence, reuse is concrete, individual, subjective, empirical, opportunistic, geometric, often irregular and haphazard; the unit and multiplier principle is abstract, social, objective, systematic, analytical, logical, artificial, algebraic.

We may also proceed in our analysis of the principles in hand by anticipating to some extent certain principles of efficiency to be treated later and by showing the relation of the unit and multiplier principle to certain other primal sources of efficiency.

Relation to Other Efficients

Thus with respect to the diversity or separation principle (see Chapter VIII) the unit and multiplier method is based externally on a certain freedom from the entanglements of space, time, and personality, and internally consists essentially in a specific distinction between the unit and the multiplier. The treatment given

later of diversity as a source of efficiency will show how each of these separations opens the way to a group of advantages, some abstract, and some concrete.

Similarly with respect to the principle of uniformity the unit and multiplier data and processes may often be standardized and thereby largely increased in power. With respect to orders of material (see Chapter VI) the unit and multiplier agency is as a rule of higher order than reuse. With respect to the categories of motion and force, it is more active and rapid. It also has a larger dialectic power. With respect to directive use it is more manageable.

It should also be noted that the principle of limitation acts in important ways in the formation of the reuse and unit and multiplier agencies. For instance there would be no need for the employment of either of these principles of efficiency if certain limitations did not exist and compel the use of such methods. Hence the study of the meaning and forms of limitation should aid us in the development and application of each of the two principles under discussion.

Analysis of Special Case

A word should also be said about the nature and classification of certain special cases of efficiency which are at least closely connected with the idea of the unit and multiplier. For instance if we bait a hook with a piece of meat and catch a fish, the question may be raised as to whether the bait may be regarded as a unit and the fish caught as the product. A similar question may be asked as to a seed which is planted and the yield which results. In the one case the bait is lost as food. In the other the seed is partly lost and partly transformed into the resulting crop; certain elements of soil and atmosphere also enter into the matter. Hence we may say that in the direct aspect the question is a complicated one, but that if we reduce the matter in each case to an amount of energy expended, and a return of energy, we find that both cases are essentially instances of unit and multiplier. The same analysis applies to a variety of like cases.

Definitions

Before closing this part of the discussion it may be well to formulate, as best we can, definitions of the unit and multiplier.

A *unit* is any entity used manifoldly in space or time or in any relation.

The *multiplier* expresses the number of manifold uses made of a unit.

The various elements which our analysis has made clear as entering into the unit and multiplier principle may all be made the means of a more effective application of the principle. Some of the improved methods thus suggested we now proceed to point out.

METHODS OF APPLICATION

Repetition of Advantages in Reuse

Many of the remarks made in Chapter II in connection with the best methods of utilizing the principle of reuse apply also to the unit and multiplier. However the fact that the unit and multiplier concept is more abstract and general than reuse gives rise to important additional considerations.

Adaptation of Unit and Multiplier to Each Situation

Owing to the fact that both unit and multiplier may take many diverse forms, also that these forms are adaptable and largely under control, it is important to study each special case where a unit and multiplier are to be employed and to select the most efficient species of each for the work in hand.

As an aid for this adaptive work it is important to realize and keep in mind in some systematic way the large number of possible forms of units, including particularly the various forms of natural, artificial, and semi-artificial units; and to treat multipliers in like manner.

Frequent Value of even Imperfect Units

Where effective units are difficult to obtain, it is important to realize the value of even imperfect ones in connection with more or less efficient multipliers. Thus, in a situation of great difficulty, rather than do nothing it is often well to make a start or beginning, however crude, and to trust that if no others are at hand, natural multipliers will act on the unit thus made, that the crude initial units will improve, and that new units and multipliers will appear.

This procedure often takes a complex form. For instance when plunged into a complex and difficult situation where comprehensive systematic methods of action are impossible, it is well to give each

available scrap of efficiency its largest possible multiplier, and to trust that later developments and coördinations will appear.

In particular in cases of difficulty it is important to realize the value of such somewhat crude multipliers as patience and persistence.

Sometimes special methods must be employed to get any unit whatever. Thus in breeding new kinds of plants, often the great difficulty is that of getting a plant to vary at all. After a variation of any kind has been obtained, means exist of developing it in a desired channel and often to high degree. In other cases the difficulty consists in getting any practical multiplier whatever. Thus a man with a new and useful idea or invention may for a long time find it impossible to obtain the means of making a wide use of his conception.

It is well to fix in mind the rule that the principle of the unit and multiplier is an important aid in all cases where it is important to make a little go a long way. Thus it applies to the problem of utilizing the power, without impairing the scenic grandeur, of Niagara Falls. For if the full volume of water were sent over the Falls during a specified hour during each day and sightseers should collect to see it at that time, all the energy of the Falls might be utilized in a mechanical way during the other twenty-three hours of the day.

Art of Adapting Units

Methods of devising units to meet the conditions in a given field may be developed into an art. Thus instead of saying that the British Empire increased 4,000,000,000 acres between the years 1800 and 1900, we may express the matter much more effectively for many purposes by saying that the British Empire during the nineteenth century increased at the rate of 2 acres per second. So instead of saying that a certain star is 20,000,000,000,000 of miles distant, we may say that it is at a distance of three light years. It is often a profitable exercise to spend one's leisure in devising units and conceiving of their multipliers, and thus acquiring the art of rapidly discovering, forming, or improving these instruments of efficiency.

A Scale of Relative Value

As an important aid in this process, it is desirable to obtain a thorough appreciation of the fact that both units and multipliers

may be of different orders of perfection or efficiency, and that it is highly useful to be able quickly to recognize and utilize those highest in power under any given circumstances. For instance the most serviceable units for general use are those which are definite, permanent, and readily reproducible.

As a special case of permanency it is to be noted that, other things being equal, old and established units are preferable, since they have been frequently tested, already have large multipliers, and any mastery which we already have of them may be reused.

Particular mention should be made at this point of the efficiency significance of the Arabic number system. When frequently used this system comes to act almost automatically in a number of ways in the extension of results. A special use of it is as an aid in the multiplicative process called generalization. In generalizing a fact we infer that what is true of one, two, or some objects in a class, is true of all. In generalizing a process we extend the process from two or three steps to an unlimited number of steps.

Devising more Fundamental Units

The power which we have of changing and adapting units often enables us to reduce a given set of them to the same common denominator, so to speak; that is, to more fundamental and inclusive units. In certain processes of mathematics, science, and philosophy, this process has been carried to an extreme degree, the result being units of great abstractness and hence of inclusiveness and power, when rightly used. For the process of progressive inclusiveness gives also an increasing systematization and interrelation of units and other sources of power.

Best Standard Method

Before closing this part of the discussion it may be well to make a statement of the best standard method for the combined use of units and multipliers in the so-called practical world. As a preliminary fact we observe that, owing to the highly organized state of society, large multipliers for any new unit of advantage are omnipresent, while only rarely does a large new unit of efficiency appear. Hence the best standard method of employing units and multipliers in combination, is to rely mainly on small units and large multipliers, but to have an eye open for large units as ex-

ceptional possibilities. Thus a machine which saved two or three drops of solder in putting a lid on a tin can, produced a fortune for its inventor because this saving had a multiplier of 700,000,000 each year. Similarly a fortune was obtained from the invention of the hook eyelet used on shoes, and another from the addition of the gimlet point to the old fashioned blunt screw. For a like reason a steel manufacturer is ready to send his machinery to the scrap heap and replace it by new, if by so doing he can save 10 cents a ton in making steel; and a cotton manufacturer will do likewise if he can save 1/16 cent per yard in making muslin.

In like manner the greatest masters of chess strive primarily not for brilliant strokes, but to make the most of each minute advantage which appears, or which they can devise. So in the highest forms of art the multipliers dominate the units.

Ideal Species of Unit and Multiplier

It is well to remember that, for general purposes of thought and life, the best units and multipliers alike, from our standpoint, are the primal efficiency principles which we are studying in this book. The specific meaning of this statement will be developed later.

Summary of Chapter

At this point it will be well to recapitulate the substance of the present chapter.

The principle of the unit and multiplier includes reuse as a special case, and hence is a more comprehensive agency of efficiency. The unit and its multiplier are often superior to reuse in efficiency because of the fact that in reuse we must take the data of life as we find them, while units and multipliers may often be created and shaped according to our needs and wishes. Innumerable varieties of both units and multipliers exist, or may be created, each endowed with its own special efficiency powers. Special mention should be made of natural and artificial units, and of predictive, reciprocal, and indirect multipliers.

The use of units and multipliers means economy and new powers of various kinds. In some cases the results take the form of elaborate systems. In other cases they are transcendent in the sense that they could not otherwise have been obtained. The study of great men and the history of human progress furnish a multitude of illustrations of these values.

In order to reap the largest possible fruit from the use of the units and their multipliers, it is necessary to follow many of the methods indicated for reuse in the last chapter. Thus it is important to realize the great variety of possible units and multipliers, and to adapt different forms of these to different situations. It is an advantage at times to arrange units in a scale according to their relative values, and to realize the great utility of even an imperfect unit or multiplier in a difficult situation. The ideal species of units and multipliers alike are the fundamental principles of efficiency which we are studying.

EXERCISE 3

1. Give an example of manifold simultaneous use.

2. Also of use that is manifold both in a simultaneous and a successive way.

Point out the unit and the multiplier in each of the following cases:

3. The use of the carpenter's level
4. Of a parliamentary motion
5. Of lieutenants by an able man
6. Of the rudder of a ship
7. The use of yeast in making bread.
8. Newton's reputed use of the falling apple in discovering the law of gravitation.
9. State which of the units employed in Exs. 3-8 are natural and which artificial.
10. Name some of the units employed in political and governmental organizations.
11. In engineering
12. In agriculture
13. In education

Name three units which are

14. Artificial
15. Natural
16. Semi-artificial

Give two illustrations of multipliers which are

17. Mechanical
18. Biological
19. Abstract
20. Predictive
21. Reciprocal
22. Negative
23. Branching or radiating

State the meaning of each of the following units and give some of the advantages resulting from its use.

24. A horse-power
25. A watt
26. A candle-power
27. What species of multiplier is implied in the statement "a thing well begun is half done"?
28. The driver of a load drawn by six horses is how many times as efficient as the driver of a one horse load?

What primary principle of efficiency enters here?

29. If a microphone makes audible an otherwise inaudible sound, state the degree of efficiency obtained by its use.
30. The process of reducing two or more fractions to a common denominator means what with reference to the use of units?

What advantages result from his process?

31. Explain as a case of unit and multiplier, the use of a bolt on a door.

32. All the various colors may be built up by combining three primary colors, viz.: blue, green, and red. How is the principle of unit and multiplier here involved? What advantages result?

33. Give an example of the multiplication of a loss.

34. Give two illustrations of publicity as a multiplier, and state the useful results which have followed in each case.

35. Give an instance where the use of a small unit and large multiplier is more advantageous than the use of a large unit and a small multiplier.

36. In what respect is a change of the direction of one's motion a unit? What is the multiplier in this case?

37. Why is knowledge gained early in life often worth many times as much as knowledge gained later?

38. State the corresponding principle with reference to mistakes.

39. All units used in physics may be built up out of what three fundamental units? Illustrate.

40. What is meant by the dimensions of a derived unit in physics? Illustrate.

41. Chemically all material objects are composed of what units?

42. In fighting battles the method used by Napoleon on land and that used by Nelson on sea were essentially the same. Explain the method and show that in effect it constituted an application of the unit and multiplier principle.

43. Explain the sense of proportion in an observer as a case of manifold unit and multiplier and give two examples.

CHAPTER IV

THE GROUP

Illustration

Just as the principle of reuse was enlarged into the broader and deeper principle of the unit and multiplier, so the latter principle may be extended so as to form a still more general agent of efficiency, viz.: that of the group or assemblage. To indicate in a brief way what is meant by this, we may continue an illustration already used in connection with reuse and the unit and multiplier. If an orator preserves a speech and uses it repeatedly, we have a case of reuse. If this speech is published simultaneously in several newspapers, the method of the unit and multiplier is employed. If a number of the orator's speeches are collected and published together in a volume, we have an example of the use of the group method as a means of obtaining efficiency.

The Group broader than the Unit and its Multiplier

The process of reuse implies a certain likeness in the steps involved. The use of the unit and multiplier is also a more or less homogeneous process. But in the group various unlike elements may be combined and treated efficiently. Thus the combination of various speeches in a book facilitates their study and analysis as a whole, as well as their preservation and manipulation in libraries.

Special Advantages in the Use of Groups

Many groups occur in a state of nature, or appear, as it were, of themselves in the world about us, and become immediate sources of efficiency. Thus the sight of a school of fishes suggests the unit and multiplier method of catching them by a net; a collection of workmen in a factory opens the way to address them all at the same time on any given subject; the fact that goods are packed in barrels may enable some one by quick handling to save them from being consumed in a fire.

A low order of force may often be utilized in manipulating groups. At the same time the idea of the one and many owing to

its great generality frequently operates in two opposite directions (see direct and inverse groups on p. 59). Even more important is the fact that the character of the ties which combine objects in groups facilitates the building of groups into extended systems. The group method is so flexible that it may be used in a crude incipient way and also as a highly finished and final instrument of efficiency.

CLASSIFICATION

Classification of Groups according to Materials

Groups may be classified according to the materials of which they are formed. Thus, inorganic groups are groups of objects like atoms, bricks, dollars, acres.

Biologic groups are illustrated by a group of cells, a field of wheat, or a herd of cattle.

Examples of social groups are the family, a convention, or a nation.

Combinations of perceptions, or ideas, are instances of more ideal groups.

It is to be noted carefully in this connection that a group of objects is often closely connected with a group of forces or of uses. Thus in a gang plow we have a group of like objects (the plow shares), and with each use of the plow the work of the plowman receives a multiplier, that is, is converted into a group.

Classification of the Ties of Groups

It is also often advantageous to characterize and classify groups according to the ties which bind or connect the elements comprising a group.

Thus a tie may be a *material thing* like a pin, piece of wire or twine, an elastic band, a basket, box, barrel, or dish.

Also a tie may be a *physical force* of some sort as cohesion, gravitation, or magnetism.

Likewise it may be a *biologic, psychic, social, ethical,* or *religious force.*

Ties may also consist of some more *abstract relation,* as of any likeness as of color, size, weight, shape, temperature, value, age, origin, development, feeling, or belief, or uniformity in any function or relation.

Special mention should be made of *spatial likeness,* that is, of concentration in space and consequent likeness of position. Some of the many advantages which often follow from this species of groupism will be pointed out later.

Of equal importance is *simultaneity,* or likeness in time.

Concentration in both space and time is illustrated by the force acting in the head of an arrow, and in the mode of action of many weapons and tools, or the force exerted by an army.

Special mention may also be made of the combinations of objects in a field of vision or in a state of *consciousness.*

Relation of Ties to the Elements of Groups

It should be noted also that in some cases the tie is something external to the elements of a group as when objects are held together by a piece of string. In other cases the tie is in itself an element or in some way embodied in an element as when a herd of animals is held together by one of the number, or a solar system by its sun. From some points of view however an external tie may often be regarded as a heterogeneous element in a group.

Complex Ties

The tie which holds a group together is often manifold or complex in nature. Thus when pupils are grouped in rooms in a graded school, the groups of pupils are determined not only by the walls and floors of the building but also by the likeness of pupils in age and development, and by the leadership and control of teachers. Similarly a family is bound together by biologic, psychic, social, ethical, and many different efficiency ties.

It is to be noticed also that one tie may be an aspect of, or may lead to, another principle of combination. Thus cohesion leads to spatial concentration; and nearness of men in space may lead to likeness of dress or opinions.

When the tie which holds a group together is strong or feeble, temporary or permanent, static, vibrant, or changing, the group may be characterized accordingly.

Classification of Groups according to their Forms

Hence the nature of the tie which holds a group together to some extent determines the form of the group. But groups may

take individual forms for other reasons, and should often be characterized and classified according to these forms.

Thus a group which contains two, three, or n elements may be termed a dual, triple, or n-fold group (or a duad, triad, or n-ad). Similarly a group may be large or small, finite or infinite. It may be one, two, or three dimensional, and may assume any geometric shape whatever.

An important distinction for our purpose is that between homogeneous and heterogeneous groups, that is, groups composed of like or unlike elements, respectively.

It is to be noted that in determining the character of a group in this respect much depends on the point of view. Thus an orchard viewed as containing apple, pear, and cherry trees constitutes a heterogeneous group, but viewed as containing trees merely (as contrasted with corn stalks, for instance) it is a homogeneous group. In other words by the use of more general ideas, a heterogeneous group may often be converted into a homogeneous one.

Natural groups are those which exist independently of any change in unit or tie wrought by human agency. Examples are the solar system, a cluster of wild flowers, the petals in one of the flowers, a herd of wild animals, or a school of fishes.

Artificial groups are those which in some respect have been formed by man, as a bundle of toothpicks, or a class of scholars.

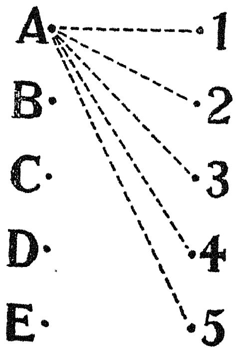
With respect to order it is useful to make a distinction between direct and inverse groups.

A *direct* group is one formed by the inclusion of two or more objects or elements into one whole or aggregate, as when a number of objects are tied together to make one package.

An *inverse* group is one formed by the expansion of a single object into many forms or elements, as when a grain of wheat produces a number of other grains, or a piece of rock is separated into its constituent elements.

A combination of the direct and inverse relations gives rise to the *dual-reciprocal* group. This consists of two groups of elements, each element in a group being connected with several or all of the elements in the other group. An illustration is a set of workmen and a set of tools so related that any workman may use any one of the tools.

This dual reciprocal relation may be represented diagrammatically



by a group of points A, B, C, D, E (indicating workmen), and another group 1, 2, 3, 4, 5 (standing for tools) and a series of dotted lines connecting each point in the first group with each in the second. In the diagram as given in the text only a few specimen connecting lines are drawn.

In this way qualities (or properties) may be regarded as forming objects and objects may be analyzed into qualities. This enables us to handle a group of qualities as a single object, or a group of objects by means of a common quality. Similar efficient group relations connect forms and materials; symbols and things; objects and places; generals and particulars; principles and details; causes and consequences. For one piece of material may take many different forms, and one form or mold may be filled with many different substances; one symbol may represent many different objects; and one object may be represented by many different symbols.

By an extension of the above process, a number of groups (as *n* groups) may be combined to form an *n*-reciprocal group. The simplest form in which this species of group can occur is that in which in a group of *n* elements each element in turn may assume the leadership of the remaining elements.

Manifold Groups

A group may be manifold in the sense that it may be characterized in many different ways. Thus a family is determined as a group primarily by certain biologic considerations, but it may also be determined by the common physiological and mental peculiarities of its members, by the fact that these members occupy the same home, and it is also often an *n*-reciprocal group because of the services which each member renders to the other members.

In concluding this portion of the discussion attention should be called to the fact that reuse is a temporal group homogeneous as to the quality of its elements; and that the unit and multiplier form a group homogeneous with respect to both the size and quality of its elements.

EFFICIENCY FUNCTIONS OF THE GROUP

To help make clear that the group should be regarded as an independent primary agent in obtaining useful results, a number of

the efficiency properties of the group were pointed out and illustrated in the opening paragraphs of this chapter. Beside these properties the group principle has other uses which invite special attention.

The Group a Source of Economy and Reuse

A group means a special field of application for the reuse and unit and multiplier principles. Hence almost all that has been said concerning the efficiency functions of these agents applies to the group also. Grouping objects is also a means of storing them in a small space and finding them readily. Hence it means an economy of space as well as of time. It may mean an economy of time in other ways. When goods are combined in packages they can be transferred much more rapidly from one place to another than if handled one by one. This leads to many other obvious advantages.

Other Direct Uses of Groups

When objects are collected in groups they may the more readily be protected from harm and loss. In this connection also we have the proverb that in union there is strength. The acquisition of knowledge is often facilitated also, since what is true of one member of a group is likely to be true in a measure also of the other members of the group.

Transcendent Uses

It is important to note that certain of the fruits of the group method are what may be termed transcendental, that is, they are qualitative and infinite, and often, apparently, could not be obtained in any other way than by the use of the assemblage principle. Thus a number of men by combining are often able to achieve an object, such as building a railroad, which no one of them singly would be able to accomplish. Other transcendent results which may follow (often in an unexpected and dialectic way) are confidence and stimulus to the builders and certain benefits to the public which result from the operation of the road.

A mere improvement in a group may lead to a like series of results, as when an increase in the number of vibrations in a wire creates an audible sound, or a concentration of the essential parts of a gas engine makes the aeroplane possible.

Complex Fruits

Owing to the general nature of the group the various classes of advantages connected with its use often occur together in a highly complex and self-developing way. Thus the combination of many small and scattered telegraph and telephone systems into one large organization brings many advantages, as increased range and frequency of use for each instrument, hence economy of service, hence improved service, and therefore a better development in many ways of the country served.

Many similar but larger benefits have resulted from the union of the various states composing the United States, with further advantages such as security from enemies and oppression.

When a number of small rural schools are combined to form one large central school, the pupils may be distributed in grades, fewer and better teachers employed, each teacher's work will receive a larger multiplier, reciprocal reuse of the combined libraries will result, outside lecturers may be employed, with a dialectic of other advantages to the various local communities concerned.

When a railroad is electrified, a large number of locomotives are replaced by one central power house. This has the advantage of replacing many engineers by one; that is, of multiplying the work of the engineer employed. The firemen on the locomotives are replaced largely by mechanical methods of handling fuel. The concentration involved also makes it possible to protect the engine from cold and loss of heat by the use of surrounding walls and other devices. It also prevents loss in the form of steam blown off when a locomotive is standing still. Many other direct and indirect advantages might be mentioned.

As a still more general illustration we may consider the relation of different forms of energy. Franklin identified lightning and electricity with manifest fruitful results. Oersted went a step further and showed the common nature of magnetism and electricity with still further advantages. Joule and Mayer advanced still further and revealed the essential identity of all physical forces as light, heat, kinetic energy, and electricity. It would take a volume to enumerate the beneficial results which have followed from this close grouping of the forces of nature. For the more vital and comprehensive the elements comprising a group, the greater, as a rule, are the advantages following from the use of the group.

Uses from other Points of View

The value of the group principle might be made evident from many other points of view as by trying to realize what life would become without the use of groups, or by noting that the history of human progress has been, in one respect, a history of improving groupages. It is also a help in realizing the importance of the group as a source of efficiency to observe the useful powers of certain special forms of the group principle as of unity, union, concentration, combination, coöperation, etc. Stating the matter in a more general form we note that every word beginning with a syllable like con-, com-, col-, syn-, syl-, or sys-, stands for a specifically realized group. Concepts, for instance, are group ideas, which, to use the comparison of William James, are a kind of seven league boots carrying us forward in our thinking with magical rapidity and ease. So groups in general give wings to the various processes of life.

"The man who will show me the one in the many", says Plato, "I will follow about like a God." All that mankind has learned since Plato's time has but added to the force of his words.

Limitations in Utilities of Groups

Certain limitations in the advantages connected with the use of groups, or of one-in-manynesses, should be mentioned before closing this part of the discussion. Thus a very extensive unification frequently brings the drawback of stifling competition and often of checking invention and initiative. A high degree of concentration may also increase the evil results of error or abuse; that is, be a source of loss instead of gain. For instance, when all the lights of a city are combined in one electrical system, an explosion at the central power house may plunge the whole city into darkness. Direct theft of a large bulk of wheat or corn is almost impossible, but if the value of the grain be condensed into a diamond or represented by a signature written on a piece of paper, such theft becomes relatively easy.

Hence the general statement may be made that groups owing to their more comprehensive nature often lead to greater dangers and losses than occur in the application of the reuse and unit and multiplier principles.

ANALYSIS AND DEFINITION

Comparison with Reuse and Unit and Multiplier

The more or less ultimate sources of efficiency in the group and its principal components and characteristics in other respects have already been stated in the preceding part of the chapter. Thus, as compared with reuse and the unit and multiplier, the group idea is less personal, implies in less degree the idea of time and succession, is more static and more remote from the idea of immediate gain. The unit elements composing it may be less uniform, definite, and homogeneous. In a word in its widest sense, the group concept is less concrete and hedged in by categories; that is, is more abstract and general.

Uniqueness in the Group

It is also to be remembered that the group, like other primal instruments of efficiency, contains a certain unique and characteristic essence which perhaps is largely beyond analysis, and which to a certain degree is the source of its efficiency.

Definitions

A definition, even though it be imperfect, is often an aid in using a concept. We may define a group as a manyness which can be treated in some respect as one.

The elements of a group are the individual objects which are combined to constitute the group.

Fundamental Nature of the Group

In the concept of the group, or of the one as related to the many, we have what is perhaps the most fundamental source of efficiency at our disposal. We obtain other approximately primary sources of efficiency by combining groups in certain special ways or by giving the parts of a group certain special forms. These additional primary agents of efficiency will be treated in the chapters which follow.

METHODS OF APPLICATION

Much that was said in previous chapters concerning the best methods of utilizing reuse and the unit and multiplier principle as sources of efficiency applies to the general group or assemblage principle. But in connection with the group as such these methods

are often to be modified, and in some cases new principles of method developed owing to the peculiar characteristics of the group.

Vast Number of Groups

For instance, in order to make the most efficient use of the group, it is important to realize the vast number of groups which exist or may be devised. It facilitates this conception to realize the immense variety of elements which may be combined in assemblages. the great diversity of means by which these elements may be bound together, and the many different forms which groups as a whole may assume. In developing these conceptions we are aided by the comparative freedom of the group idea from limiting categories, and also by the possibility of beginning with very abstract and general elements and forms, and annexing different categories, or concrete considerations, singly or in combination, in systematic ways.

Adaptation of Groups

It is also important to study the application of groups with reference to the different practical limitations which arise in different cases. The very great variety of possible groups makes such study necessary in order to select the best form of group for any given purpose. Thus it is desirable to know the peculiar advantages and disadvantages in each form of concentration or unity. For example if the extent to which magnetic energy can be concentrated in a bar of soft iron had been realized earlier, the invention of the dynamo would have been accelerated a number of years. In like manner the knowledge that the energy contained in a ton of radium equals that contained in 1,500,000 tons of coal may some day be of immense direct practical importance.

Symbolism an Aid to Groups

In cases where the unit objects under treatment must remain scattered it should be noted that the formation of more or less permanent groups is often facilitated by affixing some common mark or sign to elements belonging to a required group. Even when it is not necessary to do so, it often adds to efficiency results to follow this method, as when the soldiers belonging to a given regiment wear some distinguishing symbol.

Practice in Making Groups

Owing to the fundamental efficiency importance of the group it is highly advantageous to practice forming groups on all possible occasions till the group habit is formed. In this connection it is important to realize the many ways in which the formation of groups can be aided by the use of such simple concrete objects as pins, elastic bands, mucilage, pigeonholes, chests of drawers, or by the use of mere relative position. Similarly it is important to form the habit of making mental groups, as by cultivating the perception of similarities and differences.

It may be well to mention a few of the advantages which result from the formation of the group habit. We have (1) the direct economies of space used in storing objects; (2) the utilization of groups by the unit and multiplier and like methods; (3) the subsequent more or less spontaneous development of individual groups into systems; (4) the development of the power quickly and easily to improvise groups in any given situation.

Standard Groups

Mention should be made of the importance of the thorough mastery and preferential use of certain standard forms of groups. It is often useful for society in general to employ such groups. At the same time each individual may make special use of certain standard groups adapted to his personality and surroundings.

Other things being equal it is important to use groups in which the elements, ties, and forms are as vital and comprehensive as possible. Thus mastery of a group of the primary efficiency principles which are under investigation will be found to include a host of more detailed efficiency methods. In this connection it should not be forgotten that while a group may on the one hand be a crude incipient implement, on the other it may be a final polished instrument giving unusual speed and power to an efficiency process.

Manifold Groupings

The use of groups combined in systems will be considered in the next chapter. At the present point, however, mention should be made of the advantage of using, more or less simultaneously, several group methods in a given domain of material. Thus the process of

extracting the meaning from a book may often be facilitated by reading the book backward or beginning at the middle, or in various scattered places, and employing special methods of grouping the facts and ideas contained. This procedure not only often brings important economies but also leads to many higher results. It is often useful to use this method in connection with a more systematic use of groups, as in social or political work, or in the cooperation of the parts of a large corporation.

Definition of Efficients

In referring to the primary efficiency principles which we are studying, it will often be convenient to denote them as a class by the term Efficients. For example, the primary agents of efficiency, or the Efficients studied thus far, are reuse, the unit and multiplier, and the group. The adjective, efficiental, means pertaining to the Efficients.

Summary of Chapter

A brief statement of the ground covered in the present chapter will now be of use. A still broader and more fundamental principle of efficiency than either reuse or the unit and its multiplier is the group. Groups may be classified according to objects composing them, or according to the ties which hold these objects together, or according to the forms of the groups themselves. Special mention may be made of groups which are homogeneous or heterogeneous, natural or artificial, direct, inverse, reciprocal, or manifold.

Groups are sources of economy and new power in ways additional to those characteristic of reuse and of units. Hence the values of groups are correspondingly more transcendent and complex. The group is so fundamental in its nature that the other primal sources of efficiency may be regarded as various methods of using the group.

Here also it is important to realize the great number of possible groups and to adapt different species of groups to different uses. Especial attention is also to be paid to the use of symbolism in connection with groups, to constant practice in their formation, to the use of standard groups, and, in some cases, to the use of manifold groupings.

EXERCISE 4

Give an example of each of the following species of groups:

1. Inorganic
2. Biologic
3. Social
4. Mental
5. Religious
6. Logical (or ideal)
7. Homogeneous
8. Heterogeneous
9. Natural
10. Artificial
11. Complex
12. One dimensional
13. Two dimensional
14. Three dimensional

Give an example of each of the following kinds of group-making ties:

15. Inorganic material
16. A living object
17. Physical force
18. Chemical force
19. Biologic force
20. Mental force
21. Social force
22. Religious force
23. Ideal force

24. Give an example of a tie that is external to the elements constituting a group. Also to a tie that is embodied in one of the elements.

25. Give an example of a direct group. Of one that is inverse. Of dual reciprocal groups. Of an n-reciprocal group.

26. Find the degree of time efficiency in teaching a class of 48 persons for an hour, as compared with teaching each of the 48 persons individually for an hour. As compared with teaching them in groups of 16. Of 8.

27. Also state the time efficiency of each of the other processes in Ex. 26 as a per cent of the first process.

28. Give an example where a person's work is multiplied in value by the use of a natural group of some kind. By the use of an artificial group.

29. Give an instance where the use of groups gives speed in handling objects.

30. Where it saves property from destruction.

31. Where it results in economy of the attention or of other mental forces.

32. Where complex advantages result.

33. Where a transcendent result is obtained.

34. Give an example of the use of the group method as a crude initial instrument of efficiency.

35. Also where it is a finished, final instrument.

36. Show how the spinning jenny illustrates the group principle as a means of obtaining efficiency.

37. In one day one man by using a common plow turned over 1½ acres of land. Another man using a gang plow turned over 40 acres in two days. State the degree of wage efficiency in the second process as compared with the first. Also express the efficiency of the first process as a per cent of the second.

38. Why should goods when bought on a large scale cost less?

39. Combine 8 roses + 5 lilies + 6 pinks into a single group by the use of some more general word than those used in the given expression.

40. Give two examples of the advantages of concentration (or of the disadvantages of separation).

41. State the efficiencies connected with the use of one comprehensive electric light system for a city, instead of several thousand oil lamps.

42. Give an example where a number of scattered and moving objects are combined into a group by means of symbols or marks of some kind.

43. State the advantages of the concentration of electricity in the form of high voltage on a transmission wire.

44. Also those in the close alliance for a century and a half of the different members of the Rothschild family.

45. Ascertain the etymological meaning of the words logic, category, consciousness, and law, and show that in the meaning of each the group idea is prominent.

46. State some of the advantages of having those cells in an organism which perform the same function collected in one or two organs, as in the brain, liver, lungs, etc.

CHAPTER V

MULTIPLICATIVE GROUPS

Groups may be used not only singly, but also in combination in the form of extended systems, with a corresponding increase of efficiency results.

Illustration of Multiplicative Groups

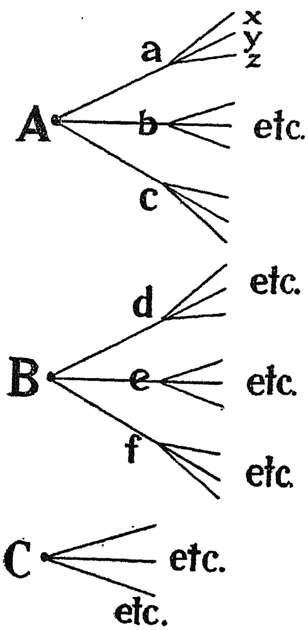
For example, if the commanding general wishes a secret order to be conveyed verbally to each soldier in his army, he does not attempt to speak personally to each individual in the army, or even to have a messenger do so. The general communicates the order to his corps commanders, each of whom conveys it to his division commanders, each of whom transfers it to his immediate subordinates, and so on till the private soldiers are reached. In this way the order is conveyed with repeated use of the advantages which characterize the group method, viz.: less labor, greater speed, practical simultaneity of knowledge, quickness and unity of action, less time for betrayal, and freedom for the commanding general to devote most of his time and energies to more important work than the communication of orders. In this case the system of groups involved consists of the group of corps commanders, the various groups of division generals, and so on.

Diagram of Essence of a Multiplicative Group System

To put the matter in an abstract and comprehensive form, if we have a group of n elements (as A, B, C, etc.), each of which is connected with a set of other elements (as A with a, b, c) forming a secondary group; each element in a secondary group being connected in like manner with elements in a tertiary group, and so on; we have a series of groups so related that the aggregate may be termed a system of multiplicative groups. This is the instrument of efficiency to be studied in the present chapter.

In the diagram the group composed of the elements, A, B, C, etc.,

is termed the prime group, or the group of the first order; the groups represented by a, b, c, and d, e, f, etc., are termed the secondary groups, or groups of the second order; the set of groups which include x, y, z, etc., are of the third order, and so on.



CLASSIFICATION

The various elements and ties which enter into the formation of multiplicative groups are, in general, the same as those mentioned in connection with single groups (see pp. 57-58). Hence the classification of multiplicative group systems with reference to elements and ties, is approximately identical with that given in the preceding chapter for groups.

It is different however with the classification of multiplicative group systems with respect to the forms which they may assume. Owing to the more elaborate nature of systems of groups, these systems appear in certain important forms which are peculiar to the category of efficiency under investigation.

Numerical Species of Multiplicative Groups

A multiplicative group system may be characterized by some number prominent in its structure, as by the *number of orders* of groups which it contains. Thus we may have a dual system, as a group of teachers and their groups of pupils; or a 3-order system, as superintendents, teachers, and pupils. An important special case is that of the infinite system, or one which contains an unlimited number of orders.

A multiplicative group system may also be characterized by some standard *number of elements* used in each of the constituent groups of the system. Thus the system as pictured in the above diagram is a grouping by threes. An important special number form is a dual system in which each group consists of only two elements, a positive and a negative one. Such a system is used in the so-

called game of Twenty Questions, where the aim is to identify a particular object by twenty successive questions each of which is answered yes or no. What is essentially this method is also employed in much detective work and scientific investigation.

Spatial Forms of Group Systems

Spatially, multiplicative systems may be characterized by the *number of* their *dimensions*. Thus we may have a one-dimensional or linear system. This consists of a number of elements arranged in a line and treated by some multiplicative method. This form is so important that it will receive special treatment later (see p. 76).

Evidently also a group system may be two-dimensional or areal in arrangement. By way of illustration we have the diagram given on p. 71, or a river and its branches, or a family tree as ordinarily diagrammed. An important case of somewhat different nature is that of an area divided and subdivided into parts, an example being a country divided into states, counties, and townships.

Similarly a group system may have three dimensions. An example is a tree and its branches. An illustration of a three-dimensional multiplicative group system of somewhat different kind is a house divided and subdivided into parts as rooms, closets, shelves, drawers, etc. Three-dimensional systems of many special geometric shapes are possible as cone-shaped, pyramidal, spherically radial, and other forms.

It is important to discriminate between group systems whose parts are *continuous* or connected throughout, and those which are *discontinuous* or made up of parts more or less separated in space. An example of the former is a river system, or a tree and its branches. An example of the discontinuous form is an army as organized into regiments, brigades, and divisions.

Other Important Forms of Multiplicative Groups

With reference to the category of *time,* multiplicative group systems may be temporary and provisional, or permanent. It is often convenient to describe genetic or causal systems as temporal, since they are determined by factors acting in time. An instance is a family tree or the consequences resulting from an event like the explosion of the battleship *Maine* in the Havana Harbor in the year 1897.

As to *order* we have species which act forward or backward, directly or inversely, by expansion or contraction, by synthesis or analysis.

Systems of multiplicative groups may differ *qualitatively* in innumerable ways. The most important distinction in this respect for our purpose is that denoted by the terms *homogeneous* and *heterogeneous*. A group system may be homogeneous (or heterogeneous) in several different ways, as with respect to the number of elements constituting its individual groups; or as to the size or other characteristics of the individual elements, or of the ties binding them; or with respect to the forms of the component groups. Thus a telephone system is heterogeneous for the reason that part of it (the system of connected wires) is continuous and part (the system of keyboards) is discontinuous.

Other cases of heterogeneous multiplicative group systems are a house and its parts; or the classification of sensations according to their quality, intensity, and duration; or a system of transportation consisting of ocean steamships, railroads, trolley lines, automobiles or carriages, and human locomotion.

Federal Species of Multiplicative Groups

Special attention should be paid to certain important individual forms of multiplicative group systems. We consider first what may be regarded as the normal or standard form; that is, the best form for general purposes. The illustration given in the opening paragraph of this chapter (p. 70) is of this type. It may be described as the federal or hierarchical species. It may have one, two, three, or n dimensions, and may often be transformed from one of these dimensional states into another. Other illustrations of it are the organization of the Roman Catholic Church, or any similar ecclesiastical, or any political system of government. Many business organizations are of the same general type. For instance in the building trade we find a combination of architects, contractors, foremen, mechanics, laborers, and apprentices.

It is to be noted that the normal or federal type may be either continuous or discontinuous. Examples of the latter are those given in the preceding paragraph. Illustrations of the continuous or branching federal type are a tree and its branches, the gas or water pipes of a city, the nerves, or blood vessels of the human body.

The use of latitude and longitude as a means of locating all places on the earth's surface may also be regarded as the use of a branching group system. The same statement applies to a system of coördinates of any sort as used in mathematics or science.

A federal system may also be continuous in certain parts or respects, and discontinuous in others. An instance is that cited above of a telephone system of wires and keyboards. An important case of somewhat different nature is that which may be termed articulate, as the system of bones in the human skeleton; or, more broadly, the system of bones, muscles, and organs in the human body; or, more broadly still, the essential parts of every biological and social organism. Articulate language, both oral and written, is largely of the same nature.

Constitutive Group Systems

The normal or standard federal system may take two other contrasted but closely related forms. The first of these is what may be termed the constitutive multiplicative group form. This consists of a domain divided into certain major parts, each of these parts being divided into smaller parts and so on. An example is that already given of a country divided and subdivided into states, counties, and townships. Other illustrations of constitutive multiplicative group systems are a city marked off into wards, blocks, and houses; a library separated into alcoves and shelves; a book regarded as consisting of chapters, paragraphs, sentences, and words; or a drama as formed of acts, scenes, lines, and words.

Marginal Group Systems

The other of the two species named above may be termed the marginal form. In this type each subgroup is something apart from and attached to an element in the group above. Thus in the organization of the Roman Catholic Church the cardinals are apart from and attached to the Pope. So in a river system, or the branches of a tree, the smaller branches are apart from and attached to the larger branches. Hence in a marginal multiplicative group system, the tie which holds the elements of a subgroup together is contained in an element of the next higher group; while in a constitutive group the tie is in some outside object or some abstract consideration.

A multiplicative group system may be part marginal and part constitutive. Thus a railroad system (marginal) may have attached to it certain cities (constitutive). So an organization may have the two forms as parallel aspects or elements. Thus we may have a continent subdivided into certain realms of ecclesiastical jurisdiction (constitutive) and these districts governed by a system of church officials (marginal). Similarly the area drained by a river system may be regarded as divided constitutively while the river system itself is marginal in structure.

Strandal Multiplicative Groups

A strandal multiplicative group system is one formed of linear elements each of which is subdivided into parts of different orders. A system of this kind may be regarded as constitutive from one point of view and marginal from another, and hence is useful in connecting one of these two types with the other. An example of the strandal type is a river and its branches. For in a river system, if the water which enters each small tributary be followed in its course through the larger branches and main stream to the ocean it may be regarded as a continuous strand in the system, and the whole river system may be regarded as a systematized bundle of strands, and hence as constitutive in its nature.

In like manner if we regard the trunk parts of a railroad system or a tree as manifolded by reuse or by swifter use, each of these systems as a whole may be regarded as strandal. Almost all marginal systems may be analyzed in a similar way and thus reduced to the strandal of constitutive form.

An important case to be considered in this connection is that of the permutations or combinations of a set of elements, as of letters (or sounds) which are combined to form words. Thus an array of letters as first written in the accompanying tabulation at the right is marginal in structure. But if the letters in the higher groups be regarded as reused the array may be rewritten in the second form, which is essentially constitutive in nature.

1st Form

$$A\begin{cases} A\begin{cases}A\\ B\\ C\end{cases}\\ B\begin{cases}A\\ B\\ C\end{cases}\\ C\begin{cases}A\\ B\\ C\end{cases}\end{cases}$$

2d Form

AAA
AAB
AAC
ABA
ABB
ABC
ACA
ACB
ACC

The advantages connected with the first method of arrangement are certain economies as of material, the more evident indication of groupages to the eye, and reuse in certain kinds of mastery. The advantages in the second form are that each completed strandal element, as ABA, can readily be identified wherever placed; that consequently selections from such a system may be arranged in linear order with any desired gaps, as in the cards in a card catalogue, or with the words in a dictionary; or that they may be arranged in various useful orders such as are employed for instance with the words of a sentence. This brings us to a fuller consideration of the linear type of group systems.

Linear Group Systems

A linear multiplicative group system has already been defined as one whose elements are arranged in a line. A straight line itself when divided and subdivided into feet, inches, and fractions of an inch is an example of a constitutive linear system. The division of time into centuries, years, months, days, etc., is a similar instance; a series of dots (or any separate objects) when arranged in a line and combined variously may form a linear constitutive or a linear marginal group system, according to circumstances. Important somewhat general instances of this class are the number signs 1, 2, 3, 4, 5, etc., written in order, or the alphabet A, B, C, D, etc. It is often an advantage to treat by the linear multiplicative group method any temporal succession of events, as thoughts in one's consciousness, or acts to be performed during a given period of time as a day or year. Similarly, as was illustrated with reference to the above diagram, it is frequently a source of efficiency to convert a domain of material into this form, as when we arrange words alphabetically in a dictionary and are thus enabled to reuse our knowledge of the order groupings in the alphabet in looking up the meaning of a word.

Quite as important as the linear is what may be termed the *semi-linear,* or approximately linear *system* of multiplicative groups. This is a linear succession of elements each of which is somewhat areal in its nature.

Illustrations of the semi-linear species are a line of print, or more noticeably the pages in a book, a card catalogue, a succession of

states of consciousness, the notation of a piece of music, or the keys constituting the keyboard of a piano.

A group system may be areal in some parts and linear in others.

Two Principal Types of Group Systems

Hence, in general we find two principal types of multiplicative group systems:

The first is, in the main, spatial, logical, deductive, full of uniformities, abstract, *a priori,* and largely constitutive.

The second is dominantly temporal, genetic, branching, strandal, full of diversity, concrete, *a posteriori,* inductive, and largely marginal.

The federal system includes the essence of both of these types.

Complex Group Systems

Multiplicative group systems may be combined and complexed in various ways. A simple instance is the combination of the root system and the branch system of a tree by means of a common trunk. Of like nature are any group systems of intake with the corresponding system of output, or related systems of induction and deduction. Also two or more systems may be combined and used in more or less parallel fashion as an aid to each other. Thus we have the multiplicative mechanism of a clock, corresponding to the multiplicative division of time into parts. Another illustration is the combination of an alphabetical name list and a number list to denote the employees of some large organization.

Manifold Mutiplicative Grouping

An important source of efficiency in manipulating a domain of material is a manifold multiplicative grouping of the given domain. A simple example of this is the multiplicative grouping in one way of the parts of machine for shipment; and of the same parts in another way, when the machine is erected for use. So the contents of a book may be grouped multiplicatively in different ways, as by chapters and also by an index. The words of a language may be grouped multiplicatively either as parts of speech, or etymologically, or alphabetically, or with respect to likeness of meaning as in a thesaurus.

Similarly the parts of a country are often grouped in various

multiplicative ways, as by political divisions, by the use of latitude and longitude, according to elevation and drainage, with reference to railroad or other transportation lines; by rainfall, climate, resources, population, culture, and other categories. So plants may be classified according to their uses, by lines of descent, or by the technical methods used by botanists, or according to the color, size, and season of appearance of their flowers.

A combination in any given business of what are called line organization and staff organization is an important example of manifold multiplicative grouping.

Similar illustrations might be cited without limit. For so simple an object as a straight line may be divided multiplicatively in an endless number of different ways.

Manifold Reciprocal Group Systems

Manifold multiplicative grouping sometimes takes a form which may be termed reciprocal. For example we may form a family tree by taking an ancestor and tracing his descendants; or we may form a family tree which spreads in the opposite direction by beginning with a descendant and tracing his ancestors back through preceding generations. When the same material is treated thus in two opposite directions, the result may be termed a dual reciprocal system. Such systems may exist with respect to a set of objects and their qualities, between forms and substances, generals and particulars, structures and functions, and other similarly related pairs of categories.

An *n*-reciprocal group system is also possible. Thus in a domain of *n* elements each element may be made the starting point or apex of a multiplicative group system which includes most or all of the given *n* elements. This *n*-fold reciprocal multiplicative relation prevails more or less among all the individuals in a civilized community; among a number of cities connected by a network of railroads, telegraph, or telephone wires. It exists among the nerve centers or even the cells of the human body; and indeed, in certain ways, among all the atoms, ideas, and entities of the universe.

An important instance of the species under consideration is that of scientific formulas, or formula-equations, such as $s = \frac{1}{2} gt^2$, each letter of which may be given the lead in turn. The result is other formulas, as $g = \frac{2s}{t^2}$, and $t = \sqrt{\frac{2s}{g}}$, each of which represents a useful aspect of the matter in hand.

Rectangular Arrays

Another important case of the n-reciprocal class of multiplicative group systems is that of a rectangular net work or array of any kind, as the rows of corn in a field, the trees in an orchard, a set of pigeon-holes, the streets in certain cities, the fibres of woven cloth, or the strands in a piece of wire netting. The manifold multiplicative nature of such arrays is illustrated by the fact that when pressure is brought to bear on any point in a piece of woven cloth or netting, such pressure is met by a special multiplicative system of cohesion radiating from the point of stress.

Some combinations of group systems are so complex that their complete analysis into component systems is difficult if not impossible. As an example of this we have rays of light radiating from a center, reflected or refracted by different objects, and variously used or transferred by them. Still more complex is the organization of the human body and mind, or the development of events in human history. It is to be noted however that in all such complex systems there appear spots and aspects of more or less explicit multiplicative structure which are sources of important efficiencies in many ways.

EFFICIENCY PROPERTIES OF MULTIPLICATIVE GROUPS

Since the multiplicative group principle is an extension of the group idea and hence of the reuse and unit and multiplier principles, it has the useful properties of these preceding instruments of efficiency in an extended and pronounced form. A system of multiplicative groups also has certain efficiency functions peculiar to itself and characteristic of its own more developed nature.

Economics due to Multiplicative Groups

An application of the multiplicative group principle naturally carries with it a greater economy of material than a single act of reuse or the employment of one group. For instance reusing a word by means of a prefix saves the invention of one new word; but the use of prefixes and suffixes in a multiplicative group way in connection with a word stem takes the place of the formation of a multitude of entirely new words. In the machinery used in transfering power in a factory, one large wheel may be replaced by a number of small wheels, geared tandem and having an aggregate

weight which is only a fraction of that of the large wheel. This economy of material in the wheels leads to an economy of space also and hence of material and grounds for buildings. A railroad system which connects a number of cities and smaller places is usually constructed as a trunk line with a number of branches and subbranches. It is easy to see that such a system requires less roadway and material of practically all kinds than would be required if each station on the whole road system were connected directly with every other station. The same principle of economy of material is found in a system of telegraph or telephone lines, and in the blood vessels and nerves of any organism. In fact this economy appears in characteristic ways in each of the species of group systems which have been discussed.

The use of a multiplicative group method and system may also be accompanied by a notable economy of energy. There is a saving of strength in speaking to a number of persons assembled as a group; much greater then is the economy which results when a message is conveyed through a large system of groups as in the opening illustration of the chapter.

If the words in a dictionary were not arranged in alphabetic and therefore linear multiplicative group order or in some other systematic way, in looking up a given word we should need to look at every word in succession till we came to the desired word. Hence on the average, we should need to examine one half the words in the dictionary, and therefore go through, say, 150,000 items of work instead of 5 or 6.

Similarly if the population of the world were not grouped multiplicatively in some way as in that indicated by the address on a letter, a messenger sent with a letter to any particular individual would need to interview, on the average, one half of the people in the world before finding the desired person.

By like methods it may be shown that the use of multiplicative groups often leads to economics of time, space, money, and resources of all kinds.

Increased Power due to Multiplicative Groups

The use of systems of groups leads not only to diminished expenditures but also to an increase of results. For instance by the

use of organized methods the products of a farm may often be greatly increased. By facilitating the consultation of encyclopedias and journals, the multiplicative group principle leads to an enlarged reuse of the work of others by any one investigator and hence to an augmentation of results. So in general by making communication and transportation of all kinds easy, the systematic application of the group principle increases the volume and elevates the character of reuse and multuse.

A system of multiplicative groups, for many purposes, may be regarded as a single group of a high order with the efficiency properties as already stated for the single group correspondingly increased. Examples are the electric lights in a city, the soldiers in an army, or the citizens of a country.

Group Systems Produce other Efficients

Multiplicative group systems are also sources of efficiency in more indirect ways. These may be better realized if we anticipate to a slight degree some of the principles of efficiency to be investigated more fully later. Thus multiplicative groups often produce the uniformities or equivalences (see Chapter VIII, p. 130) which are the basis of many new single groups or organized groupages. This may happen in different ways as

(1) By the extended propagation of original data;
(2) By discovery of the fact that different domains contain or are governed by the same group systems;
(3) By virtue of the power possessed by an infinite series of filling in a gap between two widely separated or contradictory objects.

These uniformities not only give rise to new groupages but to other results which will be presented in Chapter VIII (p. 130). An illustration of these is the fact that the recognition of a common multiplicative group essence in a tragedy, a symphony, and a system of government may lead to much marginal reuse in acquainting oneself with these, and obtaining pleasure and profit from them.

Multiplicative groupages are also sources of diversity in its various forms and hence lead to the fruits of diversity, such as delicacy of motion, tact, new units, etc. For instance all machinery

which acts with fine gradation of power will be found to be characterized by a well developed multiplicative structure.

Other similar illustrations are the deliberation and relative wisdom with which a federal government acts as compared with the suddenness and passion of a mob or even, in many cases, of a pure democracy. Likewise a multiplicative grouping of religious beliefs makes possible a unity of view in certain prime or first group matters while leaving freedom as to details, and in this way produces efficiency results of a high order. An illustration of a different kind is the diversity of function produced by the multiplicative group structure of a wire netting. In such a structure the economy of material is so great that space is left through which the air may freely pass, while flies and mosquitoes are excluded.

A multiplicative group system also often opens the way for certain advantageous directive processes (see Chapter XI, p. 192), as for substitutions and exchanges. Thus if the process of making a chair or any other article be grouped multiplicatively, certain parts of the process may be performed by machinery instead of by hand, with a great saving of time and labor.

So certain primal parts of a physiological or mental process may become automatic or instinctive; that is, be performed by force of a low order or even automatically.

Results Not Otherwise Attainable

The systematic use of the multiplicative principle frequently brings certain remote and unanticipated results. Some of these are often unique and apparently otherwise unattainable. Thus in the solution of a mathematical problem of a certain kind the 100,000 or more guesses and tests which would be necessary if no systematic method were followed may be reduced, for example, to 30 used in successive groups of say 5 each. Since a solution by the former method is practically impossible such solutions as have been obtained in the way indicated may be said to be due entirely to the multiplicative group principle. Similarly air thus far has been liquefied only by a stepping down of temperatures. So on a vastly larger scale, if human activity had not been highly organized the existing triumphs of art, engineering, and science would never have come into being.

Complex Results

Evidently the above benefits of group organization may be complexed in many ways. As an illustration we have the varied and manifold fruits of good government.

Values in Terms of Human Progress

The efficiency value of the multiplicative group principle, like that of the other Efficients may be presented in certain general and more or less pictorial ways. For example we have the fact that human progress has been characterized and largely determined by improving multiplicative group systems. This is true not only of civilization as a whole but also of each department of life and knowledge. Thus for example mathematics consists mainly of a multiplicative grouping of space and quantity; psychology of a similar grouping of mind; physical science of a like grouping of matter and force; history of the multiplicative grouping of various data with respect to time; logic and philosophy of a treatment of multiplicative groups as independent of and underlying other categories as far as possible. An improvement in each of these domains has meant an improvement in its multiplicative group essence. The human eye and ear are superior to the other sense organs because the former contain superior multiplicative group systems. Electricity became a distinct science as soon as it was developed into a multiplicative group set of relations among certain primal units and elements.

In human progress the multiplicative principle has also acted in certain comprehensive and sweeping ways. Thus each new discovery has not simply added to, it has multiplied all preceding achievements.

Lack of Group Systems Means Loss

The negative method may also be utilized in realizing the efficiency values of the principle of multiplicative groups. The African negro is not lacking in many unit elements of efficiency, but his life as a whole in his native land is barbarous because he lacks the power of forming any extended group system. Similarly if the principle of multiplicative organization were eliminated from human life in general, the world's affairs would drop back many steps toward chaos.

Familiar Words Showing Value of Group Systems

Another method of glimpsing the world of efficiencies of which the multiplicative group method is the source is that of recalling the benefits connected with particular but familiar forms of multiplicative groups such as organization, system, classification, coordination, correlation, method, and order. Every progressive or step by step process is essentially an application of the multiplicative group principle.

Limitations in Utilities of Group Systems

Before concluding this statement of the uses of systematic groupism, attention should be called to certain disadvantages which often accompany the application of this principle. Naturally the drawbacks connected with reuse or any single group will often appear in an enlarged way when many groups are used. But multiplicative group systems also have peculiar limitations of their own. In the first place a multiplicative group system often means a certain amount of added machinery which, unless it is carefully used, becomes an encumbrance or even a positive evil. Expressing the matter in another way we may say that under certain circumstances it is more efficient to use a single large group than a system of smaller groups.

Also a multiplicative group system may be a highly efficient agent in the propagation of an evil act or unit of any sort. Thus the value of the printing press has been largely negatived by the degrading purposes for which it has sometimes been employed.

An extended articulate machinery is exposed to other evils often highly insidious. For instance in a federal government where the citizens choose certain representatives, and these select other representatives of a higher order, and so on, at each step of the process danger exists of forgetfulness, selfishness, or corruption. So in every multiplicatively organized structure, weakness and danger are connected with every stage and method of articulation. It is the highly organized wheat plant not the simpler grass which is subject to blights and diseases.

Self-protective Power in Group Systems

It is to be noted however that a multiplicative group system contains within itself certain means of correcting or even preventing

its own defects. Thus the expansiveness inherent in a multiplicative system gives a largeness of view and grasp which tends to check the evils specified above. Also the multiplicative propagation of an error by increasing the results of the error quickly calls attention to the error and the need of its correction. Again, in almost every domain of material cross multiplicative group systems are possible which tend to the prevention of error and loss. Thus in forms of highly developed representative government, not only are certain cross-groupings such as the legislative, executive, and judicial systems already in use, but others such as the initiative, referendum, and recall are being adopted.

ANALYSIS AND DEFINITION

We wish now to investigate the ultimate sources of efficiency in the multiplicative group principle in so far as these differ from, or are an advance upon those already found in the single group. The way may thus perhaps be opened for increased efficiencies in the use of multiplicative groupages.

Number, Space, and Like Elements

Owing to the presence of different orders of groups it is evident that in multiplicative group systems the element of number is more prominent than in groups singly treated.

The category of space is also more evident owing to the stress laid on the relative position of the parts of a system and the resulting shapes.

The element of order is here also almost entirely new and with it some prominence is given to the category of time.

More emphasis is likewise laid on the category of force for not only are elements bound together in groups, but groups are held together to form higher groups.

Efficiental Elements

A group system implies a certain uniformity in several ways as among the elements of a group in relation to each other, among the groups which are placed together in a given order, and in the ties which bind elements or groups together. In like manner differ-

ences are supposed in, and are a necessary part of a system of multiplicative groups.

Such a system also implies something outside of itself both in general, and also from the particular fact that an element may be shown to be a node or point in which several categories of utility intersect.

A certain limitation in the nature of things is also implied in a multiplicative group system, for otherwise progress would be by a single instantaneous step, and not by progressive stages. In fact we may regard each multiplicative group system as cut or carved out by an including matrix of limitations.

A system of groups also differs from a single group and from a mere unorganized aggregate of groups by a certain inherent dynamic or dialectic property, which causes it to act both externally by expansion and internally by reorganization as in the formation of improved units, ties and subgroups, or manifold cross multiplicative groupings.

Unique Element

A multiplicative group system like each of the other Efficients contains something which is unique and characteristic; which is perhaps beyond analysis and may be termed transcendental.

Later we shall find that most of the constituents specified above are themselves capable of a similar analysis into components. Our analysis will then become a multiplicative expansion of constituents revealing a systematic array of ways in which the efficiency principle under consideration may be improved.

Definitions

At this point it may be well to make a brief statement of the efficiency characteristics of certain special well known forms of multiplicative group systems.

Classification and coördination differ from multiplicative groups in general in that they are relatively passive and inert.

Organization as such lacks the progressive orderly relation of parts which is prominent in the multiplicative concept.

The same is true, to a certain extent, of the idea of system.

For some purposes, a summary of the results of the above analysis

in the form of a definition is useful. A *multiplicative group system* may be defined as a succession of groups, all the elements in each group (except the highest) being closely connected with some one element in another group (called a group of higher order).

From another point of view, multiplicative groups form the common essence of organization and system, developed with a view to increased efficiency.

METHODS OF APPLICATION

We shall next consider the best methods of utilizing the principle of multiplicative groups as a source of efficiency.

Vast Number of Group Systems

In the first place, as with the other Efficients, it is important to realize the great number of forms which the instrument in hand may assume. The number of different group systems is evidently greater than the number of different individual groups. For not only may the component groups of a system take various forms, but their modes of combination may also differ. It is to be noted in this connection that one of the best ways of realizing and carrying in mind the great diversity of forms which the multiplicative group principle may assume is to arrange these forms in a multiplicative way; that is to apply the multiplicative group principle to itself.

Adaptation in Application of Systems of Groups

Owing to the comparatively elaborate nature of a multiplicative group system, the selection of the best form of instrument of this kind to meet any given set of circumstances is of even greater importance than when dealing with single groups. Thus for instance the invention or selection of the best possible elements out of which to form a group system often requires close study to the end that the resulting instrument may be as comprehensive and manifold as possible in its action. In general small elements or elemental groups are best adapted for this purpose. For example it has been found that the individual rather than the tribe or the family is the best unit for an elaborate social organization. Similarly we have the atom or ion in physical science, and the cell in biology. However, immediate practical limitations and relations are often of such

a nature as to call for the use of much larger units than at first would seem desirable. Thus the fact that most men read on the average but a small part of each page of a newspaper leads to the use of a large page in order that a reader may rapidly extract from the paper what he needs.

As a rule it is best if possible to have the component groups of a system contain as nearly as possible the same number of elements; that is, to have a multiplicative group system governed by a certain dominant number. This is illustrated in the decimal system of notation in arithmetic, in the metric system, and in the use of 12 as a base in certain relations of denominate numbers. In some cases a dual system, especially one consisting of positive and negative elements is highly efficient (see p. 71). A system of triads is often used in composing speeches or works of art, use being made of certain standard triadic groups such as the past, present, future; or data, means, end; or theme, antitheme, combination.

The largest number of objects which a person can grasp simultaneously in his field of vision is said to be 7. Hence perhaps we have 7 colors in the spectrum as it is usually divided, 7 days in the week, and 7 intervals in the octave. These facts illustrate the importance of group systems governed by the number 7. It is to be noted also that the printed page is made up of approximately 40 symbols, and spoken language of 40 sounds. The more the matter is studied the greater is seen to be the desirability of systems based on or utilizing certain standard number groups.

Relative Advantages in Different Species

It is also important to investigate the advantages and disadvantages of each special form of group in other respects, and often to devise special forms adapted to peculiar sets of circumstances.

Thus the federal form (described on p. 73) has certain characteristic useful properties such as the ease with which it may be grasped visually and changed in case of need into other forms.

In like manner the linear species has certain efficiency functions peculiar to itself. In general it corresponds to the linear habit of consciousness and our method of dealing with objects in time, and its use leads to reuse of those habits. Properly conceived such a system may be readily expanded or contracted and modified in

other useful ways. These advantages are illustrated for instance in a card index.

The infinite series is useful in bridging gaps, in correcting or preventing errors, and in other more technical ways.

Evidently also manifold multiplicative group systems, including parallel and reciprocal forms, have many special uses. These functions may be negative, as in preventing misuse or overuse of any one form, or positive as in supplementing each other's defects or increasing each other's efficiency. Thus when the chemical elements are systematically grouped not only according to their atomic weights, but also with respect to their spectra, and with regard to their useful properties, many checks on error and many aids to increased efficiency result.

Frequent Value of Even Crude Systems

In a difficult situation the value of even a crude multiplicative group method should not be overlooked. It is also important to remember in such connections that in view of the large number of ways in which groups may be formed, some sort of organized group instrumentalism is always possible. Thus in scientific investigation a working hypothesis, however temporary and provisional, prevents much aimless groping and frequently, indeed, seems necessary to the attainment of definite results. So in another field, the Puritan theology, however crude and imperfect it may have been, at least furnished a comprehensive and vigorous system, and hence perhaps gave the New England character its sturdy and aggressive qualities.

One is aided in using relatively imperfect systems with some boldness because of the fact that the multiplying principle is not only self-corrective, but also self-developing.

Transformations of Multiplicative Group Systems

The power to transform one species of multiplicative group system into another is often an important source of efficiency. Frequently several successive transformations of this kind must be made to meet successive changes in a situation. The different evolutions of an army during a battle or a campaign are an example of this process.

Standard units or types of transformation of multiplicative group

systems may often be employed to advantage. An instance is the conversion of a fan-shaped into a linear system. Another case is the transformation of a linear system into a rectangular array (as of pigeonholes) for the sake of economy of space and as a means to useful cross multiplicative group processes.

Similar conversions are that of a three dimensional system into a two dimensional one; a continuous into a discrete form; constitutive into marginal; heterogeneous into homogeneous; and the reverse of each of these. Perhaps also it may be said that any one kind of multiplicative group system may be converted into any other.

Important special cases are the various methods of classifying persons by means of their finger tips, with a final reduction of a branching system of classified persons into a linear card catalogue form. A process like this is sometimes termed a lexiconizing of data.

Symbolisms for Group Systems

By the principle of neighborhood or positional symbolism to be developed later (see p. 167) important special abbreviations are often obtainable in representing a multiplicative group system of some particular kind. Thus in such representations the symbols for the higher groups in a system may frequently be omitted. For example in the address on a letter, the name of the country is often not expressed, and that of the continent almost always omitted. The great power of the Linnean system of classifying biological species is due not only to the fact that it is a multiplicative group system, but also to the property that, owing to its multiplicative nature, all the names involved except two, viz.: those for genus and species, can usually be omitted. Similarly the name for each human being usually consists of only two or three words, and often only the initial letters of one or more of these words are used.

The judicious use of symbolisms not only brings negative advantages in the shape of economies of various kinds but also a dialectic of positive advantages to be presented more fully in the chapter on Symbolism (p. 164).

Calculus Methods

The simultaneous use of successive transformations and of adapted symbolisms together suggest the idea of creating certain group sys-

tems by calculus methods. Such methods consist essentially in obtaining results in a more or less mechanical way by the aid of adapted symbolisms. Thus a problem in grouping like the following may be proposed and solved. If 250,000 species of plants are classified as a multiplicative group system of 9 orders, on the average how many elements will each group in the system contain? The solution of the problem reduces to the solution of the equation $x^9 = 250{,}000$ and the result obtained is $x = 4$.

Every slide rule represents what is in essence a multiplicative group system, and the use of the slide rule is largely a mechanical method of manipulation a multiplicative group system.

A study of the efficiency nature of logic and mathematics will show that these subjects constitute particular forms or applications of multiplicative group principles, and that the calculuses in them are therefore illustrations of the calculuses possible in dealing with group systems in general.

Ideal Species in Multiplicative Groups

Finally in this connection attention should be called to the importance of keeping in mind at all times the ideal system of multiplicative groups and of realizing this ideal as nearly as possible in all cases. From our point of view this ideal species is a multiplicative grouping of the Efficients, by means of the Efficients, and for efficiental ends.

At the same time it is not to be forgotten that for immediate practical purposes the best general form is the two-dimensional federal system, accompanied by some linear species as an auxiliary.

From many parts of the preceding discussion it is evident that the application of the principle of multiplicative groups will be aided in various ways by a study of other primal efficiency agents in the chapters which follow.

Summary of Chapter

It will be an advantage at this point briefly to state results arrived at in the present chapter. Simple groups may be combined to form groups which are multiplicatively related in various ways. The different species of group systems are characterized mainly by their forms. Among the most important of the individual forms are those termed linear, federal, constitutive, marginal, and manifold.

The efficiency functions of group systems are those of single groups in a more extended form, with other useful functions added. Among these uses are economy and delicacy of action, and great intensity of force. Group systems also give rise to or aid the other Efficients in peculiar ways. Multiplicative groups also form the underlying essence common to all kinds of system, organization, and coördination. Hence a study of multiplicative groups should give a thorough mastery of various forms of system and organization.

In making use of group systems, besides realizing their great number, and methods of adapting them to special situations, it is important to understand how to use several species of systems in combination, and how to transform one species into another. Of like importance is the use of group systems in connection with other Efficients. The ideal group system is that one in which the component elements are the other Efficients.

EXERCISE 5

Give an example of each of the following species of multiplicative group systems:

1. Of three orders
2. Of more than three orders
3. Linear
4. Areal
5. Three dimensional
6. Provisional
7. Permanent
8. Direct
9. Inverse
10. Heterogeneous
11. Homogeneous
12. Constitutive
13. Marginal
14. Continuous marginal
15. Discontinuous marginal
16. Rectangular
17. Dual reciprocal
18. N-reciprocal
19. Semi-linear
20. Conically expanding
21. Parallel
22. Manifold
23. Complex

24. Name three natural multiplicative group systems.
25. Also three artificial ones.
26. Also three which are part natural and part artificial.
27. Describe some system of filing documents and explain this as a form of multiplicative groups.
28. State the different multiplicative group organizations found in a large hotel.
29. Also in a large department store.
30. On a large well managed farm.
31. Explain the multiplicative group system used in dividing the Bible into parts so as to make them readily accessible.

32. State the advantages in having the human arm jointed; that is, formed of articulate parts.

33. How is the slowness with which a clock runs down obtained and what are the advantages in this slowness?

34. Why is it that a number containing only six figures is able to express the result of the most accurate measurement that man can make by the aid of appliances thus far invented?

Give an example of a multiplicative group system which is a means to:

35. Economy of material
36. Economy of space
37. Economy of energy or force
38. Reuse
39. Uniformity of action
40. Delicacy of action
41. Substitution of a cheap force for a costly one
42. A transcendent result

43. Give an illustration of the drawbacks which sometimes accompany the use of multiplicative groups.

44. Napoleon compared his mind to a chest of drawers. State some of the advantages in a mind disciplined so as to be usable in this way.

45. Is it correct to state that an organism is high in the scale of being in proportion to the degree of its multiplicative group organization? Illustrate.

46. It is said that the commanding general of an army sometimes carries in mind and is able to recognize individually 100,000 soldiers. Show how systems of multiplicative grouping may aid in this process.

47. What are the advantages in influencing a community by influencing its leaders? In correcting the evils in a community by correctig the causes which underlie these evils.

48. State some of the various cross multiplicative groupings used in keeping a set of business accounts. What are the advantages in this manifold grouping of accounts?

Give two instances where a multiplicative group mechanism gives

49. Concentration of force
50. Concentration in time
51. In space
52. A high velocity
53. Slowness of motion

54. Why does it pay a railroad system to develop its trunk lines more thoroughly than the branch lines?

55. Show that the efficiency of the insurance methods used in business arises essentially from a multiplicative grouping of values and a surrender of certain of those of low order that those of high order may be made certain.

56. To which species of multiplicative groups does that belong which is formed by the banks outside of New York City, their correspondents in New York City, and the New York City clearing house? Explain the efficiency functions of such a system.

57. The so-called golden section has been suggested as a fundamental principle in esthetics. Show that this principle is essentially a special application of the multiplicative principle.

58. State some of the advantages which result when the materials in the

store room of a factory are arranged according to a multiplicative group system.

59. State some of the advantages which have resulted from the demonstration that all material objects may be regarded as composed of a few chemical elements grouped together multiplicatively in various ways.

60. A classification of the compounds of the elements of carbon, by the gaps which it revealed led to the prediction and discovery of certain hitherto unknown compounds. What species of efficiency result was this?

61. Show how the multiplicative grouping of knowledge is an aid to a person both in learning something about everything and everything about something.

62. State the symbolism for the different orders in a multiplicative group system used in the following: $3^\circ\ 7'\ 5''$.

63. Give an example of the transformation of a linear multiplicative group system into an areal or solid one, for the sake of economy of space. What other advantages result?

64. Give an example of the transformation of a marginal multiplicative group system into the strandal form, and point out the advantages which result.

65. If 500,000 species of plants are classified as a multiplicative group system of 8 orders, how many elements, on the average, will each of the component groups of the system contain?

Trace the historic development of multiplicative groups in the field of

66. Education 67. Politics 68. Business

CHAPTER VI

ORDERS OF MATERIAL

Illustration

It is often a source of efficiency to substitute for a given multiplicative group system a series of objects each of which represents an entire order of the given group system. For example it is frequently convenient to represent an ecclesiastical hierachy by the series, layman, priest, bishop, archbishop, cardinal, pope, or by part of this series.

Uses of a Multiplicative Series of Objects

Such a representative set of terms is particularly useful in cases

(1) Where it is difficult to form or discover an explicit and well developed multiplicative group system;

(2) Where a group system has been so highly developed that it may be made to assume a more special and peculiarly efficient form.

In these latter cases successive multiplicative degrees of groupism have become so familiar as to have acquired the aggregate of useful qualities which we often denote by the term materiality or substance. Hence a single object may be used for each order of groups, the result being a certain mechanical ease and simplicity in dealing with organizations and systems, though the process is often accompanied by compensating limitations and drawbacks.

In this way we arrive at a new principle of efficiency, broader in some ways than that of the multiplicative group instrument and narrower in others. This new instrument of efficiency we term multiplicative orders of material, or more simply orders of material.

CLASSIFICATION

Classification According to Materials

Multiplicative orders of material will be first classified according to the material in which they occur or of which they are composed.

The first to be considered are those multiplicative series found in

the physical *inorganic* world. An example of such a succession is that formed by the ion (or electron), atom, molecule, crystal, and a body in mass. It is important to observe that in this case as in many others, the progressive many-to-oneness may be regarded in several aspects or as the resultant of several elements, as those of size, number, force, and various functions. Another important general multiplicative order in the inorganic world is that of solid, liquid, gas, ether. This also is the resultant of several elements of successive groupism.

The above orders of material are found, in whole or part, in practically every domain of inorganic material. Each field of work in the physical universe also usually furnishes one or more multiplicative series peculiar to itself. Thus in the domain of astronomy are found satellite, planet, sun, constellation.

Those multiplicative series which occur in different specific fields of toolage are particularly important for our purpose. Thus in water transportation we have the raft, rowboat, sailboat, steamboat, with subordinate multiplicative progressions connected with each of the given terms. The resultant series involved may here be regarded as one of general efficiency.

Of implements used in breaking up the ground in agriculture, we have the hand, hoe, spade, plow, gangplow. Of instruments used in reaping grain the multiplicative succession is the hand, knife, cycle, scythe, cradle, mowing machine, reaper, reaper and binder. Other fields of work furnish like instances.

In the domain of *living organisms* various important series of objects multiplicatively connected are also found or have been constructed by man. Among the instances which might be cited are cell, organ, individual, society; bone, flesh, blood; annual, biennial, perennial plants; individual, species, genus, family, order, class, kingdom.

More Abstract Series

In the *region of mind,* as to cognition the succession is perception, memory, reason; as to emotion, the list is happiness, joy, blessedness, In the field of will we find impulse, volition, determination; in that of moral character occur physical courage and moral courage; or act, habit, character, destiny.

The many multiplicative group systems in the *social* and *political* world give rise to corresponding multiplicative series of objects. Thus in connection with the feudal system are found serf, burgher, knight, duke, earl, prince, king. A system of law courts at the present time is presided over and to a certain extent represented by the police justice (or justice of the peace), and by circuit, superior, and supreme judges. Many other social and political multiplicative gradations will suggest themselves to the reader.

Similarly multiplicative series are in use in various more or less *abstract* worlds. Hence we have for instance such a series with respect to position, or culture, or usefulness. In this general class the most important series for our purpose in many respects is one of degrees of efficiency. For instance a political worker at first must usually expend a large amount of effort and be satisfied with small returns. Later, after he has acquired position, he may obtain much larger results by the mere utterance of a word; and later he may produce still larger results by mere unconscious influence.

The most comprehensive and in many respects most useful multiplicative series in the world of objects is the complex resultant one denoted by the terms mineral, vegetable, animal, spiritual.

Classification According to Form

Often also it is an aid to efficiency to classify orders of material according to categories of form.

Thus some principle of space, as that of magnitude for example, may be so prominent as to give the series a certain characteristic individuality. For example we may have a multiplicative series determined mainly by the element of *size,* an example being bay, gulf, sea, ocean. The principle of relative magnitude fixes the different grades of anthracite coal, viz., rice, pea, chestnut, stove, furnace.

Similarly we have the different standard sizes of various articles of merchandise and the materials used in processes of manufacture. These gradations are sometimes due to natural causes but more often to the commercial process called standardization.

Important special cases of quantitative series are the tables of compound numbers as taught in arithmetics. For example in money we have the mill, cent, dime, dollar, eagle. Other like series are used in the measures of weight, length, area, volume, etc.

As to *geometric form* a multiplicative succession of objects is essentially a linear succession of points. Owing to the occurrence of secondary, or auxiliary, or parallel series the geometric form of the series as a whole may be slightly branching (or semi-filiar). An example is the series,

creeping, walking, riding by $\left\{ \begin{array}{l} \text{horse, trolley, railroad, steamboat,} \\ \text{bicycle, motor-car, aeroplane.} \end{array} \right.$

We here approach compound and complex orders of material.

Multiplicative series of objects may also be characterized by the element of *number*. Thus a dual series, that is one composed of only two objects is often comprehensive and important. Examples are the pairs of terms, appearance and reality, reputation and character, mind and matter, form and substance, relative and absolute.

Important triadic forms also occur. Instances are past, present, future; data, means, end; historical, psychological, logical.

The series, simple, compound, complex, is an important one, determined both by principles of number and by spatial arrangement.

A *time* element, as that of relative permanence, may also form the most essential characteristic of a multiplicative gradation of objects. For instance in law a written document is on a higher plane of importance than verbal statement, because written words are more lasting than mere verbal speech. So degrees of precedence in time may, in effect, constitute a multiplicative series. Thus we have the legal maxim that possession is nine points of law. Similarly preventive work is of a higher order than that which is merely remedial.

A *segregative series* is one which has arisen by the gradual elimination of sizes of forms of objects lying between the objects constituting the series. An example of this is the scale of sizes of type now used in printing.

Many other terms of *quality* may be applied to various orders of material. Among these are the words homogeneous, heterogeneous; natural, artificial, semi-artificial; positive, negative; increasing, diminishing; direct, inverse; contingent, absolute.

Some special explanation should be made of the last pair of terms in the above list. Owing to the shortness of human life and other species of limitation, the gap between two successive terms in a progressive series may be absolute or infinite from the efficiency

point of view. Thus in grading various specimens of a given kind of fruit, as of apples, all above a certain size may have a market value while all below have no market value. So in a scale of temperatures, all within a certain range may mean life, and all above the limit of this range may mean death. All the events in a certain future may be accessible and under control, while all in the past are beyond control.

Complex Cases

Complex orders of material of many different kinds occur. Some resultant species have already been mentioned and illustrated. Another instance is that composed of the materials used in making a fire, as match, paper, kindling wood, and coal, the component factors being size, inflammability, duration and others less conspicuous.

Similarly the stages of formal education denoted by the terms primary, secondary, collegiate, university, constitute a multiplicative series which is a resultant of such factors as the age, growth, and maturity of the pupil, and the difficulty and comprehensiveness of subjects of study.

Probably every multiplicative series is in some respects a resultant of a number of such series.

Often one multiplicative series give rise to another. Hence parallel series may occur. Thus side by side with the arrangement of bodies as solids, liquids, gases, and the ether, we have the senses of touch, taste (and smell), hearing, and vision.

Groups of Orders of Materials

Various orders of material may constitute the elements in a multiplicative group system. Thus the different series in the domain of psychology may be arranged as a multiplicative system.

In like manner it is often a source of efficiency to develop in a given domain of material a manifold multiplicative cross structure of objects. For example the different kinds of money in use may be viewed in various serial ways. Thus with respect to relative bulk we have copper, silver, gold, paper. According to security the order is gold, silver, copper, paper. Another arrangement could be formed according to steadiness of value during a given period of time. Similarly the different ways in which the facts in

a department of knowledge may be presented are denoted by the terms genetic (or historical), psychological (or inductive), and logical (or deductive), and a multiplicative series may be developed corresponding to each of these terms.

Different species of multiplicative gradations may also be combined and complexed in various more or less irregular ways. An illustration is the aggregate of such series utilized in an entire process of education.

General Multiplicative Series

The following tabular combination of multiplicative series is of far-reaching importance because of the comprehensiveness of its terms and also because of its specific applicability to many special cases by aid of slight variations.

General Multiplicative Series	Objective Side	Subjective Side
Material	Iron, wood, etc.	Muscular sense
Concrete	Sound, color, ether, mechanical and biological force	Vision
Semi-concrete	Mind as object	Mind as subject
Abstract	Civilization, religion, etc.	Reason
Efficiency-abstract	Categories, Efficients	Efficiency powers
N	N′	N″

The first column of the above table contains a general succession which is the resultant of a large number of efficiency factors to be considered in some detail later (see p. 105). Besides the efficiencies which spring from the direct use of the first column, other useful results are obtained by noting that the actual use of this column involves the use of two sets of factors, usually termed the objective and subjective respectively. Hence it is useful to develop these into two other parallel sets as given in the second and third columns.

It should be observed that the series mineral, vegetable, animal, spiritual, also corresponds in a general way to that given in column one.

EFFICIENCY FUNCTIONS OF ORDERS OF MATERIAL

Uses Compared with Those of Multiplicative Groups

Since orders of material are a modified form of multiplicative group systems, it follows that the uses of orders of material are closely related to those of multiplicative groups. In general the efficiency functions of orders of material are in some cases those of incipient and imperfectly formed multiplicative group systems, and in others those of final or highly finished systems.

Thus it has already been pointed out that in a field of great difficulty, where an explicit group system cannot be found or developed, it is possible to form an order of materials which will perform many of the efficiency functions of a group system. For instance in the series denoted as solids, liquids, gases, and the ether, complete grasp of the precise and fundamental group inclusions involved is lacking, yet the multiplicative relation approximated in the series is the source of a vast number of utilities and efficiencies. Probably if the multiplicative principles contained should be discovered and fully explained, the series as stated above would still continue to be a convenient instrument by which to handle the complex of group inclusions involved.

Economics due to Orders of Materials

The economy of material which characterizes multiplicative group systems is even more noticeable in orders of material. For in the latter that large fraction of a group system which is covered by an entire order of groups is represented by a single object. Thus in the series, individual person, man, mammal, etc., the word man not only includes the 1,600,000,000 human beings now in the world but all past and future generations of men.

Also the use of a multiplicative series of objects means an increased economy and efficiency of energy as compared with the use of a corresponding multiplicative group system. For owing to the concretish nature of the terms of a series of objects, such a series may frequently be operated by a lower and cheaper order and a smaller amount of force. In other words orders of material are characterized by their extreme triggerishness. The terms of such a series constitute handles or palpable means by which to manipulate a group system to advantage. In other words, in this respect, an order

of objects is a simple mechanical tool by which to handle a complete far extended group system.

Other Characteristic Efficiencies

Orders of material often lead to efficiency in ways perculiar to themselves. The properties of a series of objects formed by standardization are an example in point. For example the formation of certain standard parts in the manufacture of automobiles has led to large economies in the manufacture of each of those parts, and often to important improvements in the quality of the parts. Similarly the separation of freights into different groups of materials to be carried by fast and slow freight trains and by water produces important economies and efficiencies in the carriage of each of these classes of material.

Orders of material also often possess in distinctive form and in larger measure certain other efficiency properties of multiplicative group systems such as the power of self-development, of utilizing auxiliary quantities, and of opening the way to directive work of a higher order.

Fruits stated Collectively

The efficiency value of orders of material may also be presented in other more concrete and pictorial ways. For instance it may be shown that such series are at work in every highly developed department of life or action. The building material called concrete consists of the triadic series, gravel, sand, and cement. A well balanced system of diet consists essentially of starches, fats, and protein. Every great work of art contains the principle of multiplicative gradations in one or more forms. Poor writing, as Shakespeare says, is "words, words, words", that is, a mere additive aggregate; while good literature consists of words and ideas in multiplicative relations. For example the following couplet from Browning largely gets its power from the two multiplicative series of two terms each which it contains:

"Be a god and hold me with a charm;
Be a man and fold me with thine arm."

The efficiency in life insurance comes from the fact that it trans-

fers money (or resources) from a lower to a higher stage of utility; that is, from the field of luxury to the field of necessity.

Values in Terms of Human Progress

Hence also it will be found that great men are men who have known how to work with or in material of a high order. In war the superior general by a simple stroke of strategy overwhelms the mere tactician however clever. So the statesman dominates the politician.

In fact we may say that the whole of human progress has been an advance from stage to stage in multiplicative series fashion. Some of the more important of these stages are indicated in the following table:

Stages in General	Characteristic Industry	Material for Tools
Savage	Hunting	Wood, bone, stone
Barbarous	Grazing	Copper, bronze
Semi-civilized	Agriculture	Iron, steel
Civilized	Manufacture	Electrical machinery

Human progress has also been marked by the use of an increasing number of multiplicative series and by series of a progressively high order.

Limitations in Utilities

Along with the advantages which come from the appeal to our sense of materiality or thinghood in orders of material, come also certain disadvantages which must be reckoned with.

Thus orders of material are apt to harden or fossilize into tyrannical forms which limit the user of them by their premature finality. An extreme instance is the caste system of India. A more subtle and pervasive case is the use of the distinctions between solids, liquids, and gases as ultimate and complete. A milder form of this disadvantage is merely a vague lack of flexibility.

Also the brevity and extreme inclusiveness in the terms of a multiplicative series of objects often leads to certain oversights or distortions in the application of such a series.

In other words the multiplicative series as a source of efficiency is vivid and triggerish, but often crude and narrow. It has the defects of its qualities.

The orders of material which characterize our system of jurisprudence furnish illustrations of what has just been said. The careful gradation of crimes and the formulation of a certain succession of definitions, precepts, maxims, and laws, are most useful where so many varied, conflicting, and often selfish interests converge as in the realm treated by jurisprudence. On the other hand many of the distinctions and standards involved are more or less artificial and leave the way open to numerous errors and abuses.

ANALYSIS AND DEFINITION

Comparison with Multiplicative Groups

In orders of material as compared with multiplicative group systems, ties and forms recede into the background while unit elements become prominent.

Also the concept of number is more specific and distinct, as is also that of order.

With respect to force or energy, we may say that the multiplicative series is more static and passive; that is, it constitutes a near and familiar but not always aggressive toolage.

Appearances often Deceptive

It is also to be carefully noted in this connection that an order of objects, while it is essentially multiplicative in nature and action, is often additive in appearance. Thus when we climb a mountain our steps are additive in that each step increases our elevation above the sea level by the same amount; but at the same time each new step increases the area of vision in a multiplicative way. Similarly equal increments in temperature may produce multiplicative changes in any given material. If the radius of a circle be divided into equal parts the areas of the corresponding circles form a multiplicative succession, and a like property holds for the sphere and other solids. Hence it is important to remember that vital efficiency properties, not mere outward appearances, determine whether a given set of objects form a multiplicative succession. We here glimpse from another point of view the fact that the multiplicative

series contains its own characteristic sources of efficiency and hence possesses a characteristic uniqueness and individuality.

Each particular form of multiplicative series is capable of its own special analysis. The dual series denoted by the terms concrete and abstract is so important and frequently used that it seems desirable to try to determine its efficiency meaning as exactly as possible. This is best done by considering the relation of the terms concrete and abstract to certain other more or less elemental categories of efficiency.

Meaning of Concrete and Abstract

With respect to mind, the concrete relates primarily to the senses, the abstract to the reason.

In relation to force or energy, the concrete is that which has been thoroughly assimilated and reduced to the mechanical trigger state. Thus we cannot help seeing a red color, or feeling a pain when touched by a piece of very hot iron; the color red and hot iron are accordingly termed concrete.

With regard to space the efficiencies embodied in the concrete are more special, local, and individual.

With respect to uniformity, the abstract possesses in higher degree special properties of continuity, uniformity, and equivalence, as will be indicated more fully in Chapter VIII.

In relation to the group principle, the concrete tends to form the elements, especially those of lower order, and the abstract to form elements and groups of higher order and ties of all groups but more particularly those of higher order.

With respect to limitation, the concrete is more subject to limitation as we ordinarily understand the term. Thus, for instance, that species of limitation which we call impenetrability is prominent in material and concrete things. Two material objects cannot occupy the same portion of space at the same time, but two geometric diagrams may do so. In like manner a material object cannot be in two places at the same time, but it is possible for an idea to be simultaneously in a multitude of places. An acre of land cannot be owned simultaneously by two persons, but the same view of life may be possessed by many persons.

In that more advanced or pronounced form of the abstract which

we have termed the efficiency abstract, the entities involved are still freer from limitation than those included in the conventional abstract, and consist essentially of the Efficients themselves.

In the table on p. 100, by N we mean any possible sources of efficiency beyond the Efficients themselves. Hence from our point of view the term metaphysics means sources and modes of efficiency beyond concrete, physical, or phenomenal ones. Hence N stands for any possible and as yet unexplored region of efficiency.

Definition

Under certain circumstances, the following general definition will be found useful. By the term *multiplicative order of objects* is meant a series of objects each of which has a certain multiplicative relation to the preceding object in the series.

METHODS OF APPLICATION

Number of Species

The order of materials and multiplicative group principles are so closely related that much which was said in the preceding chapter concerning the best methods of using multiplicative group systems applies also to orders of material. Thus the great number of different multiplicative series of objects is evident from the fact that possible systems of multiplicative groups are so numerous.

Adaptation to Individual Situations

It also follows that it is important in given circumstances to try to determine the best available gradation of materials and often, if possible, to discover and utilize some hidden series of this kind in a given field of work. For example, in taking physical exercise it is an advantage to realize that gradations of efficiency exist in methods of exercise, and that the most effective forms for mature persons are those which with least effort produce a free circulation and change of position of the fluids of the body.

To use another illustration, in an approximately rectangular room the furniture and ornaments may be arranged in a variety of artistic ways, but in a room of irregular shape they may usually be arranged only in one best way. Hence the former shape is broadly speaking on a higher plane of excellence.

Emphasis on the Higher Terms in a Series

In using orders of material, in almost all cases it is a source of increased efficiency to lay particular stress on the higher terms in the series. Hence arises the prime value of knowing and controlling centers of influence, strategic points, psychological moments, underlying causes, means of communication, and transportation routes.

Hence also the importance of the love of art for art's sake, or of truth, or of any source of efficiency for its own sake.

Hence again the supreme importance of clinging to fundamental ethical or other principles in times and places of unusual difficulty.

This principle of efficiency is also applicable to many of the details of life. Thus the wise traveller frequently makes sure that his money is in good shape. So the writer who desires to be clear and forcible is at pains to use in the most effective way the particles which denote time, space, and other prime groups, notwithstanding the apparent insignificance of such words.

Importance of Dual Series

It is also well to note and take advantage of the great power inherent in fundamental dual series such as those denoted by pairs of terms like objective and subjective, concrete and abstract. Even the baseball player makes use of this principle when he recognizes the superior value of what he calls "inside baseball".

Self-corrective Power

In using multiplicative series of objects it is also desirable to keep in mind the fact that, owing to the power of self-correction which characterizes them as well as multiplicative group systems, even a crude series may often be used aggressively.

Groups of Series

Efficiency in the application of the principle in hand is often much increased by dealing with orders of material in groups, or even as group systems. In this connection orders of material propagated from some central form of series are of especial importance. Thus the series denoted by the terms solid, liquid, gas, ether, gives rise to the sense perception series indicated by the words touch, taste, hearing, vision; also to a progressive set of terms denoting the sources

of artificial light, viz.: tallow, oil, gas, electricity; also to the modes of transferring heat indicated by the terms conduction, convection, radiation.

Another example of parallel series is the following: country, continent, planet; local, continental, planetary winds; tornado, cyclone, monsoon.

Symbolisms

Certain efficient symbolic methods of denoting a higher term in a multiplicative series are already in use. Thus suffixes like -age, -ism, and -ize cause a particular word to have a general meaning. Examples are the words symbolism and symbolize as compared with the word symbol. Similarly a higher order of material is sometimes expressed by use of a noun in the singular instead of the plural form. An illustration is honor as contrasted with honors, or profit with profits.

Other symbolic devices to obtain a like end are the use of italics, or of capital letters of various sizes and kinds, emphasis in oral speech, the use of gestures, etc. Evidently many other forms of symbolism might be invented to indicate multiplicative gradations.

Ideal Species

With orders of material, as with the other Efficients, it is important to conceive of an ideal form and to make the utmost use of it in various ways including approximate and marginal uses.

For our purpose this ideal form is a series of efficiental terms used for efficiental ends.

Summary of Chapter

The present chapter may be briefly summarized as follows. It is advantageous, in certain connections, to condense an extended group system into a simple succession of objects, each of which forms a highly economical representation of a whole layer or order of the given system. Frequently such a multiplicative series of representative objects arises or comes into effective use in a given domain before any explicit group system has been worked out in this domain. Also such a multiplicative series may, at times, be the resultant and representative of several group systems.

As to its form, every multiplicative series essentially constitutes a line of objects, but from other points of view, a great variety of kinds of series exist, owing to the fact that such successions of objects are found in, or may be devised for, each domain of material or department of knowledge.

On the one hand, orders of material are sources of efficiency owing to the extreme inclusiveness of each object in such a series, and owing to the adaptability of these objects to directive management. On the other hand, because of the comprehensiveness of these series, they are especially adapted to deal simply and effectively with complex data such as the stages of human progress.

In making use of multiplicative series, especial care is to be taken to grasp their great number, to realize partially hidden or deceptive species, and to lay emphasis on those terms which are high in a given series.

EXERCISE 6

State the class or classes to which each of the following orders of material belongs:

1. Steel frame, brick, wood, trimmings (as found in a modern office building).
2. Percepts, concepts, ideas, laws.
3. Imitation, invention (or discovery), creation.
4. Farm, township, county, state, country, continent.
5. Fashion, principle.
6. Pint, quart, peck, bushel.
7. Water, blood.
8. $\left\{\begin{matrix}\text{Chemistry}\\ \text{Physics}\end{matrix}\right\}$ Biology, Psychology $\left\{\begin{matrix}\text{Art}\\ \text{Sociology}\end{matrix}\right\}$ $\left\{\begin{matrix}\text{Philosophy}\\ \text{Religion}\end{matrix}\right\}$
9. Bonds, preferred stock, common stock.
10. Standard, choice, fancy, extra fancy (as terms for grades of fruit).
11. Shell life, nest life, wing life
12. Food, offspring, culture
13. Starch, fat, protein
14. Physical courage, moral courage
15. Hand craft, brain craft
16. Property rights, human rights
17. Give a multiplicative series of six terms concerning periods of time of different lengths.
18. Give the multiplicative series of implements whose function is to thresh grain.
19. Give a multiplicative series of velocities in practical use
20. Of weapons used in war.
21. Of states in the United States with respect to area.

Give an illustration of an order of materials which is

22. Artificial 23. Natural 24. Resultant

Give an illustration of:

25. Two or more orders of material which are parallel.

26. Of manifold cross orders of material.

27. Give an illustration of an order of materials used in some business as a result of the process of standardization.

28. Of an order of materials that is additive in appearance but is multiplicative in essence.

29. Name a series of well known plateaus which are respectively ½ mile, 1 mile, 2 miles, 5 miles above sea level. Taken with a piece of land close to sea level in what respect is this an order of materials.

30. Give an illustration of the difference between concrete and abstract utility.

31. Point out the advantages involved in being right handed (or left handed), instead of ambidextrous.

32. Give two illustrations of the advantages of using abstract number in arithmetic.

33. Also of the utility of abstract words in language.

34. If a given process be divided into a multiplicative succession of parts, in carrying out this process it is made easy to stop when any required degree of accuracy has been obtained. What economies or efficiencies result? Illustrate.

35. From the order of materials standpoint explain the efficiency meaning of the statement "What one does not have in the head, one must have in the heels".

36. What order of materials is implied in the statement "strike me, but hear me".

37. State as fully as you can the principles of efficiency involved in the term "psychological moment".

CHAPTER VII

EXTERNALITY

Illustration

It was remarked in Chapter II that an epoch in human progress occurred when some primitive manlike creature first hit upon the device of saving labor by keeping and using again a club which he had found helpful in procuring food. It was a still earlier and perhaps more important epoch in efficiency history when a club was used for the first time without any thought of its repeated use; or when any external object was first used by some animal as an aid in obtaining food or protecting itself from its enemies, an illustration being the use of a web by a spider.

General Statement

This explicit use of an object or objects external to a given domain as a means of obtaining results constitutes the next primal instrument for obtaining results which we shall investigate. This source of efficiency often takes puzzling and extreme forms. This fact is illustrated by various statements in common use such as "The longest way round is the shortest way home", "There is a point where economy becomes wasteful extravagance", and "There is that which scattereth and yet increaseth". These as well as other aspects of the matter, show the need and value of careful study of the method of obtaining results which is under consideration.

The principle of efficiency involved in the use of objects or entities external to oneself or to a given domain, we term externality.

CLASSIFICATION

Objects Usable as Externalities

As to the objects which may constitute an externality, we may say in general that any object may be used as an auxiliary means to efficiency in some way or ways, but not in all ways. Thus the moon has often been an aid in determining the position of a ship at sea, but the rocks on the moon cannot be used in building an irrigation dam on the earth.

Evidently also in determining whether an object is to be regarded as auxiliary much depends on what is regarded as naturally and inherently a part of any given object or domain. This is a matter to be considered more fully later (see p. 123). Meantime it is well to note that among the more important objects which are often plainly used as externality agencies of efficiency are various material objects; in particular all tools and machines, and various physical forces; plants as sources of clothing, as medicines, and in other ways; animals as beasts of burden, as sources of clothing, etc.; other persons, ideas, theories, symbols, efficiency and other principles, categories like space and time; or complexes of these objects.

Many important individual objects which belong to those various classes will be mentioned in the pages which follow.

Quantitative Species of Externality

It is also important to classify externalities with respect to the forms which they may assume.

With regard to the categories of *quantity* and *space*, an externality may be large or small; near or remote; a fringe, a background, or a betweenness; local, superficial, or pervasive; and of various geometric shapes.

It is also important to remark that an externality may be of one, two, or three dimensions. A long cable is said to be the life of a ship. The cable in this case may be regarded as a linear or one-dimensional externality. A three-dimensional externality is employed when the whole of a mass of water is boiled in order to kill a few germs contained in it, or when practically all the carbon is expelled from iron in one of the Bessemer processes of making steel, so as to make it possible to introduce exactly the right per cent afterward.

An externality may also take what may be called the point form. We use this, for instance, when, in mastering a subject, we leap ahead from one leading point to another, and afterward rapidly fill in the intervening gaps or let them fill in themselves. A similar instance is the mastery of any field by first mastering its strategic points.

The importance of an infinite externality is illustrated in proofs by exclusion in logic, and much more strikingly by the value of religious ideas. A case of relatively infinite externality is that of representing zero or absence of number by the symbol 0.

What we may term matrix externality is often expressed by the word "environment".

It is frequently useful to characterize an externality by means of the category of *number*. Thus an externality may be dual in the sense that it is composed of two elements as when both water and heat are used in preparing food by the method of boiling. Similarly an externality may be triple or *n*-fold. An illustration of the latter is the repeated use of the pendulum in determining the force of gravity at a given place.

Or an externality may be manifold in the sense that it is used in many ways, or by many persons at the same time, as when a railroad or an idea is used by many persons simultaneously.

Temporal Species of Externality

A case of externality may frequently be characterized by some category of *time*. Thus a scaffolding is a temporary externality, while the finished building may be a permanent one. When time is used as a representative of what is contained in time, an externality involved in time may conveniently be spoken of as temporal. Externality of this kind is practiced by the man who is longheaded, or forehanded, or patient. Another form of temporal externality is illustrated by the manner in which the Rothschilds in London learned of the results of the battle of Waterloo several hours before any one else in the city and made the earlier knowledge the source of financial profit. All preventive and prophetic work is full of the externality of time. Another aspect of the matter is expressed by the proverb "All things come to him who waits".

Still another case of temporal externality is that of suspended operations in mathematics in order ultimately to save work by cancellation or elimination of other kinds. The same mode of obtaining efficiency applies in many other fields.

The result aimed at in any process also constitutes what is in one respect a temporal externality.

Other Important Forms

If we combine the idea of number with that of order we get the idea of reciprocity and hence of *reciprocal* externality. Illustrations of this are the two blades of a pair of scissors, man and wife, mind and body, algebra and geometry, or the fruitful interaction of theory and practice in any department as in engineering.

Similarly we may have triple or *n*-fold reciprocal externality, as that existing among the five senses, or the various organs of the body, the members of a family or community, the machines in a factory, or the various departments of knowledge and action, as agriculture, chemistry, biology, and mining.

An important special case is what is called the *comparative method* of testing a principle in history or a problem in science. Thus an important way of getting light on republican government is to study all historic republics side by side.

As to force an externality may be reluctant or *automatic*. Thus vision is characterized by certain large externalities which are automatic and irresistible, as well as by others which are voluntary. Motion often constitutes an efficient externality as when a teacher enables a pupil to realize certain properties of a cone by means of the rotation of a right triangle generating the cone, or when a teacher unravels any complex matter by causing it to unfold from certain unit principles.

Various adjectives of quality may be applied to externalities. Among the most important are the terms *homogeneous* and heterogeneous. Thus an externality may be homogeneous (or heterogeneous) in itself, or in relation to the material to which it is applied. The appliances used in keeping meat by cold storage are heterogeneous in both of the above respects. An important case of homogeneous externality is that of doing work on a large scale.

An extreme case of *heterogeneous* externality is that which may be termed negative in character. In this species the datum and externality are directly opposite in nature in some important respect. Instances are a difficulty which leads to new discoveries and knowledge, or a defeat or a misfortune which in the end imparts new power. Of like nature is the making of a temporary sacrifice for the sake of a future gain.

In relation to the directive faculty in man, externalities may be classified as *natural* or *artificial*. Thus one hand is a natural externality to the other. Machines and buildings are examples of artificial externality. The results of a mechanical or biological propagation are partly natural and partly artificial, and may be termed genetic externality. The consequences or fruits of an act or principle form externalities which may also be termed genetic. In many cases, externalities of this last kind have an important function in determining the essential nature of an act or law.

Important cases of directive externality are the preparation and use of reserves of any kind as of food, water, power, money, health, knowledge.

With respect to the principle of limitation, an externality may be vague, partial, or imperfect in any way. Thus in taking an oath in court use is made of infinite externality, but usually of this in a very obscure and general form. So mystics use unlimited externality in many forms all of which are more or less difficult to understand. The ether, as used in physics, is a case of a valuable but imperfectly understood externality in the field of science.

It is often an advantage to combine and use externalities in groups or even in group systems. Thus in irrigating a large piece of land a group of auxiliaries is used in the form of dam, ditches, pipes, tools, men, and also of time, ideas, and experience. So a microscope (or a telescope), and the various facts revealed by such an instrument form a group of related externalities. In like manner if we divide a set of externalities into two groups, viz.: natural and artificial, and subdivide each of these groups according to various spatial, numerical, temporal, and other categories, we arrive at a multiplicative group system of externalities.

Similarly an externality may be of *high or low order*. As an example of a dual series of externalities we have the saying "The pen is mightier than the sword". Space and time are auxiliaries of a higher order than material objects. The order of excellence of an externality may be determined in various ways, as by the number of sub-externalities included in the given externality, or by its dimensions, or by its general efficiency power. What is often called "the ideal" in any department, even though it may be unattainable, often constitutes a powerful and stimulating externality.

Complex Externalities

Evidently the different species of externality may be compounded and complexed in many ways. Instances of such complex externalities are the aggregate of externalities found in a large factory, or those used in connection with courts of justice in the punishment of crime or the settlement of disputes. Vastly more extensive and complex is the combination of externalities which are in operation at each stage of human progress.

EFFICIENCY FUNCTIONS OF EXTERNALITY

In studying the advantages which accompany the use of externality, it is evident at the outset that an extension of material and methods in any process means additional opportunities for the formation of various species of groups and for reaping the efficiencies which come from the use of groups.

Externality leads to Reuse

Looking at the matter in more detail, externality opens the way to important cases of reuse. For instance the application of paint to a wooden house keeps the material of the house from rotting and thus prolongs the use of the house. It is to be noted also that the house itself is an externality which prolongs the use of its contents. So the immersion of a biological specimen in alcohol leads to the preservation and repeated use of the specimen.

The combination of many business establishments into one is frequently followed by the reuse by all the rest of what is best in each one of the constituent businesses.

Externality also leads to more abstract and general forms of reuse. Thus by showing that two objects are each equal to a third object, we may show that they are equal to each other. The third object in this case may be regarded as an externality the employment of which saves the labor of an often difficult direct comparison of the two original objects. The extended use of this principle leads to the method of measurement in dealing with objects and to all the fruits of measurement.

Taking a wide view of things or doing things on a large scale frequently opens the way to numerous economies and efficiencies among the most important of which is prophetic reuse. For example the U. S. Weather Bureau in the year 1903 observed torrential rains on the Upper Missouri River and predicted a flood on the Mississippi 28 days before it arrived at certain places and thus saved many millions of dollars. The same bureau has diminished the annual loss to shipping on the Great Lakes by fifty per cent.

Leads to Multiplicative Results

The externality of outflanking an enemy opens the way to the multiplicative effect of an enfilading fire. In a much more general way the consideration of a large domain of material gives oppor-

tunity for the discovery or invention of units and multipliers of unusual excellence.

Economies

The use of auxiliary instruments results in a vast variety of economies as of strength, time, space, and resources of all kinds. At one time it took 78 men 1½ minutes to put over the helm of a large vessel; later by the aid of steam, the work was done by 2 men in 16 seconds. Every field of activity contains similar illustrations.

New Groups and Group Systems

General externality and special auxiliary objects are alike potent aids in the formation of groups and of group systems. A simple instance is the formation of a bundle by tying objects together with twine. The use of a club, or of a bow and arrow, gave the savage concentration of his strength both in time and space. The use of the gun and bullet does the same in a far greater degree. The externality of a copper wire on a railroad leads to the concentration of many locomotives into one central power engine.

So the use of a clock gives us an accurate and detailed multiplicative grouping of time. In a far larger way the telegraph, telephone, and various similar appliances of modern life make the world one large group, and also a system of groups in many ways. With reference to a special field like that of books, it has been remarked that the "literature which enlarges the outlook, also transforms the mind". Or, as Thoreau says, "Only that intellect makes any progress toward conceiving the essence of a matter which at the same time perceives its effluence."

Accuracies and Delicacies

Externality frequently produces those uniformities and accuracies which are the source of much efficiency (see Chapter VIII, p. 130). Examples are the continuity of action obtained by the use of a fly-wheel or a reserve of any kind, stability acquired by aid of ballast or a bilge keel, or accuracy by aid of extra tests or apparatus.

In like manner auxiliary appliances are often a source of useful diversity, fluidity, or sensitiveness. A simple instance is that of making food tender by the use of heat. By the aid of the bolometer a difference of temperature of less than .0001° may be detected. In

making glass the addition of soda lowers the melting point of sand from 3000° to 2400°, and lime is useful in the same way in reducing iron ore. By using the proper apparatus several thousand successive photographs of a moving object may be taken in one minute and the action of the corpuscles of the blood, for instance, traced in detail.

A Source of Orders of Materials

Similarly the use of an externality is often a means of obtaining a useful multiplicative gradation. Thus if two tracks are used on a railroad in a given direction one can be used for fast trains and the other for slow trains with many resultant advantages.

Some one has said that to get down to the pith of a matter one must go round the matter and study it in all its bearings. Expressed in other words this means that a varied externality is necessary in discovering the causes, sources, and essential efficientism in any domain. Thus a consideration of the needs, rights, and opportunities of all classes of men leads to a multiplicative series of ethical principles capped by what is at least an approximate or efficiency absolute represented perhaps by a sense of oughtness or Kant's categorical imperative.

Thus also the combination at Jerusalem as at a focus, of the various externalities represented by Egyptian, Assyrian, Persian, Greek, and Roman civilizations was instrumental in leading to the development at that place of the Jewish and Christian religions. In any field of work breadth of view as to ends aimed at is important in obtaining efficiency of a high order. Efficiency for selfish ends is not efficiency at all in the long run.

New Powers of Management

A broad view of things is also a source of efficiency in that it leads to a higher and more effective use of man's directive powers (see Chapter XI, p. 192), and is productive of many valuable utilizations and substitutions. It gives a wider range of material to select from, more space in which to act, and many aids in transferring objects in space. Thus a lampshade enables us to direct light to a place of need and thus more fully to utilize the light. By the use of salt, vinegar, tin, or cold storage, we can preserve food and direct its use with respect to the categories of both time and place.

Externality is often a source of efficiency by enabling us to substitute a material or force that costs little for that which costs much, as in the use of water-power instead of hand-power to saw lumber.

Externality also assists in getting results by enabling us to apply energies and resources at important centers. Thus Napoleon by taking a large view of the field of operations was able to strike the enemy in the rear at Marengo and thus to win the campaign by a single battle. So in general a large view of matters reveals centers of influence and also the means of reaching and mastering these centers and thus obtaining multiplicative results.

Prevention of Loss

The use of externality often produces important results in what is at first a negative form. Thus a broad view of a given matter may prevent error, sickness, or even loss of life. The use of an arbiter to settle a dispute renders prejudice inoperative. The breadth of view gained by education and travel is a safeguard against hasty generalizations, or overuse of any one method. A somewhat similar qualitative result springing from the use of externality is the conversion of an obstacle into a stepping stone, or of a defeat into a victory.

Unique Fruits

Some of the results obtained by the use of externality are unique or transcendental in the sense that apparently they could not have been obtained without the use of this principle in some form. Thus the aeroplane enables a man to fly. The yellow fever parasite is so small as to be invisible in the microscope yet it can be entirely controlled by indirect means. The same is true of the germs of several other diseases, as rinderpest, and pellagra. By the use of the Roentgen rays the action of the heart and lungs may be seen and photographed. By the same means when bismuth salts have been added to the food the progress of this food during the whole process of digestion may be watched. Religious conversions as produced by the action of infinite externality upon the individual form an illustration of beneficial results of an incomparably higher type.

Complex Fruitages

It is scarcely necessary to add in this connection that the above classes of efficiency functions may be compounded and complexed in an endless variety of ways. For instance externality in the form of an anticipation of future rewards often begets various efficiency advantages. Such an anticipation, for example, arouses a feeling of joy in the worker which increases his efficiency in a number of ways as by stimulating his various activities, increasing his speed, causing him to search for new knowledge, and perhaps leading him to invent new methods.

Greater Use of Externality in Higher Organisms

If we wish to look at the fruits of externality in a more collective and pictorial way we find, for instance, that the higher we go in the scale of organisms the greater is the use of auxiliaries. The spider has its web, and the bird its nest, but man has his cities, railroads and philosophies.

If we examine the various definitions which have been made of man for the purpose of discriminating between him and the lower animals, we find that the essential part of each of these statements is the conception of man as an animal using externality in some explicit and conscious way. Thus man has been defined as the tool using animal; as an animal looking before and behind; the cause using animal (das ursachen Thier); a rational animal; the animal requiring the superfluous.

Greater Use by Greater Men

Furthermore it is to be noted that the greatest men are those who have used the most and highest externality. A study of great men in the schoolboy stage has shown that their most common characteristic is a tendency to break out of ruts and discard conventions. The use of this and other forms of externality increases as they develop.

Time and tide, it is said, are always on the side of great men. As a matter of fact such men have such an outside and comprehensive vision of affairs that they are enabled to utilize time and tide for their own purposes.

Human Progress Marked by an Increasing Use of Externality

Hence it is not surprising that each stage of human progress has been marked by an increasing use of externality in its various forms. The first club or stone used thus has developed into the present mighty volume of machinery of all kinds, material, linguistic, social, artistic and logical.

Furthermore an examination of each department of activity and of each element of civilization reveals the same law of increasing auxiliaries. Thus the knowledge required of the expert agriculturist has increased till at the present time he must use the main principles and some of the details of chemistry, physics, botany, biology, engineering, meteorology, commerce, and social psychology.

Lack of Externality Means Loss

Another method of gaining an appreciation of the importance of externality is to try to picture what life would be without certain of the auxiliary instrumentalities now in common use. If, for instance, in navigation we take away steam and gas we reduce transportation by water to the use of sail vessels; if we take away masts and sails, we have left boats propelled by oars; if we remove wood and skins, man must propel himself in the water by swimming.

One reason why man has not been able to analyze the force of gravitation into constituent elements is that this force is so pervasive that it has thus far been impossible to get an outside point from which to view it; that is, it has been impossible to apply any comprehensive externality to it.

Likewise in the higher mental world, the man without externality is the narrow, selfish, and therefore blighted man.

Familiar Words Showing the Value of Externality

Still another way of obtaining an insight into the power of externality is to recall the efficiencies connected with words or phrases which denote certain more or less familiar forms of externality, such for instance as tools, machinery, breadth of mind, reserves, solidarity, outwit, undermine, correlations, larger horizon.

Limitations in the Utilities of Externality

Finally in this relation mention should be made of the limitations and often drawbacks connected with externality as a source of efficiency. Externality, of necessity, means an added amount of machinery to carry and often the annexed toolage does not pay for itself. It then becomes mere red tape or worse. Too much light makes a confusing glare or even produces blindness. Wealth without some higher externality often becomes a curse. At times a doubt has been expressed whether modern machinery as used thus far has, on the whole, improved the condition of mankind. At least we can say the burden of material mechanism carried by modern civilization is as great as it can bear. Another class of danger connected with the use of outside auxiliaries is the dwarfing of one's powers of self reliance. Other aspects of this evil are expressed by the proverb "Too many cooks spoil the broth", and "What is everybody's business is nobody's business". In case an externality results in a more extended multiplicative group system, the drawbacks presented in a preceding chapter connected with such systems will also be present.

It is well to note also that the dangers connected with an externality often correspond in size to the useful power of the externality. The efficiency possibilities of unlimited externality for instance in the best religious concepts seem to be boundless yet in many ways the strength of religion has proved to be its weakness. Seemingly as a consequence of the prodigious values in religion have come pride, reluctance to use other sources of efficiency, hostility to general culture, cruelty, and general persecution. Hence it has been said, "The vision of a spiritual ideal is the most important single element of efficiency, but one of the most dangerous".

Self-corrective Power in Externality

It should be observed, however, that externality contains within itself the power largely to correct its own defects and evils. For the high order of multiplicative power which results from such externality also means in large degree the power of self-correction and self-development.

ANALYSIS

In order to get a more precise idea of the nature of externality and hence to increase the advantages in its use, we shall now make

a more thorough examination of the elements which enter into it than was made in the opening paragraphs of the chapter.

Space and Like Elements

A spatial element seems necessary to the idea of externality. For the term implies outsideness to what is given or customary, and the concept of something outside or beyond is a spatial one.

It is to be noted, however, that in the more abstract forms of externality the idea of space is refined till in some respects this idea seems to disappear altogether. An instance in point is the use of an abstract quality or method of some kind as an auxiliary. Physically a quality may be as innerness, while in the domain of mental attention it is outside of what was included in the consciousness of the doer at the start.

The element of time is also present to some extent in externality. For, at least with respect to the conscious apprehension of them, the auxiliary used follows that which was first tried without the externality, and hence we have in each use of externality some flavor of succession or order.

The element of mind or consciousness is also usually present, since the use of something beyond what is customary implies a certain element of effort or surprise. Looked at in another way, externality implies first an enlargement of the mind and then of the material used or considered.

The meaning of externality may also be expressed in terms of the group. From this point of view externality means an enlarged group, the result being obtained by removing some limitation in the datum group.

Relativity of Externality

From the preceding discussion it is evident that the element of relativity is prominent in determining whether a given object or process is a case of externality or not. What is externality from one point of view may not be so from another. To the savage chief the use of a messenger would be natural and the use of mails an externality, while to a modern business man who has come to regard the use of the mails as natural, the use of a special messenger would usually seem an externality. Similarly the use of salt in one's food, or of medicine, or of clothing, may or may not constitute an externality according to circumstances.

The two extreme points of view, in this respect, with regard to externality should here be stated. Thus from the point of view of a single monad, the rest of the universe is an externality. But from the universal point of view, there is no such thing as externality, but all efficiency is an internal process in the universe as a whole. These extreme conceptions are useful since they are themselves, with respect to the customary use of this agent, special cases of externality. However, under usual conditions it is an advantage to divide a situation into what is ordinary and what is extraordinary or external to the ordinary.

It is also well to remember that in externality as well as in each of the other Efficients there is a characteristic or unique element, something perhaps beyond analysis.

Auxiliaries vs. Externality

It may also be well in this connection to point out the essential difference between the terms auxiliaries and externality. Auxiliaries are usually subordinate in size and importance to the main datum or method while an externality may be superior to the datum and may dominate it. The term auxiliaries also implies a more direct and immediate use than does externality.

In conclusion we have the following approximate definition:

Externality is a method of obtaining results by the application of something outside of the customary data.

Also the outside object thus used is often termed an externality.

METHODS OF APPLICATION

Externality as a Group

Viewing externality as a larger group, and hence as essentially a higher use of the multiplicative principle, is an aid to the efficient use of externality in several ways. For it leads to an extended use of this instrument by making evident the great value of this method of work, by revealing the vast number of available auxiliary objects, and by putting such objects in an effective form.

Adaptation in its Application

Taking up the matter in more detail, externality is so broad a principle that in its use, relativity, or special adaptation in peculiar cases, needs to be studied carefully.

In many situations of great difficulty, rather than to use no externality, it is well to use the principle in some crude form. This may be a blind kind of externality like patience, or an unformed kind like agitation, or publicity, or the mere recording of facts while waiting for laws to appear. Compromise is often a complex form of crude externality since in it patience is exercised and results from two or more simultaneous series of developments are awaited.

Externality by Doubling

A simple but widely applicable form of externality is that which consists of doubling a given object or repeating a given process. Nature has employed this source of efficiency in giving man two hands and two eyes. Man utilizes the same method when he installs two engines in a power house, or checks the result obtained from a computation by repeating the calculation.

Preference to Highly Developed Externalities

In using externalities it is evidently important to learn to recognize those auxiliaries which are highly developed and therefore unusually effective and to give the preference to such species. In this connection it is to be noted that a high externality is usually more or less abstract, and, for our purpose, closely related to the Efficients. It has also in many cases acquired a tendency to act, so to speak, of itself.

Even when an additive externality would answer a given purpose it is often well to use a highly multiplicative process giving larger results than are needed and to apply the extra results to the apices of some other group system.

A small or superficial externality is often dangerous but one cannot possess too much externality in its higher forms. One reason for this is that the higher forms carry with them in an unusual degree their own corrective, as well as more aggressive principles of development.

Educational Sources of Externality Powers

In the process of acquiring externality powers it is well to remember that the study of a certain subject or some special form of experience is often particularly fitted to give mastery of some special kind of externality. Thus geography gives a grasp of space,

geology of time, astronomy of space and time, chemistry of quality, physics of force, psychology of mind, and history of time and mind.

The effective application of the principle of externality is also often greatly aided by the use of certain of the primal sources of efficiency to be studied in the chapters which follow. Thus the analysis (see Chapter VIII) of a domain into different well selected elements often opens the way to obtaining important externalities by a mere extension of certain of these elements. Also appropriate symbols (see Chapter X) are an aid in grasping large externalities and in applying them. The use of symbols, for instance, may lead to standard methods of transforming and adapting auxiliaries and obtaining results from them by semi-mechanical means.

Ideal Species

It is ever to be borne in mind that for our purpose the most important externality is that of the aggregate of the Efficients combined in the most effective way and kept ready for use at any moment. It is to be noted also that each individual person may advantageously develop this general form into a special form or shape adapted to his peculiar circumstances. It will also be recognized that the use of the above ideal constitutes a use of externality of externality.

Synopsis of Chapter

The principal results arrived at in this chapter are as follows. Not only may the organization of given material produce efficiency in its use, but an equal or even greater increase of efficiency may often be obtained by going outside of the given situation and applying certain auxiliaries to the data in hand. Evidently the use of outside material may be of innumerable kinds since it may include an endless variety both of objects and also of the forms in which these objects may be assembled or viewed.

The fundamental source of new power in externality is that it, from its very nature, opens the way to the formation of new and larger groups, and hence means larger and higher forms of reuse, multipliers, groups, and group systems, with the various consequent fruitages of each of these. Of particular importance are the new

possibilities of directive management which arise. Many valuable results otherwise impossible are also obtained. Hence human progress has ever been marked by an increased use of externality of various kinds.

So powerful is this source of efficiency, that it is especially important to realize the great number of forms which it assumes, and the frequently large value of even its crudest and most elementary species. At the same time the greatest stress should be laid on its highest forms, and careful study be made of the sources from which these may be obtained.

EXERCISE 7

State the class or classes to which each of the following externalities belongs:

1. An axe
2. Sunlight
3. Business capital
4. Darkness as a revealer of stars
5. A relay circuit on a telegraph line
6. A name
7. Heat used to expand a wagon tire before placing the tire on the rim of the wheel.
8. Heat used in making ice.
9. A line added to a diagram to aid in proving a proposition in geometry.
10. The third dimension of space, as utilized by a man riding in an aëroplane.
11. An ambuscade.
12. Boiling water to kill the germs in it.
13. During the blizzard of the year 1888, a telegram was sent from New York to Boston by the way of Paris. What externalities were employed in this process?
14. Give an example of each of the principal kinds of externality.

State the principal kinds of externality

15. Gained by travel.
16. Afforded by an education in the ordinary sense of the term.
17. Used by an intelligent farmer.
18. By a teacher.
19. Employed in the manufacture of pottery.
20. In mining coal.
21. State the different kinds of externality involved in mental consciousness.
22. Which of the mental powers (as memory, etc.) supply noteworthy externalities?
23. What externalities are used in writing a letter and sending it by mail?
24. Explain altruism as a form of externality and the efficiencies which result.

25. Give three instances of the use of color as an externality.

26. Also of the use of heat in this way.

27. Also of water.

28. State the kinds of externality used by an ordinary bird.

29. In a certain factory 1000 yards of a given kind of cloth can be manufactured at a cost of $800, and 50,000 yards at a cost of $32,000. The efficiency of the first process is what per cent of that of the second? What externalities are here the source of efficiency?

30. Before the use of power machinery in making muslin, 1000 operators could produce 10,000 yards in a certain time. After the introduction of such machinery 50,000 operatives in the same time produced 4,000,000 yards. Express the degree of human efficiency here involved. What externalities were the source of this efficiency?

31. By direct computation obtain, to three decimal places, the numerical value of $\frac{5}{\sqrt{3}}$. Also compute this value after first rationalizing the denominator of the given fraction. State the degree of labor efficiency involved. What externality is used in obtaining this efficiency?

32. If a man without a machine can plant 1 acre of potatoes in 1 day, while a man with a machine can plant $3\frac{1}{2}$ acres, state the degree of human efficiency in the second process and the source of the same.

33. Of what advantage is it to a boxer to keep in more or less constant motion?

34. State the advantages in the use of a large storage battery in connection with a city electric railway.

35. State the difference between parsimony and economy.

36. Point out the advantages in using a somewhat larger furnace than is actually required in heating a house.

37. Also in the use of a fountain pen as compared with an ordinary pen.

38. State as fully as you can the externalities now used by physicians which were not used by them one hundred years ago.

39. What kinds of externality are involved in publicity as to governmental affairs, and what are the advantages which result?

40. Give an example where the use of externality results in a loss of efficiency.

41. Show that reuse always implies externality.

42. Give three examples of groups formed by the use of externality.

43. What kinds of externality did Vergil use when he wrote many lines in the morning and cut them down to a few by evening, and what were the advantages involved?

44. Give an instance where the use of crude externality rather than of none is advantageous.

45. Instead of telephoning a given message it is sometimes more advantageous to send the message in a written form. Why?

46. What are the advantages to a business man in keeping a considerable cash reserve on hand?

47. In carrying on commerce with another country (especially one low

in the scale of civilization) it has been found useful to learn the history, geography, and customs of the country. What externalities and resulting advantages are involved in this procedure?

48. State the externality involved in realizing the future rewards of present work and the advantages of such realization. State also some efficient ways of making such externality vivid to different classes of people, as to pupils, or workmen.

49. What advantages result from reasoning a principle out to its logical consequences before applying it? What forms of externality are here involved?

CHAPTER VIII

UNIFORMITY AND DIVERSITY

Illustrations of Diversity as a Source of Efficiency

The separation of an object into parts with a view to the individual treatment of these parts is often a fundamental source of efficiency. Watt's invention of a separate condenser changed the steam engine from a toy to the most effective machine in man's possession. Flight by the aeroplane was made a practical success by dividing the wings of the machine into parts so that they could be warped with such exactness as to make stability possible amid shifting air currents. The triumphs of modern surgery come from regarding the human body not as a lumpish whole but as made up of separate cells, and also from keeping the germs of infection away from wounds.

The source of efficiency thus illustrated, we term diversity. More complex and abstract examples of its value are the beneficial results arising from the separation of church and state, from individual ownership of property, and from the distinction between what is personal and impersonal.

Illustrations of Uniformity as a Source of Efficiency

In like manner, and often in a closely related way the principle of uniformity or sameness is a highly important aid in obtaining useful results. Thus the fact that the same units of money are used in all parts of a given country saves the traveller much trouble and expense in having one kind of money converted into another. The perception by Newton that the moon falls toward the earth by the same law that an apple does led to the establishment of the law of gravitation with the many useful consequences that have resulted. Similarly the equality of men before the law results in a general sense of security which is productive of personal and social efficiency in many ways.

Aids to other Efficients

The principles of diversity and uniformity are not only themselves direct sources of efficiency, but they also act in important ways

through the other Efficients. Thus in order to teach a group of children at the same time, certain uniformities of age, development, and interest must exist among the individual children composing the group. In other words special uniformities and diversities enter into the formation of a group, and hence into such particular forms of the group, as reuse, the unit and multiplier, multiplicative groups, and orders of material. In fact we shall find that uniformity and diversity are the raw materials, so to speak, out of which groups and the various Efficients are formed.

In particular it is well to observe at this point that a uniformity (and its accompanying diversity) in some cases form an incipient, or rude germinal group, and in other cases a specially efficient final form of group.

It will be noted that psychologically the perception (and treatment) of diversities precedes that of uniformities; while logically the reverse is often the case. Hence in some places we treat diversities before uniformities, and in other cases follow the reverse order.

CLASSIFICATION OF DIVERSITIES

Relations of Uniformity and Diversity

Before the classification of diversities and uniformities is attempted, it should be stated that a uniformity always implies a diversity (and vice versa), but that when one of the two is prominent as a source of efficiency the other may be inconspicuous or practically hidden. It is especially important to bear this fact in mind whenever, for certain reasons, we shall temporarily treat the two categories separately.

It is useful to realize that diversities may exist among all possible material objects, such for instance as those catalogued in the classification sections of the preceding chapters. It does not seem necessary to repeat these lists at this point.

It will be advantageous, on the other hand, to consider in some detail the classification of diversities as to their form.

Spatial Forms of Diversity

As to space and its related categories a given diversity may be partial, entire, or partial in some special way, an example of the last class being a mottled form. An illustration of efficiency ob-

tained by a positional diversity is the fact that the sewing machine was made possible by the transfer of the eye from the blunt to the sharp end of the needle. A similar instance was the placing of a rudder in front of the aeroplane. Spatially also a diversity may be internal, in that it exists among the parts of an object as among the parts of a locomotive; or it may be external, in that it exists between one object and another external object, as between two locomotives.

Numerical and Group Forms

A given diversity may be dual in that it contains two elements as when objects differ in both shape and color; or triple, as when motions differ in direction, velocity, and duration; or manifold, as when two given objects differ in size, weight, color, form, position, motion, density, utility, and with respect to other categories.

Hence it follows that diversities may occur in bundles or groups. Important cases are those variations which in the process of evolution occur in aggregates or groups and which are termed saltations or mutations.

It is evident also that diversities may be grouped multiplicatively in various ways. Thus the differences between an amoeba and a man may be treated as a multiplicative group system of differences.

Other Species of Diversity

Again, diversities may be artificial or natural; active or passive; permanent or temporary; relative or absolute.

A temporary diversity is a diversity of diversity; that is, a diversity of the second order.

In general it is most important for our purpose to remember that one diversity may be of higher multiplicative order than another, and as a consequence more vital and useful in many ways. Thus the diversities which characterize a set of accounts kept in the card form are in general of a higher order than those found among accounts kept in the ledger book form. So a difference in the mental power of two men is many stages above a difference in the color of their eyes.

In brief we may say a diversity may be of high order because it is manifold, or of several dimensions, or because it occurs in material of a high order of efficiency, or exists between materials of

widely different orders, or because the given diversity produces especially important efficiency results.

Difficult and Complex Forms

Certain diversities are so peculiar as to be difficult to grasp and characterize. Illustrations are the differences between the properties of certain alloys and those of their constituents, or that between matter and mind, and that between uniformity and diversity themselves. Some of these diversities may be so ultimate that we lack externality by which to determine them; others of them are doubtless due to some untraced action of the multiplicative principle.

The various species of diversities may be complexed together in many ways. An example of a complex diversity is that between two personalities, or two civilizations, or two philosophies.

CLASSIFICATION OF UNIFORMITIES

As has already been stated, for every case of diversity there is a corresponding case of uniformity. But in certain instances the diversity involved may be prominent while the related uniformity is obscure, or vice versa. Hence cases often occur where the one of these sources of efficiency is noteworthy, while the other is inconspicuous. Thus certain differences between the ideas of space and time are evident but the connection or likeness between these two concepts is more difficult to conceive.

It does not seem necessary to catalogue and illustrate those species of uniformity which plainly correspond to species of diversity, such as uniformities of color, size, shape, weight, etc. We shall limit ourselves therefore to a consideration of the cases where uniformities are especially prominent and are relatively independent of the diversities related to them.

Dual Uniformities

Besides the dual uniformity which consists of uniformity in two respects such as color and size, another form of dual uniformity is that which exists between two objects, as between two peas. Among the most important dual uniformities of this class, are those equivalences between objects or systems which are termed equalities, equations, or identities.

Truth in one of its aspects viz.: as a correspondence between fact and symbol is a dual uniformity of this general kind.

Orders of Uniformity

Cases of uniformity of higher order, like those of diversity, call for especial attention. Thus the uniformity with which the earth rotates on its axis is more important than uniformity in the speed with which a man walks. So equality of opportunity for different men is of a higher order than equality of physical weight.

Important special cases of uniformity are those which are due to some outside cause, and which may often be controlled through this cause. An example is a likeness in the mental habits of pupils due to certain qualities in a teacher, or uniformity in family traits due to heredity. Mention should also be made of the uniformity which arises when one set of forces is generated in order to balance or counteract another set.

Approximate Uniformities

It is well to notice also that certain so-called uniformities are due to limitations in our powers of perception. An example is the apparent uniformity in the surface of a table. It is often profitable to regard such a uniformity as a first group in a multiplicative group system.

COMBINATIONS OF UNIFORMITIES AND DIVERSITIES

Evidently uniformities and diversities may be combined in a great variety of ways. Only the most important of these will be here considered.

Combinations Related Spatially

(1) In a given uniformity, the *diversities* may be *internal.* An example is the uniformity of length during wide changes of temperature in a bar formed of the metal called invar, due to the fact that the bar is an alloy of two metals, nickel and steel; or steadiness of bodily health due to the action of different organs in different ways; or an equivalence relation of any sort between two objects which differ in their composition or details.

(2) The diversities which are related to a given uniformity may be *external* to the uniformity. Thus comparative uniformity of the temperature is often maintained within a building by dividing the walls which surround the building into several air chambers.

(3) A special variety of (2) is of sufficient importance to call

for independent mention. The allusion is to that form in which differences are *distributed* more or less *symmetrically* about a mean or average. Illustrations are a series of groups formed by setting up certain standards and including in a given group all the cases that fall within certain limits above or below a given standard. Similar cases are the grouping of places on the earth's surface by the use of latitude and longitude, and the many varieties of symmetry in mathematics, art, and common life.

Combinations Otherwise Related

(4) The uniformity and the diversity which are combined may be of such a nature that the category of external or internal does not apply to their relation. As an example we have that combination of diversity in space with uniformity in time which is found in markings on finger tips as used in identifying criminals.

(5) Finally we have those complex cases where uniformities and diversities are combined in various ways at the same time. Thus in a card index certain diversities are internal and others external to the uniform size of the cards. Some of the uniformities and diversities involved are also temporal in nature. The result is a manifold and varying system of groups, productive of much efficiency.

EFFICIENCY FUNCTIONS OF DIVERSITIES

Since uniformities and diversities are incipient or special forms of groups and other Efficients, the efficiency functions of uniformities and diversities are best expressed in terms of those of the other Efficients.

Diversities as a Source of Reuse

In the first place it is to be observed that a diversity is often the primal source of a case of reuse. For instance to recur to a former illustration the separation of the engine of a steamboat from the hull together with the transfer of this engine to a tugboat leads to repeated use of the engine, and hence to the various economies and efficiencies which characterize such reuse. To consider a more general case, some diversity in the form of separation of parts is evidently essential to all marginal reuse. Also a high degree of diversity often leads to a high degree of marginal reuse. Thus the

condyle on which the head of a bird rotates allows much greater freedom of motion than that on which man's head is poised; hence the work done by a bird in putting its body in a given position is reused marginally to a much greater extent than is such work when done by a man.

Diversities also open the way to other forms of reuse. Thus by eating a meal composed of foods which differ as to the time of their digestibility, the gastric juice and whole digestive apparatus may be reused in the process of assimilating the meal.

Diversities lead to Groups and Group Systems

Similarly uniformities and diversities lead to the formation of units and multipliers and of groups in general. These groupage results often take many special and highly important forms. An instance is the discovery by Joseph Henry that oiled silk will insulate an electric wire and that consequently great lengths of such wire may be wound around an iron core in concentrated form, later results being of course the invention of the electric telegraph and the dynamo. In like manner the tubular boiler is a case where a diversity of material gives rise to a concentration of force in both time and space and as a result has made the locomotive, for instance, a practical success.

Not less important is the formation of multiplicative group systems by the utilization of diversities. For example the separation of the continuous manuscript roll into the equal parts called leaves led to several of the group systems which we find in a book. So owing to chemical analysis the way has been opened to the systematic group combination of elementary substances, the result being the innumerable substances now in use which have been prepared by chemical synthesis.

A diversity often leads to the formation of an order of materials without any intervening multiplicative group stage. Thus the change of seasons in the Temperate Zones results in a multiplicative gradation of seasons and of corresponding life activities, winter being the abstract efficiency season. The superiority of the inhabitants of the Temperate Zone is doubtless in large part due to this fact and its efficiency consequences.

Lead to New Uniformities and Diversities

One diversity frequently leads to the formation or recognition of another diversity and its fruits. Thus difference in soil should mean difference in methods of cultivation and of products, and hence to increased profits on the average.

Similarly a diversity may lead to a uniformity and its fruits. For instance the fluidity which characterizes water causes the top of a body of water to form a level surface and manifests itself in other useful uniformity properties. Diversified farming makes profitable returns more certain. Investment of funds in several different ways adds to the likelihood of always having some income.

An Aid in Directive Work

Diversity in its various forms also facilitates directive efficiency processes. Thus even the plant in order to transfer starch from one part of its structure to another converts insoluble starch into sugar which can be dissolved and made fluid. So in business fluid capital of all kinds is directible. In another line of action, separation or distinction of parts opens the way for the substitution of what costs little for what is expensive. Also the separation of canned goods for instance from surrounding causes of decay leads to the efficient directive action of these foods both in time and space. The recognition of diversity makes it easy to realize the needlessness of throwing out the baby with the bath, and later the possibility of developing the child without breaking its will.

Absolute Values of Diversity

The use of diversity often leads to certain absolute values; that is, to advantageous results which transcend the data incalculably, and may be unique in that they are not otherwise attainable. Thus if the division of a ship into compartments leads to the saving of the lives of the passengers, the result is of the kind just described. Similarly the isolation of persons sick of infectious diseases may lead to like inestimable advantages.

Noticing differences in weight in samples of supposed nitrogen was the means of discovering the chemical element argon and later of other elements. The plasticity of maize made it possible to develop varieties which ripen in the short summer of comparatively cold climates. A diversity of parts in a landscape or work of art is frequently a source of both esthetic charm and commercial value.

Complex Utilities

The above efficiency functions may be combined in various complex ways. An example is the great variety of useful results which comes to the farmer who makes it a rule to keep the soil thoroughly pulverized about certain growing crops. These complex developments may take extraordinary and unexpected forms. Thus from the chance observation that light makes a smudge on silver has sprung the whole science and art of photography. From the fact that rubbed amber attracts a feather has arisen all of electrical science. Of similar complex and dialectic character are the advantages which have resulted respectively from the ownership of property in severalty, from the division of labor, and from the atomic theory of matter.

Words Illustrating Usefulness of Diversities

The benefits obtainable by the use of diversities may be realized in another often effective way, viz.: by realizing the useful results connected with certain more or less special and familiar forms of diversity such, for instance, as analysis, variety, solution, independence, detachment, ductility, delicacy, sensitiveness, freedom. The advantages connected with diversity may also be realized in a negative way by recalling the evils and losses suggested by such words as fixity, monotony, and fossilism. A concrete example is a railroad accident caused by color blindness in an engineer.

Limitations to Values of Diversity

The presentation of the efficiency functions of diversities should not be closed without a statement concerning the dangers and evils connected with the use of diversities. Too much diversity, or misplaced diversity, may result in the impairment or destruction of groups, or in difficulty in their formation. An example is the diversity which characterizes a routed army. Another illustration is furnished by the evil results of too pronounced a division of labor. Often extreme diversities result in what is called license or chaos.

EFFICIENCY FUNCTIONS OF UNIFORMITIES

Uniformity as a Source of Reuse

Uniformities also have their characteristic efficiency functions. Thus the fact that the ocean is approximately the same the world

over leads to reuse in all other places of knowledge of it gained in any given place. The same statement applies to the atmosphere, a geological stratum, or human nature. Permanence in time, gained for instance by printed or phonographic records, or by monuments, leads to the reuse of work and facts. The discovery by Van t'Hoff that liquids and gases have important properties in common has led to the reuse of our knowledge of one with respect to the other in important cases.

Basis of Groups and Group Systems

Similarly a uniformity may be the basis of a group formation of some sort, and hence be the source of the advantages which come from the use of the group. For example, if a field is approximately level, this uniformity opens the way to the use of a gangplow in cultivating the field. Uniformity in the size and quality of a manufactured article leads to the wide use of that article. Justice and honesty are the sources of many similar beneficial results.

The invariability in the time occupied by the earth in rotating once on its axis is the basis of our multiplicative group system of time measurement with its many ramifying utilities. Similarly various social, political, and indeed all group systems are founded on certain uniformities.

Other Uses of Uniformities

Uniformity and continuity, when realized, often make the use of certain kinds of externality comparatively easy. An example is the manufacture of an article on a large scale when the demand for the object is certain and permanent.

Uniformities frequently open the way to profitable directive acts. Thus uniformities in the parts of a watch or automobile facilitate the substitution of a new part for any part that may be broken or worn out. If we know that a certain volume of gas will give as much energy as a ton of coal the substitution of one for the other is often the source of valuable efficiencies. The fact that a network of protoplasm connects the cells in a plant makes possible the operation of botanical grafting.

The use of uniformities as of diversities, in certain cases has transcendental or absolute value, such as those of saving life, or giving knowledge otherwise unattainable.

Complex and Self-developing Values

The advantages resulting from the use of uniformity also often take a complex and self-developing form. Such for instance are the uses made of the straightness of a ray of light, or the advantages arising from uniformity in weights and measures, or in language, or from the correspondence of an organism to its environment. Sometimes these benefits are in part negative. Instances are the results obtained from applications of the law of conservation of energy, or from the principle that action and reaction are always equal.

Words Illustrating the Usefulness of Uniformities

We are aided in realizing the aggregate of efficiency values in uniformities by noting those connected with certain special categories of uniformity as for instance with honesty, justice, constancy, reliability, perseverance, durability, etc.; or by trying to grasp the inefficiency that would characterize the world life if carried on without the aid of these and similar categories.

Limitations to the Utilities in Uniformities

The use of uniformities however often brings certain drawbacks or compensations. This fact may be realized by noting for instance the evils connected with such words as monotony, fashion, custom, panic, socialism. Ambiguities are an illustration of another class of evils connected with the uniformity category. An important case of the misuse of the principle of uniformity is what is termed hasty generalization. This, in brief, consists of jumping to the conclusion that what is true in one or a few cases is true in all cases of a given class.

The most important error of this kind is that of regarding some special form of uniformity, such as reality or substance, or more specifically matter, force, mind, or some other species of reality uniformity taken as fundamental and inclusive, as the only route to efficiency results. This view often leads to the further error of regarding uniformity in some form (as in that often termed truth) as more fundamental and important than the uses of uniformity. Errors of this class will be treateed in more detail later.

Human Progress an Advance in Uniformities and Diversities

After what has been said as to the various efficiency values of uniformity and diversity, it is not surprising to note that progress

in each department of knowledge and action has been marked by a progressive development of uniformities and diversities, both in number and in degree of efficiency power. The child's mind begins its unfolding process by the perception of differences and similarities, and progresses by penetrating to and grasping diversities and uniformities which are ever more comprehensive and vital. The beginning of abstract and formal science is probably to be found in the first perception by man of the difference between fixed stars and planets, and all subsequent astronomical progress may be regarded as a mere development of continuously better uniformities and diversities, leading to the generation of ever more efficient systems of mutiplicative groups. A similar process has characterized progress in every department of human life and thought, and hence progress as a whole.

EFFICIENCY ANALYSIS OF UNIFORMITIES AND DIVERSITIES

In order to make the most efficient use of uniformities and diversities it is important to analyze these concepts as closely as possible, especially in terms of the other Efficients.

Relations to the Group Principle

With respect to the group principle, in general every diversity means a new unit and hence often an order of units; every uniformity means a new tie, or combination of units. Hence, from one point of view, the diversity and uniformity aspects are the two halves of the group concept.

Also it has already been remarked that uniformities and diversities sometimes serve as crude, incipient groups, and at other times as final highly finished groups. Hence from these various points of view uniformity and diversity are to be regarded primarily as a stage in the multiplicative system of things.

This method of regarding uniformities and diversities explains why an animal, as a zebra, may be marked in a very peculiar way, and yet be protected by its environment. For owing to our habit of seeing things in groups, if the animal's markings fit, not its environment in all respects, but merely some one of the many group systems of which the environment is composed, we shall be likely to see the animal as a part of this one system and not as distinct from its surroundings. The multiplicative view of uniformity and diversity

as contrasted with the vague, lumpish view often taken has many other advantages.

In uniformities and diversities the group principle involved is built around with certain auxiliary categories, or efficiency handles, which should at least be mentioned.

Relations to other Concepts and Efficients

As to the number idea, we find an implication at least of duality in every uniformity; for a uniformity implies a comparison with some other entity taken as a standard. Similarly a difference implies at least two compared objects. This duality in a uniformity or diversity has certain uses since, in effect, one of the two objects constitutes an externality to the other.

Externalities of another nature are often implied in a uniformity or diversity. Thus the evenness of the surface of a body of water is the resultant or intersection of two opposite sets of forces, those which draw objects together, and those which often cause objects to repel each other. So the peculiar individuality of any object, as of a piece of gold, is due to the convergence in the object of several qualities or properties which are present in many other objects besides gold, and hence in a measure are external to the gold. Similarly any quality of an object may often be regarded as the focus of several widely existent elements.

Uniformities and diversities imply or depend on certain limitations. Thus objectively every uniformity is hedged in by certain diversities, and correspondingly every diversity by uniformities. Subjectively, the efficiency of uniformities and diversities is limited by the degree in which we are cognizant of them. Hence in this and other ways much relativity arises in their application.

Certain other relations between uniformities and diversities should be mentioned. Thus with respect to order, as already stated, diversity as a rule comes first psychologically, while uniformity is first logically. With respect to the principle of orders of material, diversity is more concrete (therefore individual and qualitative), uniformity more abstract. With respect to force, diversity is more active, and uniformity more passive.

Diversity relates primarily to quality, uniformity to quantity.

In relation to the idea of efficiency, diversity is directly essential to this idea (since efficiency means surplusage) while uniformity is an indirect route to it.

Definitions

The terms in current use for various kinds of uniformity and diversity are loose, overlap, and are often difficult of precise definition and coördination. However in the course of different discussions it is frequently necessary to use some of these terms, and it is therefore important to define them as accurately as possible with reference to uniformity and diversity as sources of efficiency.

The following are among the most important terms of the uniformity class.

Uniformity means primarily oneness of form, amid various diversities such as those of position, material, color. Hence it is primarily a spatial uniformity. However its meaning is often refined and extended so as to include almost every kind of oneness. Thus we may speak of objects as uniform in color or in any other quality.

Equivalence means primarily sameness with respect to weight; stated more generally it means, equally availing. Hence it is primarily a dynamic term. As often used however it has outgrown its original limited significance.

Continuity means literally a holding together (against opposing forces). Hence it has spatial and negative dynamic aspects. It is a relation without gaps, that is, an unbroken relation, and usually contrary to certain appearances. It may also be one, two, or three dimensional.

Similar efficiency descriptions might be given of other terms belonging to this general class such as accuracy, analogy, adaptation, conservation, consistency, constancy, consecutivity, certainty, equality, evenness, homogeneity, immobility, identity, likeness, monotony, monism, necessity, oneness, persistence, parallelism, parity, reality, reliability, resemblance, similarity, stability, simultaneity, sameness, truth, unity.

Of terms in the diversity class the following call for particular mention.

Diversity means literally a bearing or turning apart. Hence the term originally had a dynamic as well as a spatial aspect. But in its present generalized form it may be used to express almost any difference.

Change means diverse forms of the same object occurring in succession. Hence it involves an element of time. Its meaning in other respects is vague and general.

Similar efficiency descriptions might be given of the other diversity terms in current use such as of alteration, conversion, deviation, discontinuity, dissimilarity, flexibility, fluidity, freedom, heterogeneity, instability, manyness, mobility, modification, mutation, plasticity, relativity, sensitiveness, unevenness, unlikeness, variation, variety.

METHODS OF APPLICATION

Group View an Aid

The view of uniformity as not mere vague lumpish likeness and of diversity as not accidental or capricious difference, but of both as essentially groupish in nature, and when taken together, as forming certain useful stages in the multiplicative system of things, is an important aid in using these instruments in the most efficient way.

Great Number of Forms

Thus the multiplicative view is an aid in realizing the vast number of diversities and uniformities which exist or may be generated. For example in order to form some idea of the different kinds of apples which are possible we may consider the following product:

$$(\text{no. possible sizes}) \times (\text{no. shapes}) \times (\text{no. colors}) \times (\text{no. flavors}) \times (\text{no. durations}).$$

It is also well to remember that some of the above factors are also multiplicative group systems. Such a method of treating diversities enables the farmer for instance to forecast the great variety of ways in which a horse or fertilizer may be defective, or a teacher to grasp the immense diversity in young minds, and of methods and devices which may be employed to meet special needs.

Illustrations of Extreme Cases

In reinforcement of the above conception it is important to bear in mind certain facts or principles like the following: The divisibility of matter is practically without limit, for the molecules in a cubic centimeter of gas if placed in line would go many times round the world and would take 265 billions of years to count and yet a molecule is composed of many ions. Also different kinds of matter interfuse so that it is said that "a perfectly pure chemical element is the dream of the ignoramus". Also relations of kinds of matter are

in a constant state of change since motion is the normal law of the universe. Even the force of gravitation on the surface of the earth is constantly changing as the earth shifts its position with reference to the sun and moon. The number of diversities man can create or conceive of in like manner is without limit since for instance the grey matter of the human brain contains 5,000,000,000 cells each of which apparently may be a center or modifier of ideation in many ways. A cubic millimeter of blood contains 5,000,000 red corpuscles, or the blood in a human body on the average contains 22,500,000,000,000 of them, and with changes in these, the bodily health and mental processes vary.

With respect to uniformity no one as yet has been able to discover a variation in the atomic weight of any chemical element. In color photography of a certain kind, two successive laminae in a gelatine plate are separated by a distance of only one one hundredth millionth of an inch, yet those laminae retain their exact position during the process of applying chemical solutions to fix them. We also have the general principle of continuity, often embodied in the statement that nature never proceeds *per saltum.*

The principle of multiplying relations also enables us to realize the extraordinary power in a small diversity. Thus a difference of one-tenth of one per cent in the amount of carbon in steel may render worthless an otherwise good piece of metal.

The Multiplicative Principle an Aid in Other Ways

The multiplicative principle also aids us in utilizing crude and imperfect uniformities and diversities when better forms are not available. For, as has been pointed out in other connections, the multiplicative principle in a measure is both self corrective and has the power to develop the imperfect into something better. An illustration is the use in combination of our crude theory of atoms (or diversity) and the ether (or unity) and the vast number of useful results thus obtained. A still broader case is the employment together of monistic and pluralistic theories as toolage aspects of the universe.

Externality an Aid to Greater Results

Externality is often a powerful aid in discovering a uniformity or diversity in a difficult situation. For within a large area of facts,

one is more likely to find a spot or spots where some important diversity or uniformity is unmasked and exposed to view than in a narrow area. So subjectively the externalities afforded by a broad education or by wide travel are fitted to break up the permanences of habit and tradition.

To look at the matter in more detail, the externality of time has enabled men to discover the change in position of many so-called fixed stars, and to learn that mountains rise and fall, and that some rivers are more stable than mountains. So the externality of travel and study enabled Darwin and Wallace to establish the fact of the variation of species and to assign an adequate reason for the same. By the aid of the gelatine culture plate Koch was able to isolate different species of bacteria. Externality in the form of a fly wheel may give uniformity and continuity to the motion of an otherwise unstable machine.

A more abstract form of externality in this connection is that whereby we get a diversity by the use of another intermediate diversity as when we form a conception of a new kind of space by analyzing the ordinary space concept into its essential elements and varying or changing one or more of these elements.

Treatment of Difficult Cases

In forming or discovering diversities in difficult cases it is often a help to try to obtain a diversity with respect to certain important categories taken in succession. Thus in such cases attempts may be made successively to form, for instance, diversity as to space, quantity, number, or time; or a diversity of velocity, force, order, color, mentality, or any other quality or category. A similar remark applies to uniformities.

It may also be well to remark at this point that the discovery of diversities usually precedes that of uniformities.

In this connection something should also be said as to the importance of determining whether to give a uniformity or a diversity the leadership in a given situation, which species of each to use, and how best to combine them. For instance it is sometimes necessary to decide whether it is best to put all of one's eggs in one basket (as in making investments); or to determine whether, in a given case, to live each day by itself, or to treat life as a whole. Externality, especially the externality of consequences, is useful in such cases.

Other Efficients as Aids

The various categories of efficiency which have been investigated in the preceding chapters, as well as those which are to be studied in the chapters which follow, may also be used to advantage in connection with uniformities and diversities. Thus it is well to remember that it is advantageous to master and reuse certain important types of uniformity and diversity, and also that every case of reuse produces a uniformity along its path. Again by the application of predictive reuse we realize that certain uniformities and diversities are in a sense omnipresent. Thus we may argue that, in order to survive, every organism must reach a certain level of efficiency. Hence we may anticipate that every existing organism possesses its own characteristic instruments for obtaining results; that this instrumentalism differs widely in its forms but approximates a certain uniformity of power.

The method of multiplicative groups is also an aid in grasping (or inventing) a large variety of uniformities or diversities and in selecting that particular one in the array which is best fitted for a given purpose. It also leads to the use of uniformities and diversities in other and more aggressive ways. Negatively it prevents the ignoring and dread of using the profusion of uniformities and diversities which exist.

It is also important to apply the principle of orders of material to uniformities and diversities as an aid in discriminating between higher and lower forms. Thus the caste system of India is characterized by a uniformity and diversity both of which are of a low order. For the uniformity involved is so concrete and fixed that it interferes with useful change, while the diversity is of such a nature that it prevents uniformity of reward for efficient action. Another illustration of somewhat different kind is the contrast between the kind of diversities which are utilized when a drug is dissolved and absorbed through the stomach and general fluids of the body, and the diversities which are involved when the drug is first ionized outside of the body and then driven by an electric current into that particular part of the organism where it is needed. The former method because of the many unnecessary disturbances which it produces (that is, its lack of uniformity) must be regarded as of a distinctly lower order of efficiency than the latter.

In connection with the limitations which hem in the useful functions of uniformity and diversity (see pp. 138, 140) it is well to

remember that in many domains a certain average or normal degree of uniformity and diversity are desirable. Thus in certain relations the peculiarities or bizarre features of the English system of weights and measures make it more highly educative than the more uniform metric system. A similar remark applies to the English language as compared with an artificial language such as Ido or Esperanto.

Ideal Species

Finally, it is well to realize and work toward the ideal system of uniformities and diversities. For our purpose this consists of uniformities and diversities in one or more of the Efficients, applied in efficiental ways, for efficiental ends.

It is also well to have constantly in mind the best form of uniformities and diversities which is directly applicable to affairs in general. This consists of uniformities and diversities combined as a multiplicative group system and in such a way that each uniformity will interfere with the diversities to the least possible extent, and vice versa. Contrasted with absolutism at one extreme and laissez-faire individualism at the other, the form just described constitutes a federal system characterized by the utmost freedom and the utmost law and order. Practical success of a high order in any calling consists primarily in realizing the enormous number of diversities involved, the great number of uniformities or ways in which groups may be formed, and in uniting and utilizing these diversities and uniformities by means of multiplicative group systems of a higher order.

Summation of the Chapter

A condensed statement of the principles discussed in this chapter will now be of service. The perception and use of uniformities and diversities form a source of efficiency which is often of fundamental importance. Owing to their general and comprehensive nature, the principles under discussion exist in a multitude of forms, though much study is often necessary to uncover certain of these. The simple and complex forms which they assume are alike numerous.

Uniformities and diversities in some cases, in essence constitute crude, incipient groups; in other cases they are a highly finished final form of the group. Consequently in all cases they have the

efficiency properties of groups in special forms. They are especially useful as first steps in developing more elaborate efficiency organizations or processes. Often, apart from any consideration of the group idea, uniformities and diversities lead directly to reuse and other forms of efficiency and even to transcendent results. Numerous more or less familiar words like analysis and mobility illustrate the uses of diversity; and similarly words like justice and reliability suggest the values in uniformity. The progress of the individual and that of the human race alike, are marked by the perception and use of more and more vital and comprehensive uniformities and diversities.

The general multiplicative principle is an aid in realizing both the vast number of kinds of uniformities and diversities and the extreme forms which these may take. The self-developing and self-corrective powers inherent in the multiplicative principle also constitute an aid in using crude forms of uniformities and diversities. It is important to be able to realize the highest types of resemblances and differences, and to use these with the utmost effectiveness. Especially important is it to realize that the highest species of all is the combination of those uniformities and diversities which are individually most efficient, which aid each other the most, and interfere with each other the least.

EXERCISE 8

1. Give an example of each of the principal species of diversity.
2. Also of each of the principal species of uniformity.
3. State some of the advantages in having two engines in a factory instead of a single larger engine.
4. Give a similar instance where duality is the source of efficiency.
5. State some of the advantages in being able to separate water into small particles of vapor.
6. State the uniformities and diversities which enter into the structure of a Zeppelin airship.
7. State some of the advantages which follow from the fact that the action of one force, as gravitation, is independent of that of another, as light or heat.
8. In some factories the accounts for the year are kept in terms of thirteen periods of four weeks each (instead of the twelve calendar months). State the advantages in this.
9. Ascertain some of the peculiarities of different kinds of Jena glass, and the corresponding efficiency properties.

10. Define each of the following terms with reference to the principle of diversity, and give an example of the efficiency value of each: flexibility, plasticity, mobility, adjustability.

11. In a very dry summer at the Kansas Experiment Station, on ground prepared merely by disking, only 4 bushels of wheat per acre were produced. On ground deeply plowed and thoroughly pulverized, the yield was 38 bushels per acre. If the cost of cultivation per acre in the two cases was $.75 and $4 respectively, and the wheat produced was worth 90c per bushel, what would have been the gain from the more thorough pulverization of 1000 acres?

12. Give in detail some of the advantages in thorough pulverization of the soil in raising crops.

13. Describe the block system of managing a railroad, and state some of its advantages. What elements of uniformity and diversity are here involved?

14. What are the advantages to a large railroad company in having all the stationery used in its offices reduced to a few standard types?

15. State some of the advantages in knowing that the different forms of energy in action on the earth's surface are, with slight exceptions, transformed sun force.

16. Give some of the advantages of regarding a given form of work, as that of laying brick, as a succession of parts or motions.

17. In what ways does the use of machines give uniformity and accuracy? Show that this uniformity is sometimes excessive.

Give an example where a diversity leads to

18. Marginal reuse
19. Prophetic reuse
20. Formation of a group
21. Of a multiplicative group system
22. Another diversity
23. A uniformity
24. A useful directive act
25. A transcendent result
26. Complex self-developing results

27. Give examples where uniformities produce the results indicated in Exs. 18-26.

28. Give an example where a diversity is obtained by the use of externality.

29. Where a uniformity is obtained by externality.

30. State the advantages in preparing lumber in pieces of small thickness and nailing these together when heavier pieces are needed. Also state the disadvantages.

31. State some of the different ways in which the lights of light houses differ, and also state the resulting advantages. Why are light ships or buoys sometimes used instead of light houses?

32. By what means is the effect of solid space produced by a flat picture?

33. In terms of the Efficients studied thus far, state the efficiency advantages of honesty.

34. Give an example where an excessive amount of diversity results in loss.

35. Another where diversity misused in some other way results in loss.

36. Where an excessive amount of uniformity results in loss.

37. Where uniformity misused in some other way results in loss.

38. State some of the disadvantages which result from the fact that the gold dollar fluctuates in value. Suggest some means by which this evil might be diminished, if not altogether remedied.

39. How are the values of the uniformity and diversity principles illustrated by the use of the wheel?

40. What uniformity and diversity principles are illustrated in a cream separator?

41. Why is it that a snake can swallow an object larger than its own head (thus a boa constrictor can swallow a calf)?

42. If each State in the United States should have a different system of weights and measures point out some of the disadvantages which would result.

43. How does the principle of diversity help give sea power the superiority over land power?

44. Explain standard time with reference to uniformity and diversity principles.

45. Arthur Young says "the magic of property turns sand into gold". State the source of efficiency which is involved in this case.

46. Give an example where two diversities are combined to form a uniformity.

47. Also where two diversities when combined make a greater diversity.

48. Discuss the value of accuracy in terms of the other Efficients.

49. State some of the advantages of isolating oneself or one's employees from distractions when at work. Give also instances where such isolation results in loss.

50. Give an example showing the value of knowing that action and reaction are always equal.

51. What is meant by uniformity of the laws of nature? Illustrate. Also give three illustrations of the efficiencies which result from the utilization of this uniformity.

52. Illustrate the advantages which result from the fact that the atomic weights of the chemical elements do not vary.

53. State the successive kinds of freedom which have been obtained by the people of the United States, and the new groupings and other advantages which have resulted from each.

CHAPTER IX

EXPENDITURES AND RESULTS

General Statement

In Chapters II to VIII primary attention has been paid to methods by which to attain increased efficiency; that is, to the relations or apparatus which intervene between an expenditure and the result obtained, rather than to the expenditure and result themselves. It will be an advantage also to give some primary consideration to the data and results. For instance, it is often an aid to efficiency to make it a rule to strive after those results which are most inclusive and multiplicative in value while at the same time making the data employed as inexpensive as possible; in other words, to realize that the efficiency of a process is in proportion to the gap between the expenditure and the result. A comprehensive review of the various aims and ends of processes will be useful as a means of forming even an approximate scale of values in any situation. Since the result of one process may be data of another process, the study of possible expenditures and results should also facilitate the linking of such processes together so as to form more or less comprehensive systems for obtaining desired ends. If even a rude common essence of values could be obtained, other equally or even more important advantages would follow. Accordingly the expenditures and results of efficiency processes will be the main object of investigation in the present chapter. Primary attention will be given to results aimed at, expenditures being considered largely by implication.

CLASSIFICATION

Results Grouped according to Materials

We shall first classify the aims and results of efficiency processes according to the materials of which they are constituted. Proceeding thus we obtain the following groups of aims:

Physical objects or *forces,* including, for instance, land, buildings, mines, tools, machinery, light, heat, mechanical power, speed, and all other species of so-called material inorganic wealth.

Biological results, as plants and animals and their useful products, as food, warmth, shelter, life, offspring.

Personal or *psychological* results, as pleasure, power, health, leisure, knowledge, education, comfort, welfare, well-being.

Social results, as position, fame, friendship, love, service; social means to these ends, as social and political organization; and complexes of these results, such for instance as are included under a term like civilization.

Ethical and *religious* results, as character, the approval of one's conscience, and religious conversion.

Abstract formal results, as space, time, freedom, reuse, groups, and the various categories and Efficients both as means to ends, and also, from certain points of view, as ends in themselves. In the past the uniformity or equivalence aims which come under this head, such as peace, truth, and reality, have been unduly emphasized as ultimate ends, though perhaps they cannot be too greatly emphasized as means to ends.

Evidently one of the above results (especially some item in one of the later classes) often includes or quickly brings others. Thus good government often means wealth, health, education, and happiness to many citizens.

Results Grouped according to Forms

The results of efficiency processes may also be classified with reference to their form.

Thus with respect to *size* results aimed at may be large or small; one, two, three, or n-dimensional. With reference to other spatial categories, they may be local or omnipresent; may have various shapes; may occupy different positions, as when they are internal or external to data. With reference to *number,* they may be single or manifold.

In this connection it is especially important to observe that they may be *direct* or *indirect*. Thus adaptation to one's environment is not so much an end in itself as a means to an end. From our point of view, as has already been remarked, equivalence results (as truth, reality, accuracy, justice) are not ultimate, but proximate and intermediate. A mere means to a result often becomes of such importance that it comes to be regarded as itself a more or less ultimate end of action. This being the case, from certain points

of view it is often useful to sum up the equivalence and more ultimate ends of action in couplets like the following: light and sweetness, truth and love, science and sentiment, the universe as mechanism and the universe as idea, certitude and utility, realism and idealism.

With reference to *time* results may be temporary or permanent, and may come quickly or slowly. Similarly they may be reciprocal (with the data) ; positive or negative; simple or complex. Instances of complex results are all the beneficial changes included in a case of wholesome growth, as for instance the change from the acorn to the oak, or the results of civilization.

The beneficial results of an efficiency process may occur in a *grouped* or multiplicatively grouped form. An illustration of grouped results is knowledge as including health, wealth, fame, service, and happiness. Often in this connection relativity is present. Thus each item in the list of the results just mentioned may lead to or include the others. Thus health often leads to knowledge, wealth, happiness, etc. In this manner groups are obtained which may be regarded as sub-groups in a larger system.

By-products

A somewhat important method of classifying results in this connection is that which makes a distinction between direct results and by-products. Thus cattle are slaughtered primarily for the meat which they supply as food. But when the process is rightly conducted a host of by-products is also obtained such as hides, oils, oleomargarine, stearine, tallow, soap, fertilizers, medical extracts, dye stuffs, buttons, glue, bone, charcoal, some hundreds in all. The externality of carrying on work on a large scale often makes by-products particularly important. In highly competitive work, the by-products may become the chief source of profit. Much relativity frequently comes into play in determining what is a main and what a by-product. The side results are often abstract and scarcely observed, but at the same time of prime importance. Such for instance are experience gained, particularly knowledge and power concerning efficiency processes.

Scales of Results

As a continuation of this line of thought it follows that the distinction between results as of higher or lower order is of the first importance. A graded series of results may be obtained with reference to various categories. Thus with reference to the perpetuation of the race, the aims and ends of processes in the order of their importance are self-preservation (including food, shelter, life); self-propagation (offspring); happiness; self-sacrifice.

The matter of arranging all results in a general and inclusive multiplicative scale will be considered in connection with the analysis of surplusages as a whole and their comprehensive marginal multiplicative grouping.

Before taking up this topic, it will be of advantage briefly to analyze certain terms which are often used to express the results of processes. One of the most important of these terms is the word value.

Value and Worth

The term value as currently used has two widely different meanings, which are often confused, viz.: (1) an equivalence meaning and (2) a surplusage meaning. Thus in the phrase "the value of a gallon of oil" the first or equivalence meaning of the word value is the amount of money that may be obtained by selling the oil (i.e., money value); or the amount of heat generated by burning the oil (i.e., fuel value); or the amount of coal which when burned will produce the same amount of heat as the oil (i.e., coal value).

The second or surplusage meaning of the words is the superiority of oil with respect to another kind of fuel (which is equivalent in heat value), as the superiority of the oil in concentration, lightness, rapidity of combustion, and power to give a war vessel a large radius of action and consequent capacity to win naval battles. In like manner it may have meanings of these two kinds in other ways and in comparison with other substances, for instance as a kind of lubricant or an insecticide, or as a source of coal-tar and its products, or as a means of calming stormy waters at sea.

In this book the term value is used as far as possible in the second or surplusage sense.

The term worth has two similarly contrasted meanings. These

are illustrated in the sentences, "This house is worth $2000", and "Mr. X. is a man of great worth".

ANALYSIS AND MULTIPLICATIVE GROUPING OF THE AIMS OF EFFICIENCY

If we attempt to tabulate the aims and ends of efficiency processes in a comprehensive, hierarchical, and completely filial form by the customary method of progressively including sub-aims in larger and larger aims, we are hindered by the complexity and cross relation of many of the value concepts involved and by their qualitative incommensurability. However, we may make what is, in some respects, an even more useful systematization of the aims and ends of efficiency processes by analyzing efficiency aims into a scale of constituent elements and, by use of these elements, building up values or aims into a marginal multiplicative group system, general and undeveloped in some respects, but serviceable in many more or less fundamental ways.

Four Elements in Valuable Results

The first and most fundamental constituent in the result aimed at in any efficiency process is that of non-equivalence with respect to the datum or expenditure, or a certain disproportion between means and ends. This non-equivalence may be positive or negative, that is, may represent gain or loss. It may be quantitative or qualitative. When it is quantitative it may be named a surplusage. When it is qualitative (as when physical exercise produces good health) it is useful to realize it as a fact, to regard certain cases of it as units or elements, and to reuse, group, and extend them systematically.

The second constituent in the results with which we are particularly concerned is a certain positive or plus quality. It often requires much externality to determine this, yet it is an important source of efficiency to recognize it as a distinctive element in results sought after, and to regard it in its most fundamental forms as primarily impersonal or subpersonal. Thus groups as such and as a whole have values and uses which are practically universal and only occasionally in the systematic scheme of things take personal form.

The third principal constituent in efficiency results is that supplied by the element of personality in its various forms. It is this which, when annexed to positive non-equivalence, causes it to become happiness or well-being.

The fourth class of constituents are those more or less concrete elements which when annexed cause positive non-equivalence to take such special forms as food, or offspring, wealth, or fame; or which cause personal positive non-equivalence to take the form of special sensations of pleasure as that of warmth, or some form of pleasure which is related to any special organ of the body or faculty of the mind.

It is not necessary that all of these four classes of constituents be present in a given result. Thus if a process produces a new and better system (or groupage) only the first two constituents are present in the result obtained. In a sensing of beauty or sublimity only the first three are present.

This analysis at least makes possible an approximate multiplicative group system of values in any given situation and opens the way to many directive and other efficiency processes in the treatment of such values.

Illustrations of the Four Elements

Specific illustrations will aid in making clear the meaning and general uses of the above analysis and the multiplicative group synthesis thus made possible. A careful examination of the acts of a boy, for instance, shows that the dominant source of pleasure in a boy's nature is fundamentally that of generating or obtaining a surplus as such, any immediate concrete gain to himself being a secondary matter. Many of his acts of mischief are performed not because he delights in destructive deeds as such, but because he finds his greatest pleasure in a mere disproportion between means employed and the results which follow. Hence by sufficiently tactful treatment he may, if he is normal in body and mind, almost always be led to conduct which will give this same sort of pleasure, but will also be profitable in other ways to all concerned. For a like reason a child is often more interested in working out and operating a puzzle which has no concrete use than by making money by raising crops in a garden. In like manner the able busi-

ness man frequently finds more pleasure in business as a game, that is as a mere means of obtaining marvellous results, than in the yacht or paintings which financial profits enable him to buy and enjoy. So, also, many persons read the newspapers primarily for the new sensations which they thus experience, that is the new pleasure which they obtain from the perception of new and strange diversities, externalities, and groupages, rather than from the direct concrete utility of the new facts which are assimilated. The dominant surplusage principle in both of these cases is sound as far as it goes but it needs careful safeguarding and directive treatment in connection with the other constituents of efficiency results.

A somewhat higher case is that in which a more fundamental pleasure is found in harmony, beauty, and sublimity, or in living a highly moral and broadly sympathetic life (that is, in highly developed forms of groupage and externality) than in food or money, which indeed are often mere detailed consequences of the primal efficiency principles involved in the former cases.

Advantages in the Use of the Four Elements

The efficiency advantages which flow from the preceding analysis and systematization of values, call for some more formal and extended statement.

In the first place it is to be noted that the first and more fundamental and therefore comprehensive constituents in values (viz.: positive non-equivalence or groupage) are practically omnipresent and in a vast number of cases may be had at little or no cost; also that by obtaining them thus the more specific forms, those which are usually expensive when sought for individually, may be obtained at slight cost by the use of the marginal reuse. Thus every occupation, however humble in its conventional rating, as cooking or boot-blacking even, is saturated in many ways with the various Efficients which may become sources of the highest pleasure when properly understood. For all persons also the sight of a sunset costs nothing, and when to this is annexed some knowledge of the principles of esthetics and art gained from books or art galleries at small cost, highly specialized forms of pleasure and value are obtained at slight expense. By pursuing methods like these the way is also often opened for the attainments of results more or less in groups instead of singly.

It follows also that the prime source and method of social efficiency is to generate or create positive non-equivalence constantly and in all possible ways quite regardless of all personal considerations, or at least putting all such considerations in a secondary place. New light is thus thrown for instance on ethical principles. For the method of life thus indicated includes as a detail the doing of good to those who have annoyed us. At one stroke it eliminates revenge, selfishness, heartless fun, pessimism, and substitutes something better. It puts constructive work and plusness in all forms in the foreground actively and constantly. Personal considerations are set apart but are always within hailing distance. It makes it possible for each individual to enjoy the primal essence of all things not only without interference with others but rather by primarily aiding others. It follows, of course, that this view makes one expectant of and ready to utilize all new forms and kinds of useful results.

Other Advantages

One or two matters of detail should also be mentioned in this connection. If positive non-equivalence is the primal essence of values it follows that a method, at least approximate, is thus supplied of ranking and ordering the various means of obtaining results in a given field and hence of accelerating and otherwise improving the attainment of these results. Thus in educational work those studies, as a rule will be most valuable which give the greatest surplusage and groupage, or the greatest groupage powers. The same remark will apply to other callings or departments of life and work.

A still more general result is that the above treatment of values gives a comprehensive, sound, and vital scheme of efficiency, all parts of which reinforce each other.

Drawbacks and Their Remedy

At the same time mention should be made of the fact that the above method of viewing and treating the results aimed at in efficiency processes has certain drawbacks. One of these is a frequent coldness and lack of force due to the absence of personality. Another is the fact that putting non-equivalence (instead of some

form of uniformity) in the first place may open the way under some cases to incoherence and lack of system as when each person in a community tries to form his own system of values. However, a closer view of the matter will show that these defects are largely superficial. For the outgoing power inherent in the multiplicative principle will be in the end remedy the first of these defects and the corrective power which is also inherent in the multiplicative principle will check and remedy the second.

Advantages Illustrated by Human Progress

As a general illustration of the advantages to be gained in efficiency processes by making subpersonal positive non-equivalence the primary and most fundamental element in the results sought, it may be well to point out that during the last five hundred years human progress in general has largely been determined by the progress of science, and that the most important factor in the increase of scientific knowledge has been that of making the subpersonal point of view fundamental. Thus modern chemistry arose after the spirits of the medieval alchemist had been discarded. The astrologist regarded the planets as animated and personal. Even Kepler viewed the earth as analogous to a huge animal, and Newton was able to establish the law of gravitation only by disentangling the problem from all immediate considerations of personality. On the other hand science perhaps has erred at times by trying to eliminate the category of personality altogether, instead of putting it, at least for the time being, in a secondary place. That is, it has reacted too far from the medieval point of view. Apparently also it has erred frequently by exalting mere equivalence (or "truth") above the fruitful results of equivalence, that is above positive non-equivalence and the various special forms which this assumes. Hence the results obtained by the scientific method in the past would seem to be but an imperfect glimpse of those obtainable by a broader and more exact view of efficiency.

Relation to Chapter I

Before concluding the present chapter it will be well to consider briefly its relation to Chapter I. In Chapter I the main discussion related to quantitative homogeneous efficiency. Cases of hetero-

geneous or qualitative efficiency were treated only by the ratio or comparative method. In the present chapter the intention has been to treat qualitative efficiency by a more direct method, viz.: by a comprehensive view of such cases, an analysis of them into a progressive series of elements, and the synthesis of them into a more or less complete multiplicative group system. This method includes the methods of Chapter I as special details and hence gives a more comprehensive and aggressive view of the whole matter under consideration.

Summary and Outlook

The substance of the present chapter may now be briefly stated. The study of the aims and ends of efficiency processes is important both in itself, and also as a means of improving these processes. Aims and ends may be classified as to the materials composing them (viz., as mineral, psychological, social, etc.); and also according to their form. An important distinction in the latter respect is that between the main result and a by-product.

Aims and ends of all kinds are analyzable into four principal elements, viz.: (1) non-equivalence (between datum and result); (2) a positive quality; (3) the element of personal application; (4) all other constituents. By means of this analysis we find that, with respect to efficiency processes, the most important and omnipresent constituent is positive non-equivalence. The subpersonal nature of this aim enables us to strive to create and utilize values in an unlimited variety of ways and with great freedom. In particular it prevents much waste, opens the way to the utilization of many comparatively costless sources of pleasure, and to an ordering of and grouping of values in cases where such a process would otherwise be difficult or even impossible. It also gives that breadth of view without which the highest and most continuous efficiency processes are impossible. Hence it is not surprising, that, as the world has advanced this view of the aims and ends of conduct has become more and more prominent.

In the present chapter and those which precede it we have discussed the meaning of the term efficiency the principal general methods by which efficiency may be obtained or increased, and the ends and aims of efficiency. In the chapters which immediately

follow we shall investigate certain more special methods of obtaining results than those considered in the preceding chapters. The first of these more individual efficiency agencies relates to man's cognitive faculty. Others that follow are related more particularly to his powers of volition or action.

EXERCISE 9

Give an example of each of the following classes of the fruits of efficiency processes:

1. Inorganic material	6. Imaginative	11. Spatial
2. Dynamic	7. Moral	12. Reciprocal
3. Biological	8. Religious	13. Negative
4. Emotional	9. Social	14. Indirect
5. Intellectual	10. Temporal	15. Grouped

16. Multiplicatively grouped

17. Give a set of results which form an order of materials series as to their relative importance.

18. State the different beneficial results (including efficiental ones) involved in shelter.

19. In wealth 20. Knowledge 21. Power 22. Health

23. Give a case where a means to an end has become, in effect, an end in itself.

24. Give a by-product in the manufacture of butter. Of pig iron.

25. Give a set of desirable results each of which, in some respects, includes the others.

26. Name some of the by-products which accompany the manufacture of kerosene for illuminating purposes.

27. Name the by-products of some other process with which you are familiar.

28. Give an instance where a by-product has become more important than the original principal product.

29. Give an instance of the conversion of a source of loss into a source of gain.

30. Explain the different possible meanings of the word value in the phrase, "value of a bushel of wheat".

31. Give an example of the use of the word "worth" both in an equivalence and in a surplusage sense.

32. Why is it that a man who receives, at the end of a year, 5 per cent in addition to his principal, is often regarded as receiving an equivalent for his investment; a man who receives 8 per cent., as making an efficient use of his money; and a man who tries to obtain 300 per cent (as by investing his money in mining stocks), as attempting to obtain something for nothing.

33. Discuss the various ways in which pleasure in one's work is a source of efficiency and give illustrations.

34. State the various kinds of pleasure which a carpenter may experience when building a house. Also state in some detail the efficiency results of such pleasures.

35. Make a similar statement concerning a teacher and his or her work.

36. Discuss some of the various devices which an employer may use in causing his workmen to take pleasure in their work.

37. Also those which a person in a given calling can use in developing his own interest in the same.

38. Discuss the subject of attention in connection with efficiency.

39. Name a number of efficiental sources of pleasure which are widely present and cost little or nothing.

40. Explain the distinction between: "value in use" and "value in exchange", as used by political economists.

41. The term truth has come to include both equivalence relations and also the surplusage fruits of such relations. State certain truths and also the equivalence and surplusage elements in each of these.

42. State the various useful results, both immediate and more remote, which may be the outcome of a successful teacher's work.

43. State the same for money expended in improving the water supply of a city.

44. Ascertain what is meant by the term "epigenesis" and show how it illustrates the idea of surplusage.

45. Explain what is meant by synthetic judgments, and in what respect they illustrate the principle of surplusage.

CHAPTER X

SYMBOLISM

Introductory Illustrations

When a merchant in Chicago wishes to buy goods in New York, by writing a letter he can often save himself the expense of a journey to New York. Or, going a step farther by sending a telegram he can diminish in an important degree the time required for obtaining the goods. This latter advantage often leads to other advantages, such as the rapid reuse of capital or the making of a sale that would otherwise be lost. When a scientific investigator makes a discovery, by publishing the fact in a journal he opens the way to its immediate and wide reuse. Also he often finds that in the process of stating his discovery in terms of concepts in general use, he develops and extends the new fact or principle. When a surveyor enters in his notebook a drawing of the route surveyed and also numerical and descriptive facts relating to it, he prevents mistakes which he might make if he trusted to mere memory, frequently saves himself the labor of making a resurvey, and also puts the facts obtained in such a form that they can readily be grouped in various forms and made the basis of efficient action by himself and others in different ways, as in building a railroad.

The above instances illustrate the fact that the use of symbols may be an important aid in obtaining useful results. Accordingly symbolism is the next primary agent of efficiency which we shall study.

CLASSIFICATION

Cases of symbolisms might be grouped and named according to the objects or materials represented by the given symbols, but such a classification is not important for our purpose. It is well to observe however that any object or entity may be denoted by a symbol and usually by many different kinds of symbols. Extreme but important cases are the representation of nothing (absence of material) by some sign, as the absence of a number by the zero mark; or of an unknown object or number by a sign, as x; or of a partially known object by a symbol, as in certain cases in chemistry.

Objects Usable as Symbols

More valuable for our purpose is the classification of symbolisms according to the material of which they are composed.

In the first place we find many *material inorganic objects* employed in whole or part as signs or marks. Examples are dress, flags, ink, paper, pencil marks, paint, idols made of stone, certain fetishes, incense, clouds, the wind, the ocean, the barometer, thermometer, etc. Such symbolisms are further illustrated by quotations like the following: "He is pure gold", or "He is a star".

Also a *physical force,* as gravitation or electricity, may be used as an expressive representation of some other kind of power. Thus we speak of a weighty idea, of a magnetic person, or say that a man is a live wire.

Of especial importance in this connection are light, color, and sound. Examples of the employment of light and color in symbol form are the black and white of the printed page, color as it appears in paintings, flowers, the warning colors of animals, and certain signals in railroading. Color often has abstract and conventional meanings. An instance in the use in art of red and green to denote the concrete, and of blue to express the abstract.

Among the sounds which are recognized or used as symbols are thunder and various other sounds made by inorganic objects, the sounds made by birds and other animals, the cry of a baby, the yell of a savage, the drum signals of the African, spoken articulate language, and an organization of sounds such as is found in a symphony or fugue.

A *living object* or a combination of living objects may often form a useful symbol. Thus green grass denotes the presence of water, and a buzzard the proximity of carrion. Living objects are used as symbols in a more abstract way, as when we say, for example, that a certain man is a lion or an eagle.

In like manner it is often advantageous to use a part of a living object as a symbol for the whole object. Examples are the recognition of a plant by means of its leaves or flowers, or an elephant by his trunk. So a beard is sometimes a convenient symbol for a man, and a criminal may be identified by the marks on his finger tips. So a smile or a gesture may express a state of mind.

Similarly a *mind,* or a mental state, attribute, or product frequently

forms an effective symbol. Thus a feeling of pain often represents danger or loss; and a certain feeling of pleasure indicates gain or advantage.

Certain sensations are in some way representative of the outside world and must be interpreted as such. Thus some sensations of taste or smell indicate foods, and others represent poisons.

So certain acts symbolize certain degrees of knowledge or states of feeling. Also a mental peculiarity may represent the past education of a person, or his early environment, or, in a measure, his parentage.

An *entire personality* may also be a symbol as when an ambassador represents a nation. In certain cases a personality may represent more or less impersonal data. Instances are the use of a personal concept like angels, devils, demons, gods, to represent a natural force.

Spatial Objects as Symbols

The different spatial categories furnish material for symbols in various ways. For example a *group of points* is frequently used to represent a set of objects of any given kind. Or one point may stand for a number of points or objects. Such a point is often termed a focus. The Braille printed alphabet for the blind consists entirely of aggregates of points. So a *line* may be made to stand for an object in various useful ways, or for a motion, a velocity, duration, relation, etc. Specific illustrations are supplied by various diagrams used in scientific works. Similarly we have lines of force and lines of cleavage. The bounding lines of an area, as of a country, frequently form a useful symbol for the whole area. Similarly a solid object may be denoted by its bounding lines. The letters used in writing and printing are made up essentially of lines. Punctuation marks are points and small lines which indicate to the eye the main parts of the group structure of a sentence.

An *area* may represent the product of any two factors. Thus the distance passed over by a moving body may be represented by an area, in which one dimension stands for the time occupied by the motion and the other dimension represents the velocity of the moving body. In common life and perception the visual surface of a physical object is used in countless ways as a representative of

the whole solid object. Similarly a *volume* may be used to stand for the product of any three factors. Thus interest on money may be represented by a volume whose three dimensions are principal, rate, and time. Also in a less definite way volume or size is employed to express power, influence, or efficiency in any form. So a part of a geometric object is often used to represent the whole.

Shape or form may have an almost innumerable number of different symbolic meanings. The closely allied category of position also has many symbolical uses. For instance superiority in power or influence is often represented by a higher position of some kind, as a seat on a platform in a public meeting. In music gradations in pitch are indicated by a series of positions on a printed scale; also simultaneity of sound is denoted by placing on the same vertical line the marks which represent the given sounds. Position is also used as a principle of symbolism in other highly conventional but important ways, as in the arrangement of words in a sentence, of digits in a number such as 78562, or of symbols in various combinations of which the following are illustrations, 5^2, 52, 25, 2^5, ${}_25$.

Neighborhood Symbolism

The important case where the meaning of a symbol is largely determined by its surroundings we shall name neighborhood symbolism. Thus in 78562 the meaning or value of the 7 depends mainly on the number of symbols which follow it. A similar case is that of a word like bill which may have a number of distinct meanings and whose particular meaning is determined in any given case by the context, that is, by the position of the given word with reference to adjacent words. As an illustration we may take the word bill in the two sentences, The woodcock has a long bill; and, I will not pay this bill.

Other Abstract Symbols

Number is used as a symbol in many ways. Thus it may express volume, size, position, power, or efficiency of any kind. An example of the latter kind is contained in the statement, An opportunity in early life is not one but a thousand.

Similarly *time* and its various categories often constitute useful symbols. Thus time may represent space as when we say that two

cities are eight hours apart. Time may also represent growth, for instance a person's age indicates on the average a certain stage of development. A more complex case is the use of time as representative of various forces operating in time, as when we say that time cures all ills.

Motion is a useful symbol in a variety of ways. The motion of a cloud indicates the direction and velocity of the wind in the stratum of air which contains the cloud. The flight of a person is often a sign of guilt or of fright. Certain kinds of motion indicate the presence of life. The motions of a workman are expressive of the amount of bodily energy he is expending.

Motion symbolism may take an elaborate, articulate form. Examples are wigwagging as practised in the signal service of an army, various kinds of dancing, finger reckoning, and the various forms of deaf and dumb gesture language.

The *order* in which one well known set of objects is customarily arranged often constitutes a simple and effective means of indicating the order which holds among a less familiar set. Thus on a geological map the order in time of the geological strata is often conveniently denoted by the order of the colors of the spectrum. Similarly on a topographical map the parts of the ocean are frequently denoted by blue, lands little above the sea level by green, and mountains by red, the various intermediate colors and shades of the spectrum being used to express various intermediate degrees of elevation of the land surface. In like manner the order of letters in the alphabet, and of numbers in the number scale are frequently used as symbols for other cases of order, as of order in a multiplicative series scale.

Efficients as Symbols

Entities of a still more abstract and general kind may be employed as symbols. In general, any Efficient, or any part of an Efficient, may be used as a brief and comprehensive symbol of other things. Thus an element may represent a group, or one group may stand for any other group or form of groupism. Any diversity may be represented by a diversity of any other sort as by an irregular mark like < or ~. So we often represent any uniformity by any other more convenient or powerful uniformity. An important case, already re-

ferred to in different places is that in which an equivalence of some particular kind stands for ubiquitous available surplusage, as when peace is used as a symbol for constant potential happiness. So any instrument may be a symbol for the fruits of that instrument.

Absence as a Symbol

An extreme case of efficiency in symbols is that in which mere absence of anything represents some particular object or entity. Thus empty space may be a sign for some number agreed upon; x when used for $1x^1$ is a twofold illustration of this principle. Absence or silence may indicate dislike, forgetfulness, suspicion, or have any prearranged meaning. Thus we have the saying "Silence is more eloquent than words." A convenient and widely applicable symbolism of this sort is the representation of the absence of obstacles and the resulting speed or power by a vacuum. Thus a railroad system may be conceived of as a branching vacuum connecting important places which are stores and sources of positive surplusage.

Symbols Formed of Complex Materials

Evidently the above materials out of which symbols are formed may be compounded and complexed in many ways. An illustration is the combination of symptoms in the human body which indicates health or the presence of some particular disease. This combination may consist of a coating on the tongue, feelings of pain or pleasure, sounds of different kinds, lines in the face, color of the complexion, temperature, position of the bodily parts, action of the pulse or heart, absence of appetite, changes of growth or decay.

Symbols Classified according to Forms

It is often also an advantage to characterize certain symbolisms according to their forms or some other prominent quality.

Thus with respect to personality and directive processes it is often useful to discriminate between symbols which have been devised by a person for his own use (*private* or subjective symbolisms), and those which are *objective* and general.

In a still more general way we can distinguish symbols which are *artificial* like the alphabet and those which are *natural* like the colors of a bird or the beard of a man.

Spatial and Like Forms

A spatial element is often prominent in a symbolism. In particular, it is useful to note that, from the relative point of view, a diagram (or other symbol) may be larger than the material represented, as in diagrams of molecular structure, or smaller, as in most maps, printed language, etc.

A symbol or symbolism may be of one, two, three, or no *dimensions,* examples of the last species being the use of a point or empty space as a sign of something. An important special dimensional case is what may be termed the semilinear form of symbolism. This consists of a set of symbols dominantly linear but with some width varying according to circumstances. As examples, we have a line of print, or mathematical symbols, or the symbolisms which are found in a person's stream of consciousness.

As to *time* and allied categories, various symbolisms may be characterized as temporary (provisional), permanent, survivals, prehistoric, prenatal, or hereditary.

With reference to the category of *number,* the specific character of a symbolism may be determined in several different ways. For example the number of its fundamental symbols is often an important distinguishing characteristic of a given system. Thus the Arabic system of notation for numbers contains ten symbols. This numerical element in a symbolism varies greatly owing to the fact that in certain cases greater efficiency is obtained by using only a few primary symbols, and in other cases by using many.

Manifold Symbolisms

Also we may obtain what we may term a dual, triple, or *n*-fold symbolism by symbolizing a given domain of material in two, three, or *n*-different ways. In a mine a danger signal often consists both of the ringing of a bell and of the flashing of a light of some kind. The policemen of a city are denoted by their names, by their numbers, and as a whole by their uniforms. A country may be described in common language, or by maps, or diagrams, or tables of numerical data, or a series of pictures, each method having its own special uses, and all combining multiplicatively to produce expressiveness of a high order.

A reciprocal symbolism is a combination of two objects or of two domains of material such that each may be used as a symbolical representative of the other. An example is furnished by the music and words of a good song.

Symbolisms Characterized by an Efficient

An important class of symbolisms with respect to the category of force are the records automatically made by certain machines.

A species of symbolism is often best characterized by that Efficient which is most prominent in its form or structure. Thus it is frequently best to regard a case where the whole is represented by a part as a unit and multiplier symbolism. A special instance of this form is a prize of any kind given as a representative or specimen of other larger rewards to come later.

Similarly certain important species of symbolisms have a marginal reuse form. Thus a conductor on a railroad train instead of recording by pencil the fact that a certain passenger has paid his fare saves time and labor by simply making a hole in the passenger's ticket.

A symbol may be systematically grouped in its inner structure; and in like manner individual symbols or symbolisms may be combined so as to form a multiplicatively grouped aggregate. The fact that a sentence as used in ordinary language is essentially a multiplicative group structure is indicated by the term "articulate language".

Orders of Symbolisms

One symbol may often be represented by another and frequently briefer and more efficient symbol; hence symbolisms may be of different orders. Thus ordinary language may be converted into shorthand (as used by a stenographer in reporting a speech), or stated in terms of the Morse telegraphic code, or expressed in some species of cryptography. Hence all of these devised systems are of the second order. When a telegrapher reads a message by sound, the set of sounds utilized by him constitutes a symbolic system of the third order.

One symbol may also be of higher order than another owing to its aggregate superior efficiency. Thus among civilized peoples, a word like Jehovah is superior in significance to a wooden image in representing a divine principle.

Scale of Departure from Appearances

The classification of symbols (or symbolisms) according to the degree of their departure from the object represented, is of funda-

mental importance. In this respect we have the following principal types:

(1) Cases where such departure is slight, as in photographs, maps, geometric outlines, or in verbal contractions, as the use of U. S. for United States. In such symbols the transformations involved consist essentially in a reduction of size, or a syncopation (or in some cases of an expansion) of the object represented. Often also a slight reluctant departure from the form of this object is involved. The largest departure from phenomenal appearance allowable in this respect is found, perhaps, in the representation of a solid object by a drawing on a flat surface.

(2) Diagrams in which there is not only a change in size, but also a considerable change in form, as in a diagram of the human heart with the parts rearranged so that the process of the heart's action can be studied in consecutive detail. Similar cases are a diagram of the structure of a molecule, or of the parts of a sentence.

(3) Symbols in which the departure from the form of the object represented is preferential and aggressive. Instances are the word "man" as compared with what the symbol represents; the use of x to denote the volume of a box; and the sign $=$ instead of the word "equals". All ordinary verbal language, whether spoken or written, shorthand, telegraphic codes, etc., are of this class. In this connection, special mention may be made of symbols which involve an unlimited or infinite departure from so-called reality. Examples are the representation of infinity by the sign ∞ and zero by 0. So certain religious teachers have enforced the inferiority of matter and evil by declaring them to be non-existent.

Transcendent Symbols

Symbols often appear in a more or less unanalyzable, mysterious, and even transcendental form. An instance is the combination of marks on a plant by recognizing which, even before blossoms have been produced, a plant breeder like Luther Burbank is able to select from 10,000 individual plants of a given variety the two or three which will develop specified desirable qualities. Certain physicians have like powers in reading symptoms of diseases, and certain other persons in reading minds and characters. Often a work of art, as a painting by Turner, is a complex and a more or less transcendental symbol.

Complex Forms of Symbolism

It is evident that the above forms may be compounded and complexed together in various ways. An instance is found in a scientific book which contains not only ordinary language but also mathematical or logical formulas, pictures, diagrams, and numerical tables. Its statements and formulas often also contain implicit expressions of truths and facts not yet fully unravelled and perhaps beyond our full comprehension.

Two General Types of Symbols

Before closing the discussion of species of symbolisms, it will be well to note that it is often useful to group different kinds of symbolism into two main classes as follows:

I. *Geometric, graphic, or pictorial* symbols. These are preferentially single (or integral), concrete, depart reluctantly from appearances and appeal primarily to the sense of vision. They are characterized by vividness, immediacy, and force of action.

II. *Algebraic* symbols. These are characterized by an aggressive departure from appearances, are, in general, abstract in nature, and composed of numerous small parts which are combined in multiplicative group fashion. They are related to the hand and higher mental faculties as well as to the eye. They are marked by their comprehensive grasp, ease of formation, and many-sided efficiency.

These two general classes of symbols may also be briefly described as *direct,* and *indirect.*

Efficiental Essences

An important form of the first of these two classes is what may be termed the *efficiental essence* of an object or domain. By this is meant the Efficients which together form the dominating structure of the object, or to which it can be reduced, and which often can advantageously be used in place of the object or domain. A simple example is the set of lines which is often used to represent a box. Another is the group of fundamental ideas which form the essence of a book. A more complex case is the aggregate of special efficiental methods which characterize any particular nation, as the English, or which may be regarded as representing a given type of civilization.

EFFICIENCY FUNCTIONS OF SYMBOLS

The efficiency functions of symbolism are presented to best advantage by stating them in terms of other leading principles of efficiency.

Symbols a Means of Economy

Thus many of the advantages connected with the use of symbols appear in the form of economies or groups. For instance a symbol usually contains far less material than the object represented. A hundred dollar bill weighs much less than one hundred gold dollars and still greater is the disparity between the bill and one hundred bushels of wheat. Economy of this kind is often practically infinite, as when a house is represented by certain diagrams, or by the word "house".

Closely connected with economy of material is economy of force or energy. For example it is much easier to transfer to a distant place a picture of a house than the house itself.

This economy often takes a wide and complex form. An illustration is the saving of energy involved in reading a book of travels, instead of making the actual journeys described in the book.

Closely connected with economy of material and energy is economy of time and all that time represents. In fact the proper use of symbols may beget an economy in each and every element employed in producing results. These economies are often combined in complex forms. For example the colored petals of a flower form a sign to the bee, which leads to a saving of pollen in cross fertilization, hence to a saving of energy to the plant, and to a saving of time in the evolution of the plant into a higher form.

An Aid in Foming and Using Groups

Symbolisms are also a means of producing more positive inclusions or groups than economy. Thus the symbols which represent objects can often be collected into a small portion of space or time where it would be impossible to collect the objects themselves. An example is the summation in figures of the coal deposits of the United States as compared with a physical combination of all this coal in one heap. Signs and signals also lead to concentrations of strength and action, as in games, or war, business, or governmental processes.

A Means to Reuse

The employment of symbols often leads to reuse on a large scale. A printed description or picture of a building often produces extensive multuse of the principles embodied in the building. If it were not for oral or printed language or pictorial representations of an object, only those actually witnessing an event would ever know of it.

This useful function of symbols is increased by the ease with which they can be preserved on account of their conciseness. This quality makes it possible readily to inscribe them on lasting materials or to shelter them. Symbols lead to reuse not only in these ways but also because they facilitate the action of the imagination and reason.

When we say that a certain orator is a star, we employ reuse of still another kind. In this case the reuse involved is that of work done in assimilating the meaning of a star. This gain also takes the form of added force and vividness in the process of communicating ideas.

Symbols Auxiliary to Multiplicative Group Systems

The use of symbols often facilitates the formation of multiplicative group systems. An example is the organization of an army by the aid of uniforms, and various flags and standards. So widely separated objects may often be grouped multiplicatively by affixing proper tags or marks to them. Many systems of biological classification are obtained essentially in this way. An important illustration from the business world is the coördination of several lines of work in a machine shop or other establishment by means of carefully prepared charts. This process often in effect constitutes the combination of several linear group systems into a comprehensive areal one. Similarly the tabulation of the results of any large and many-sided investigation facilitates the discovery of laws governing these results, and the use of checks, tests, and various cross multiplicative groupings. Also the multiplicative grouping of a given domain of material in terms of a certain few units taken as fundamental is facilitated by expressing these units in symbolic form. Thus as soon as a national consciousness has adopted certain ideas as standard or primal and expressed these in words, other experiences and ideas will be expressed in terms of these standard ones, and the national life will be correspondingly systematized.

An Aid to Other Efficients

In like manner the employment of symbols develops orders of material. Thus in business processes the construction of efficiency curves and charts facilitates the working back to causes and sources of efficiency or waste and the formation of improved processes of work. In some cases, the symbolism itself acts as a higher grade of material with reference to the objects represented.

In the use of symbols, uniformities are developed by the intentional making of different symbols of the same general size, by the reuse of symbols, by the discovery of like efficiental essences in different domains, or by the permanence conferred on the data represented. Symbols also furnish diversities in the form of differences between the symbol and the object represented, and frequently also in the spatial detachment of the symbol from the object. These diversities are often the sources of important characteristic advantages. In certain cases a name or symbol confers a kind of substantial individuality upon an otherwise vague and uncertain entity. Other persons before the time of Priestley had obtained oxygen but he is regarded as its discoverer, in part because he first gave it a name.

The development of uniformities and diversities in connection with symbols also leads to the fruits which are characteristic of uniformities and diversities.

Symbols Mean Various Externalities

Symbols are not only themselves a useful auxiliary but they facilitate the application of other externalities in connection with a given domain of material. Thus by their conciseness they open the way for the formation of short tabulations or other brief combinations of various materials, and for the use of reciprocal externality among these materials and the application of outside externality to them in group ways. An important special case of this source of efficiency is the use of maps and charts of various kinds. The use of symbols frequently leads to externality in the form of suspended operations, and consequent economies such as are involved in cancellations for example.

Also the process of contracting material into symbols often leaves vacant space where externalities can be annexed, or indirect routes or methods followed.

By taking advantage of the detachment of symbols, the mind also acquires a certain freedom and largeness in its operations which are attended by many useful results.

The externalities inextricably inwoven with our visual habits and those due to the permanency and easy motion of symbols may also be mentioned in this connection.

The above species of externality often occur in complex form. Thus in solving a given problem, the use of symbols often enables one to plunge into the middle of the subject, and to work in several directions at once, using many auxiliaries simultaneously.

Facilitate Directive Work

The smallness and lightness of symbols facilitate the directive use of them and hence, in effect, of the materials which they represent. The useful results of such directive use are discussed in Chapter XI (p. 192). Symbolism, especially in the form of the efficiental essence of an object, often leads to important directive efficiency acts of other kinds. This consists in regarding an object as a multiplicative group structure in which the efficiental essence is the prime group. Often the essentials of a desired object can be found in some accessible or cheap form, and the supplementary details treated in some special way.

Unexpected and Self-developing Results

The use of symbols often also leads to certain unexpected and self-developing results. These dialectic results often spring from the unrealized manifold nature of symbols, from their concentration, reciprocal action, and the various uniformities and diversities which characterize them, or to which they give rise. This dialectic development is often vague and uncertain, in other cases it is standard and to a certain degree dependable as in the processes of mathematics or other calculuses.

It is well to note in this connection that the dialectic power in a calculus in large measure may be resolved into the dialectic properties of symbolisms outlined above. Thus the approximate uniformity in the size of the symbols employed in an algebraic equation leads to many important results which we cannot stop to point out in detail. The same is true of the uniformity in the size of the letters of the alphabet, and of Arabic numerals. For these and like reasons, in

mathematical work a symbolism often seems to move forward to new results by some intrinsic, uncanny, and irresistible power. Hence the mathematician Euler, for example, remarked that his pencil seemed at times to surpass himself in intelligence.

Transcendent Results

Some of the fruits obtained by the use of symbols are transcendental and absolute, in the sense that they cannot be measured. An instance is the saving of lives of the passengers on a sinking ship by the aid of wireless telegraphy. Others are unique in the sense that apparently they could not have been obtained in any other way. Thus it is scarcely conceivable that certain highly developed forms of multiplicative groups could have been obtained or kept in operation without the aid of symbolisms.

Another illustration of a like high order of fruitage is the pleasure afforded by a symphony or great painting. Still another of widely different character is the power conferred on us by symbols to use together toolages apparently contradictory, and to get the advantages belonging to each with a further dialectic of advantageous results. An illustration is the use in combination of the atomic and ether theories of matter.

The above functions of symbols may be complexed in many ways. Illustrations of such complex useful results are given in the paragraphs which follow.

Increasing Use of Symbols

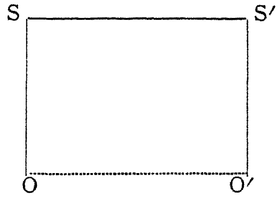
The value of symbolisms may be realized in other more pictorial ways, as by noting that increasing civilization has been marked by an increasing volume and variety of symbolisms. This value may also be made evident by trying to conceive what the modern world would become if its various languages, signs, and symbols were removed. Evidently the most elementary processes of life would be greatly diminished in efficiency while, apparently, the higher and complex processes would crumble and disappear.

Words which show the Value of Symbolisms

The value of symbolisms may be realized from another point of view by recalling and massing together the utilities implied by such words as language, plans, scheme, outline, diagrams, metaphors, paintings, sculpture, photography, pictures, art, telegraph, telephone.

Dangers and Limitations

The dangers and limitations connected with the use of symbols are especially prominent on account of the abstract, detached, and often highly transformed character of this instrument of efficiency. The following diagram will be an aid in grouping and grasping these drawbacks. In the diagram O stands for the object symbolized; S means the symbol for O; S′ is the symbol after it has been transformed or shifted in position for some purpose; and O′ is the object again but often in a new relation or condition. This diagram makes clear that there are three main classes of dangers connected with the use of symbols:



(1) Those in going from O to S (that is, from the object to the symbol);

(2) Those in going from S to S′ (that is, from the symbol to the symbol in a new position or state);

(3) Those in going from S′ to O′ (that is, from the symbol back to the object).

Errors in Going from Object to Symbol

In class (1) are included all those cases where the symbol fails to correspond to the object which it is supposed to represent. A common example is the over-capitalization of a stock company. The lack of correspondence between fact and symbol may be due either to unconscious error or to intention. In the latter class would come various misrepresentations, lies, and swindles. An extreme case is that where a symbol is wholly deceptive and stands for nothing, or what is worse than nothing, as in some cases of forgery.

Errors in Manipulation of Symbols

As illustrations of class (2) we have the errors made in transcribing and combining symbols in mathematical calculations, or those made in processes of reasoning carried on in oral or printed language. Details concerning such errors are given in Chapter XVI. In this class also belong losses due to difficulties in preserving symbols till the time is ready for their translation back into the data represented. As instances we have the cases where paper

money and various memoranda and securities have been destroyed by fire, flood, insects, or rodents before the valuable symbols named could be converted into some concrete and permanent form of wealth. Alterations made in symbols for the purposes of deception, as when a check is raised, also belong to this class of dangers and losses.

Errors in Going from Symbol back to Object

Class (3) includes all errors and misrepresentations made in the process of converting the symbol into the material represented, or some datum more or less like this material. An important case in this class is that which assumes that the object will have all the properties of the symbol, as that a bridge will be as strong in proportion to its size as in the model which has been made to represent the bridge. Another special case in this class is the ignorant selling of stock certificates for less than they are worth, or signing any important document under the impression that one's signature has an entirely different meaning from the actual one.

An extreme case in this class is the error of regarding the symbol as not representative but ultimate. Instances are the common error of mistaking paper money for real money, or of exalting a religious image or ceremony above the reality represented. This conception of the symbol as an objective reality rather than as an instrument for attaining results often takes partial and elusive forms. Thus many of the difficulties in the mastery of mathematics come from regarding such symbols as

$$\sqrt{-1},\ x^{\circ},\ \frac{dy}{dx},$$

as objects rather than as instruments.

The Tongue of Efficiency

On the whole, however, it is evident that the advantages connected with the use of symbols far outweigh the disadvantages. In fact symbolism may be termed the tongue of efficiency, and in some respects also its brain.

SOURCES OF EFFICIENCY IN SYMBOLISM

Our study of symbolism up to this point has made it clear that a symbol is a special form of group, and that the utilities which flow from the use of symbols are special forms of the benefits which

accompany the employment of groups. Hence as an aid to using symbols in the most efficient way it is important to investigate as closely as we can the relation of symbols and symbolisms to the group idea.

The Symbol as a Group

The word symbol means something thrown together; that is, a contraction, abbreviation, or syncopation. Hence primarily symbol means a group formed by the process of inclusion. Special mention should be made of the fact that this contraction is made in such a way as to bring in front of the user and within the scope of a single act of vision, data that may have been lying in many directions and at various distances from the observer.

The process of contraction by which symbols are formed may be carried on in such a way as to make the resulting symbol a multiplicative group structure; or later, by use of externality, the symbol may be developed into this form. Also, as has been pointed out, different symbols are frequently combined so as to form a systematically grouped aggregate. The symbolic multiplicative group system is often parallel to a multiplicative group system of material in the object represented.

A Symbol as a Unit

In some aspects or cases, a symbol is a unit which has various multipliers or multuses. For example a gesture which causes an entire audience to rise to its feet is a unit motion with respect to the motion of the audience which follows. So a word in some respects is a unit with respect to the object represented by the word. A symbol may also be a multiplier. Thus a gold reserve is in effect multiplied many times by the large amount of paper money circulated with reference to it.

Relation of Symbolism to other Efficients

With respect to the principle of orders of material, symbols are usually high in the scale of efficiency, and often indeed belong to the efficiental abstract grade.

Externalities of various kinds are present or implied in symbols. Beside the various species of related externality spoken of in the preceding pages mention may be made of the fact that a symbol

implies a wide range of application, and also certain auxiliaries to safeguard the use of the symbol.

In addition to the various uniformities and diversities which have already been considered as prominent in symbolisms, special mention should be made of the exact correspondence, or uniformity, which should exist between symbol and object, and between symbols representing the same object.

Some element of limitation is always a contributing cause to the formation and use of symbols. Thus if it were not for our limited grasp of objects in space and time, we should not need to use symbols. This limitation factor varies greatly in degree and form. Hence a special study of it will be an important aid to the most efficient use of symbols in different cases. For this special study see Chapter XV (p. 265).

Relativity of Symbols

This brings us to the general matter of relativity in the nature symbols. With reference to different individuals and to the same individual under different circumstances there is often much variation as to what is to be taken as a primal datum of experience and what is to be regarded as an artificially formal symbolism. Thus the colored surface of an object from one view point is an original datum of experience which may usefully be denoted by symbols (described in words, or expressed in units of area); from another standpoint this colored surface is but a symbolic representation of the certain molecules and forces.

A symbol also contains an element of unique individuality, not statable in terms of other concepts or primal modes of efficiency. This often becomes the source of unexpected uses and values.

Definition

As a summary of the results of our study of the nature of symbols we may make the following definition:

A *symbol* (or symbolism) is a group (or system of groups) whose function is the efficient expression and transfer of other groups and which acts mainly through the eye and ear.

METHODS OF APPLICATION

A realization of the essentially multiplicative nature of symbolism is an aid in many ways in making the most effective use of signs and

representatives. Many of these methods are obvious and we shall discuss only those which are most important or most likely to be overlooked.

Vast Number of Possible Symbolisms

By an application similar to that made with the other Efficients, the multiplicative principle aids us in realizing the great number of possible materials and combinations of material which can be utilized in making symbols. It also aids us in conceiving the vast number of forms which symbolisms may assume. The multiplicative group principle is likewise a powerful help in systematizing these materials and forms and holding them ready for use.

Potency of Symbolisms

The great power in the multiplicative principle enables us to realize also the extreme extent and efficiency to which even a limited or crude species of symbolism may be developed. Illustrations are the power of the tactual symbolisms employed by a person like Helen Keller, and those special forms unconsciously employed by untutored lightning calculators. As a more abstract illustration we have the power of the equation $ax^2 + bxy + cy^2 + dx + ey + f = o$ to represent all the properties of the straight line, circle, ellipse, parabola, and hyperbola.

Adapting Symbolisms to Special Circumstances

Owing to the great variety of forms which symbols and symbolisms may take, and the wide differences in the circumstances attending their use, in any specified case it is of especial importance to study the limitations and possibilities involved in order to select or devise the most desirable symbolism for use in the given situation. In this process much use of externality is often desirable. Thus in making a graph, or realizing an efficiental essence, especially in a difficult case, externality is frequently helpful. This may take the form of getting at a distance from (or holding at arm's length) the material considered, or of viewing it from all sides, or of allowing time to work upon it. Also when a wide domain is studied, the efficiental essence is apt to come to the surface in certain spots, or a natural symbolism to be revealed.

Cases illustrative of the advantage and need of special symbolisms

under different circumstances are the use of adapted marks and signs on clay tablets in Babylonia and Assyria, on papyrus and granite in Egypt, and parchment and marble in Greece. A modern instance is the difference between day and night signals on railroads. An illustration from the world of commerce is the fact that cotton and wheat may be sold by standard (that is, technical verbal description as Grade A, etc.), while wool cannot be sold in this convenient way but is bargained for by sample.

Relations of Graphic and Linguistic Symbolisms

As to the use of more general kinds of symbolisms under different circumstances, it should be observed that algebraic symbolism (Class II, p. 173) is preferable as a general or standard method of expression, while graphic or pictorial symbols are to be used locally or occasionally for the sake of special advantages. Examples of such local use are the employment of flags in the management of an army, or of diagrams in the teaching of grammar.

Careful study must often be made as to when and how to combine these two main classes of symbols. A rhythmic or alternate use of the two will often be found very advantageous.

Form and Color in Symbolisms

Also in devising and using symbols it is to be noted that, in general, the form of a symbol is a more effective element than the color. This is one reason why algebraic symbolism, as a rule, is superior to the graphic form. For a somewhat similar reason spoken language is more efficient than gesture language as a standard method of expression. Vocal language also, since it employs certain organs in the mouth and throat, has the advantage of leaving the arms and hands free for use in other ways.

Standard and Special Symbolisms

In the field of objective symbolisms (that is, of those used not for private purposes, but in connection with other people) it is desirable to give the preference to standard and conventional symbols. At the same time it is important to acquire the power and form the habit of improvising symbols for one's private use, since such symbols may often be highly efficient in special situations.

Preference to Symbols of High Order

It is scarcely necessary to say that it is desirable to make all symbols and symbolisms of as high an order as circumstances will permit. In the first place the material to be symbolized should be selected and transformed so as to make it as efficient as possible. Again individual symbols should be as efficient as possible both in form and in other ways. As examples of superior symbolisms in this respect we have the phonetic alphabet as compared with the Egyptian hieroglyphs, the Leibnitzian $\frac{d^2y}{dx^2}$ as contrasted with the Newtonian $\ddot{y}$, and the use of a word like Jehovah in expressing infinite external multiplicative power as compared with a heathen idol.

Also in a symbolism of high order the individual symbols are combined in effective ways, as for example in a multiplicative group system. By this method, for instance, a few symbols rightly used are made to take the place of a large number combined in an inefficient way. An example is the English alphabet as compared with the Chinese syllabic system. A more specific instance is the Linnean method of naming species of plants and animals. Before the time of Linneus one name of common blue grass was "gramen pratense paniculatum majus latiore folio poa theophrasti". In the Linnean system of nomenclature this name is reduced to "poa pratensis". The swift and vast progress of botany and zoology since the time of Linneus is largely due to the simplicity and power which characterize the binomial method of naming species.

A high order of symbolism often takes the form of a symbolism of a symbolism, as when shorthand is used instead of ordinary language. Again it may take the dimensional form, as when a solid is represented by a flat picture, or in some cases by a set of points, or even by one point.

In this connection it is to be remembered that neighborhood symbolisms (see p. 167) are of a high order of efficiency, and that they can often be utilized not only in private provisional symbolisms, but also in giving added economy and power to certain applications of objective language.

Similarly a manifold symbolism is of a high order of efficiency. An example is that of expressing the fact that a given bottle contains poison by printing the word poison on the label, annexing a

picture of a skull and crossbones, and making the surface of the bottle rough to the touch.

Auxiliaries in Making and Using Symbolisms

In applications of symbolism, other forms of externality than those already indicated are often extremely useful. For instance the use of some auxiliary often makes it possible to obtain pictures or other representations of an object not otherwise possible. A striking instance of this are the skiagraphs of the bloodvessels of the brain obtained by the injection of mercury into these bloodvessels, and the application of X rays. In other cases extra and, from the strictly logical point of view, superfluous symbolism frequently results in quickness and vigor of action with a consequent dialectic of values. Instances of this are the method of denoting the end of a sentence by a period or dot and also by the use of a capital letter at the beginning of the next sentence; or the employment of difference of shape as well as size in forming a symbol as when A is used instead of *a* as the capital form of small *a*. A combination of color and form in a symbol when either alone would be adequate is a like instance.

A more general and important case is supplied by the manifold and various, even profuse, symbolism in a language like English as compared with a minimal language like Esperanto or Volapuk. Here, as has been said, "luxury is economy", and, we may add, efficiency.

This apparent excess of symbolism is useful not only as a direct source of efficiency but often also in a negative way as a means of preventing error. For example it often furnishes simple means of checking a given statement or process.

Utilization of the Self-developing Power in Symbolisms

Especially important also is it to recognize and utilize the self-corrective and self-developing property which is peculiarly characteristic of symbolism as a source of efficiency. This principle will be found particularly useful in cases of limitation and difficulty, as when we express in language all that we know about partially understood matters and find by development of this symbolism that our language proves to possess a far wider expressiveness than we had supposed. In its higher forms this principle is illustrated by the extraordinary power of certain symbolic forms of deductive reasoning.

Ideal Species

It follows from what has been said in the preceding pages that our ideal form of symbolism would be a representation of efficiental data in efficiental form for efficiental ends.

Summary of Chapter

An illustration of the great efficiency value of symbols is the fact that making a written record of a result opens the way to the reuse of the given result in many lines. Among the most obvious kinds of symbolisms are written and oral language, pictures, maps, tabulations, motions, and combinations of these. More extreme species are neighborhood symbolism, and the expressive use of absence or emptiness. Symbols are thus not only composed of a great diversity of materials, but they also occur in an endless variety of forms. Among the most important forms of symbolisms are the different linear and areal species, and those which are built upon each other in succession. The two most general types are the geometric and algebraic (or the direct and indirect). Its efficiental essence is often a highly important representative of an object or a domain of material.

The economy attained by using the symbol of an object instead of the object itself is often practically infinite. Other values which frequently result are speed, organization, ease in directive management and in the application of externality, the fruits of these, and other useful results which are often transcendent. Each particular form of symbolism has its own special efficiency powers. However, owing to the detached and abstract nature of most symbols their use is exposed to peculiar dangers which must be guarded against.

Owing to the great variety of existing or possible symbolisms and their extreme power, great care is often needed in order to select or invent the particular species which, in a given case, will be most effective. Especial attention is to be paid to the use of different kinds of symbolism in combination, as in the alternate use of the two principal kinds. The application of externality is useful in different cases. Preference is to be given to standard forms of high order, the highest form of all being that which is as efficiental as possible both as to data, form, and aim.

EXERCISE 10

State the class or classes to which each of the following symbols or symbolisms belongs:

1. A smile
2. A flag
3. Footprints
4. Ditto marks
5. A trade mark
6. A weather vane
7. Sounds made by sleigh bells.
8. The word rock in "He was a rock in defence of his position".
9. A curved line showing the fluctuations of the price of wheat from day to day.
10. The sign + as used to express addition.
11. A coat of arms as used in heraldry.
12. The cry of an infant when in pain.

Give an example of the use of each of the following as a symbol:

13. Some bird
14. Some quadruped
15. Some flower
16. Some tree
17. Heat
18. Flame
19. Some metal
20. Water
21. Some other fluid
22. The air
23. Smoke
24. A leaf
25. A physical force to represent a mental power
26. State the advantages involved in the use of each symbol given in Exs. 1-12.
27. Name all the audible symbols you can.
28. All the visible ones.
29. All the mental ones.
30. Give an example of order used as a symbol.

Give an example of the following classes of symbols or symbolisms:

31. Natural
32. Artificial
33. Semi-artificial
34. Negative
35. Neighborhood
36. Inherited
37. Dual
38. Triple
39. Multiplicatively grouped
40. Semi-linear
41. Of second order
42. Of third order
43. What different kinds of symbolism are combined in a rocket fired as a signal?
44. In a flash light as used by a light house?
45. Name some of the most important signs of spring, and state the class or classes of symbols to which each of these belongs.
46. If it costs $3000 per year to maintain a given light house and $150,000 worth of property is saved by means of the light house, state the degree of efficiency involved.
47. During one year the expenses of a certain business were $10,000 and the returns were $16,000. The next year $1000 additional spent in advertising increased the profits of the business by $4000. State the degree of efficiency of the business the second year as compared with the first. State the degree of efficiency of the additional money spent in advertising.
48. State the advantages in the use of a set of signals in playing football. Also the disadvantages.
49. Discuss the principles of symbolism involved in moving pictures. Also the efficiental and other advantages which result.

50. Answer the question "What's in a name".

Give an instance where the use of symbols leads to

51. An economy of material
52. An economy of force
53. An economy of time
54. Reuse
55. Multuse
56. Formation of a group
57. Formation of a multiplicative group system
58. Formation of an order of materials series
59. Self-developing efficiency results

State approximately the degree of efficiency involved in the use of

60. "Five" as a substitute for "four and one more".

61. 6792 for "six thousand seven hundred and ninety-two".

62. $4 + 2 = 6$ for "four and two are six".

63. In 1,987,653 the figure 3 expresses three units and it also helps the figure 5 to express tens, 6 to express thousands, etc. How many items of work does it do in all? How many kinds of symbolism are combined in this instance?

64. Give an example of symbolism of the third order with a statement of the efficiencies connected with it.

65. Give two examples of the use of protective coloration in animals, and explain its benefits.

66. What is the smallest number of points needed in order to represent a straight line? A plane surface? What is the degree of efficiency in each of these cases?

Name as many advantages as you can connected with the use of

67. Maps
68. The electric telegraph
69. Wireless telegraphy

70. What is the semaphore and what principles of symbolism are embodied in it?

71. State the different forms of symbolism which are often used in making an exhibit of the expenditures of a city, and the results obtained. What are the advantages of such symbolic summations?

72. Discuss the advantages in the use of a trade mark.

73. Give two examples of graphic (or geometric) symbolism. Also two of algebraic symbolism.

74. Give an example where graphic and algebraic symbolism are used to advantage in alternate succession.

75. State as many different kinds of symbols as you can which can be used in advertising a business, and illustrate each.

76. By numbering the parts of a ship and keeping a model of the ship at some shipyard, it is often possible quickly to repair the ship while at a distant place. Explain how this may be done and what principles of symbolism are utilized in the process. Name some other objects besides ships to which the method is applicable.

77. State what is meant by routing a piece of work in a factory, the advantages involved, and how routing is aided by symbolism.

78. What kinds of symbolism are involved in social etiquette? What advantages result? What disadvantages?

79. Give an example of a symbol which has erroneously come to be used as a reality.

80. Discuss the disadvantages in the use of symbolisms implied in the statement, "talk is cheap".

81. Give two illustrations of errors or losses arising from the use of symbolisms.

82. State as many checks as you can on the abuse of symbols.

83. Express as well as you can the unused wealth of symbolic power in the alphabet.

84. Explain the Dewey method of classifying books and the principles of symbolism utilized in it. Indicate other fields in which this method of classifying objects is used or is usable.

85. In what respects do actions speak louder than words?

86. It was the habit of Clerk Maxwell from early childhood to try to discover what he called the "go", or the "particular go", of any machine or instrument which he saw. In terms of the ideas of the present chapter explain the meaning of the word "go" as thus used.

87. Discuss the relation of symbolism to efficiental essence. For instance, which of the two terms is broader?

88. Why should the letters in a written word be connected or run together, while those in a printed word are distinct?

89. Explain the principles of symbolism embodied in some system of print for the blind.

90. Give an example of some kind of equivalence used as a symbol for surplusage or profit.

91. Give an instance where the removal of an object or domain of material to a distance aids in revealing its efficiental essence.

92. Give an instance where the form of a symbol is more efficient than its color. Also give one where the reverse is true.

93. Discuss the relative values of audible and visible symbols.

EXERCISE 11

Review

1. Of 4000 eggs which were placed in a given incubator, 2400 hatched. State the degree of efficiency involved.

2. A girl can write 15 words per minute by hand and 60 words by typewriter. Express in two ways the degree of efficiency involved. State some of the sources of this efficiency.

3. Make up and solve a problem of your own concerning the degree of efficiency in a given case and the sources of this efficiency.

4. Instead of paying for each article as purchased, it is often an advantage to run an account at a store, and to pay for purchases occasionally by check. State the Efficients involved in the latter method.

5. What principle of efficiency is utilized in the wedge form of attack in warfare?

6. Why is it often an advantage for a baseball player to train himself for batting by swinging Indian clubs many times heavier than the bat which he is to use? What principle of efficiency is involved?

7. Explain the addition of numbers in arithmetic as a combining of multiplicative groups. What advantages result?

8. Treat in like manner the multiplication of numbers.

9. Explain division of numbers with reference to the principles of groups.

10. In a certain county clerk's office under the elective system in a given year the receipts were $13,968 and the expenditures $23,928. Later under the appointive system the receipts were $31,355 and the expenditures $20,200. Compute the degree of efficiency involved. State some of the possible sources of the increased efficiency.

11. What principle of efficiency is most prominent in the following precept: Be sure you are right and then go ahead?

12. What principles of efficiency are involved in the case of a straw which shows which way the wind blows?

13. What principle of efficiency is involved in the use of a baggage check for a trunk, and what advantages result?

14. In language study what efficiencies are involved in using the principal parts of a verb as representative of all the forms of the verb?

15. A boy, sick of the hook worm disease was found to be 12 per cent efficient. If a normal boy could work 8 hours a day, how long could the sick boy work?

16. In the business world what are the signs of the beginning of a period of commercial prosperity? State some of the advantages of being able to read these signs.

17. Discuss the efficiency values of artistic surroundings in a store or factory, especially of those symbolisms which convey an appreciation of the Efficients in some way.

18. What principles of efficiency are involved in Emerson's statement "The key to every man is his thought". Give illustration of the uses which may be made of this statement.

19. As a result of the use of Dalton's chemical units, exact methods of making sulphuric acid have been substituted for guess work and rule of thumb. State the Efficients involved and the useful results obtained.

20. Discuss the relation of the classification of objects to the principles of multiplicative grouping.

CHAPTER XI

DIRECTIVE

Illustrations

It has been estimated that by stocking the rivers of France with black bass brought from America the value of the fish taken annually from the French streams has been doubled. The transfer of the eucalyptus tree from Australia to other lands as California and South Africa has produced even more important results. These instances illustrate the efficiency value of man's power to change the position of objects including himself. Often this principle takes various complex and apparently qualitatively different forms, such as the changing of the forms of objects, the selection and cultivation of plants, and the cooking and preservation of their products. But the primary and fundamental principle involved in all such cases is that of changing the position of objects or parts of objects. This principle of directive action constitutes the next fundamental source of efficiency which we shall study. We shall denote it by the single word directive.

The ways and forms in which this category of efficiency comes into play are so numerous and so often complexed with other Efficients that we shall call particular attention only to those cases where the directive principle is prominent as compared with the other prime sources of efficiency which are involved.

CLASSIFICATION

Objects which may be directed

Cases of directive action may be classified according to the materials moved or directed in them. In general we may say that anything may be moved, changed, or directed in some respects, but no object can be moved in all respects. For example we cannot change the position of the moon in its orbit, but by shifting our position on the earth we can change the position of the moon in the sky with reference to other heavenly bodies. We cannot change the amount of rain which falls from a cloud but by irrigation or the processes

of dry farming we can in part change the route by which water travels from the cloud to the ocean with very beneficial results to mankind. We cannot live over the past directly, or make two plus two equal five, but we can do both of these things in some indirect sense.

As a result of what has been said it is evident that a special science of directive action may exist in each of the principal fields into which human affairs are usually divided. Thus we have civil engineering, mechanical engineering, electrical, social, educational, and similar departmental species of management.

Other methods of classifying directive acts are, however, more important than this for our purpose.

Directive Agents

One of these is a grouping of directive cases according to the directive agents, that is, the directors involved. All the various forms of what is called a living matter have some directive power both over themselves and over some objects external to themselves.

Thus various *plants* are able to turn themselves (as a whole or in part) toward the sun (heliotropism), or toward moisture (hydrotropism), or toward the earth (geotropism), and in other ways. Highly specialized forms of such powers are illustrated by the sensitive plants and the Venus flytrap. Plants also have power to draw food from the earth and air, and also to move such food for considerable distances in opposition to the force of gravitation. Important special cases are the power which protoplasm has to direct sunlight and which certain plants have to fixate nitrogen.

Individual animals have many different ways of moving themselves and external objects. A primitive case is that of the sea urchin covering itself with sea weed to protect itself from its enemies.

An aggregate of individuals may also constitute an important directive agency. Examples are a flock of birds, a pack of wolves, a society, nation, or corporation.

Similarly it is often useful to regard certain parts of individuals as independent directors. Examples are the cells and organs in a living body, and the different so-called faculties of the mind.

More abstract entities also have what constitutes for our purpose directive power; illustrations of these are the spirit of the age, the weltgeist, or any aggregate of efficiency agencies.

Also in efficiency processes allowance must be made for unknown, or imperfectly known, or even transcendental efficiency agencies.

It is important to note in this connection the wide range in directive powers both in the individual agent, and in different directors when compared with each other. The oyster when young has considerable self-directive power, but when mature has very little. With man the reverse is true.

In self-directive the material moved and the agent are one.

Directive Acts Classified according to Form

Most important of all for our purpose is the classification of directive acts according to their form.

A simple case of *spatial* directive is that of changing the position of an object. Changes in the position of the parts of an object may result in a change of shape. With regard to the category of space (or quantity) a directive act may also be narrow or comprehensive. More important still it may be a separating or a collecting act; that is, it may be an act of analysis or one of synthesis. It may be partial in that it is directive of some part of an object, or in that it is a transfer of an object in some specifically limited way. It may be indefinite as when a situation is merely agitated or stirred up, the motive being a vague hope that something useful may result; or definite, as when every step and the reason for it are carefully calculated.

An act of directive may be characterized according to the category of *number* in various ways as by the number of objects moved, the number of directors, or the number of parts in the given act.

A simple case of a temporal directive act is the transfer of an object from one point in *time* to another. An example is the saving of money to be used at some future date. A much broader case is the conservation of national resources. With respect to time a directive act may also be brief and temporary, or of long duration. The adjectives prophetic, preventive, or well timed are also often applicable. An important temporal species is that termed initiative, that is, the act or power of making a start (or doing all that the beginning of an act implies) under any given circumstances, especially those which are difficult.

As to *motion* and *force*, directive acts may be characterized as feeble, delicate, energetic, automatic, slow, or swift.

An important species is what may be termed *reciprocal* directive. In the simplest form of this case each of two agents moves, directs, or influences the other or something belonging to the other. A large part of trade, exchange, and commerce belongs to this form of directive action.

With respect to the categories of *mind,* various cases of directive may be characterized as conscious, unconscious, and subconscious; or even voluntary and involuntary. Thus certain kinds of personal influence are unconscious and often involuntary.

As to *quality* a directive process may be homogeneous or heterogeneous either as to material, directors, or form. It may also be continuous or discontinuous. An important case of heterogeneous directive is that in which the result is noticeably different in some respect from the datum, as when carbon is converted into diamonds, heat into light, or the useless into the useful.

Directive in the Form of Inaction

Another important species of directive is that which may be termed negative. This consists essentially of absence (in whole or part) of action on the part of the agent, or in his motion away from an object, or in his keeping an object from motion, while at the same time some other process which in the end will prove beneficial is allowed to go on. Special cases of negative directive are inaction, patience, persistence, removal, prevention, rejection, withdrawal, self-sacrifice, restraint. The utility of negative directive is illustrated by Napoleon's maxim never to disturb an enemy while he is making a false move, by the frequent success of what is called the Fabian policy, and by all cases of the removal of waste.

Grouped Directive Acts

It is often advantageous to perform directive acts in groups. Thus in a well organized shop by pulling a single lever many machines may be set in operation simultaneously. A case of unit and multiplier directive is that of changing the course of an entire vessel by a movement of the rudder only. Also when a ship is steered in the region of a favorable trade wind, owing to the continued action of the wind the initial directive act receives a multiplier of still higher order. When a gun is fired the directive work of aiming the gun and pulling the trigger is multiplied in complex ways.

It is even more advantageous to combine directive acts in multiplicative group systems. An important case is that of superintending a group of persons each of whom exercises directive functions over a group of other persons and so on. Illustrations are the supervision and direction of an army by a general, of a railroad by its head, and of the government and people of the United States by its president.

Other less homogeneous cases of multiplicative group directive are the work of irrigating a large tract of land, or the shaping of public opinion by means of a variety of agencies.

Directive Acts of High Order

It is an important aid to efficiency to be able to recognize a given directive act or process as of high or low order, and more important still to be able to determine its exact place in a given scale. In such a determination various factors must be considered. Thus an act may be of high order if it moves highly efficient material, as the germs, sources, or underlying causes of things, or if it influences leading or governing men. This kind of directive is recognized in the proverb, the hand that rocks the cradle moves the world.

A case of somewhat different kind is the control of the business of a country maintained by means of a domination of its transportation routes.

A more abstract case and often a very useful one is directive of the marks or symbols which represent objects instead of the objects themselves.

Highly efficient directive is also often obtained by moving an object to an apex or controlling point in a system or domain. Thus the dog jumps at the throat of his victim, and the bee tries to sting the eye. Von Moltke went from Denmark to Berlin in order to practise the art of war. Jesus went to Jerusalem that he might the better influence the religious thought and practice of the world.

Other directive acts of high order are those of converting what is harmful or useless into what is valuable. On this same plane of high efficiency are directive acts which cost little or no effort, examples being negative directive and results obtained by unconscious influence. Acts which proceed in an automatic or self-regulating way are of the same high grade. In such cases we are often utiliz-

ing work done by other men at some distant time or place or in some obscure way.

On the highest plane of all perhaps are directive acts which produce groups or other fundamental efficiency machinery of some kind. Several of the above elements of high efficiency may of course be present in a single act or process.

From the preceding discussion it follows that directive acts in a given department may often be arranged as a multiplicative series. Thus in military evolutions we have tactics and strategy, the terms being placed in ascending order. In like order in agriculture are found the cultivation of nearby and familiar plants; the search for and cultivation of the best existing plants to be found anywhere in the world; and the breeding of new and better species of plants.

In work in general we have as a multiplicative series, imitation, invention (or discovery), creation.

Indirect Management

With respect to externality directive acts may be characterized as direct or indirect. An example of indirect directive is the removal of an obstacle in order that another object may be moved. A still more indirect case is that of undermining an obstacle in order to remove it.

Another form of indirect directive is that of moving an object by means of power exerted through some auxiliary object, as when a man moves himself by moving the locomotive on which he is standing, or by attaching a weight to a window sash utilizes at a later time (when raising the sash) the energy generated by the falling of the sash. Another form of directive characterized by externality is work performed on a large scale.

Complex Species of Directive

Evidently the above species of directive may be compounded and complexed in many ways. Examples of complex directive are any comprehensive piece of engineering, the directive processes involved in agriculture, or in the manufacture of various articles, in education, or in the various departments of life and work in a highly civilized community. Plainly the work of the so-called efficiency engineer, when fully developed, is not only of far-reaching importance but of cumulative complexity.

EFFICIENCY RESULTS OF DIRECTIVE ACTION

General Illustrations of Uses of Directive

If we bake clay and thereby make a dish, we perform a directive act which has a variety of advantageous results. One result is the separation of the moisture from the clay (diversity), with the consequence that the dish may be permanent in shape (uniformity). Hence result other diversities such as the separation of food into portions and of each portion from the table and other contaminating matter. Also the dish thus formed is capable of much reuse. A still more general efficiental result is an increased refinement in the multiplicative group organization of society. Looked at from another point of view, the making of dishes leads to an improvement in human health, an increase of happiness, and often to prolonged life. The latter class of results is however largely determined and dominated by the former, and primary attention should therefore be paid to the former class.

A Means to Group Formations

A directive act may result in the formation of a group in a single and direct way, as when a useful assemblage of articles is obtained by packing a number of articles in a satchel. But directive acts always result in group formations in much more general and manifold ways. Thus when any object is moved from position A to position B the result will be the separation of the moved object from all points and materials near A, and also its greater proximity to those near B. Hence many groups will be dissolved and others will be formed; or stating the matter more generally, many new uniformities and diversities will be generated which will serve as the raw material for group formations.

It is also to be noted in this connection that the selection of an article means the rejection of all other articles, and that the formation of one group means the formation of an infinite negative or matrix group.

Economies

In various cases the result of the above process is the formation of groups which are especially efficient in different ways. Thus important economies may result. An example is the utilization in gas engines of the gases given off by a blast furnace with the consequent

economy of fuel in the factory. When salmon are left to propagate themselves in a state of nature more than 98 per cent of the ova or young fish are destroyed by other fish or by birds. In artificial hatcheries only 15 per cent are lost. Or the result may take the form of a saving of time. Thus it is estimated that Luther Burbank in the breeding of new plants has attained results in a single life time which, if left to the processes of nature, would have required millions of years.

A Means to Other Efficients

In like manner the gain may take a unit and multiplier form. Thus when a window sash is raised in order that a breeze may blow through the room, the work of raising the sash may be said to be multiplied by the breeze. Telegraph operators express a like result when they say that multiplex telegraphy produces "phantom lines" of wire on which messages are carried.

The application of forced draft to a tubular boiler produces both a concentration of motion in a small portion of time (increased velocity) and also a concentration of force in a small portion of matter.

The importance of the diversity and uniformity results of directive is illustrated by chemical analysis and synthesis, and the products obtained by means of them. Examples are the artificial production of camphor, quinine, and salvarsan, the specific for syphilis. An important special form of uniformity which may result from a directive action is that obtained by dovetailing together two or more irregular objects so as to make one that is uniform in some way.

The result of directive may also be the production of an article of a higher plane of excellence in an order of materials scale. Examples are the superior qualities of cheese, butter, and tea obtained by the careful manipulation of ferments.

Directive acts or processes may result in the formation of a new and more powerful symbolism or calculus, or it may open the way to other directives. Thus the manufacture of iron supplied iron as an aid in later directive processes. Or it may produce new externality. Thus travel gives breadth of view and knowledge of new auxiliaries.

Self-developing Fruitage

The dialectic fruitage of directive is especially prominent. Thus the man who first combined steel and concrete could not foresee all the advantageous results which would follow. In a much more fundamental field those who first obtained mathematical results by moving geometric objects and applying them to each other could not forecast all the principles of geometry which would be discovered as a consequence and the many applications of these in engineering and various departments of science.

In this connection the fact is to be emphasized that in directive acts we are either dealing with material objects or with entities the substantial aspect of which has been made prominent. Hence the result of a directive act has that peculiar permanency which is characteristic of the categories of materiality and substance and which often constitutes a new dimension of efficiency. Actually performing a process (as compared with the theoretical achievement of results) also has a valuable reflex effect on the doer. He is compelled to realize and observe limitations in the world about him, and learns to recognize and avoid certain common errors. Also owing to limitations in his hands, eyes, and nerves, and to other limitations in his powers of conscious grasp, and owing to still others in objects, he must divide a concrete directive process into parts and perform the whole as a linear or semi-linear series or group system. He thus acquires group neighborhood symbolism and other habits adapted to the efficient manipulation of such a series. With practice these powers in time often become so well disciplined that they adjust themselves to given conditions more or less automatically.

Transcendent Results

Many of the results obtained by the directive method are absolute or infinite in the sense that in value they immeasurably exceed the data employed. In a sense also they are transcendent in that they cannot be analyzed so as to be statable in terms of the data or of other Efficients. Also some of them are unique in that they cannot apparently be obtained without the use or aid of directive.

As illustrations we have the saving of life by the directive use of mosquito netting to exclude the malaria carrying mosquito; or by the use of antiseptic surgery; and the making visible of otherwise

invisible objects by the invention of the microscope and telescope. A further illustration is the artificial fertilization of the eggs of certain species of animals by the use of chemicals, or even by the prick of a needle.

Hence many directive acts become in effect what may be termed acts of creation. Thus Watt for efficiency purposes created the steam engine. Afterward the use of the steam engine gave coal most of its value. So lignite which when burned directly is relatively worthless becomes valuable by being converted into gas.

Special Fruitage of Each Species of Directive

Each particular species of directive action has its own peculiar efficiency functions in addition to many of the more general ones given in the preceding discussion. One of the most important forms of directive in this respect is the species termed initiative. An act of this kind gives a certain priority in time to one's work and this priority often means an inclusiveness or groupism which ultimately leads to many valuable results. Initiative also means a breaking into new ground and hence a new diversity, often a new unit, radically new externality, and other efficiency materials or results.

Advantages of Management in Each Department of Life

An important way in which to realize the vast amount of fruitage already obtained by the use of the directive principle, as well as to appreciate the complex forms in which these fruits appear, is to try to grasp the results from this source in each department of life and activity. For instance the value of directive in agriculture may be realized in a general way by raising the question as to what would be left of this department of activity if all directive were eliminated from it. In manufacturing it is estimated that the one form of directive called scientific management, alone has doubled the output obtained from a given number of workmen in establishments where it is used. A more comprehensive way of realizing the sum total of the values of directive is to note that human progress throughout has been marked by an increasing amount and progressively higher order of management. For instance each epoch of history has been a new epoch in directive processes, examples being the periods marked by man's control of stone, wood, organs of speech, iron, fire, and electricity.

Looking at the matter from another point of view it has been stated as a law that "other things being equal, progress is in proportion to control". Thus man's control in the fields of chemistry and physics being greater, progress in these sciences has been much more rapid than in botany, zoology, psychology, and sociology. Similarly the supremacy of the Anglo-Saxon in the world's civilization at the present time is largely due to the great energy which is characteristic of this race, and which is especially noticeable when contrasted with oriental fatalism.

Words Illustrating Value of Directive

In grasping the efficiency importance of the directive principle, it is also an aid to note the values connected with certain special and relatively familiar species of directive as analysis, synthesis, substitution, utilization, management, engineering, superintendence, trade, exchange, control, and executive work in all forms.

The increasing employment of directive and the growing appreciation of its value are indicated by the increasing use of such terms as social engineer, educational engineer, light, chemical, or efficiency engineer.

Limitations to Utilities in Directive

Certain limitations in the fruitage of directive acts arise from man's circumscribed control of objects. Illustrations are man's inability to cause rain in the time of droughts, or to compel a horse to drink though we can lead him to the water.

The direct losses connected with cases of directive take two principal forms, thus we have

(1) Unconscious or unintentional losses, such as are indicated by words like mistake, and often also by the words misdirect, misuse, misapply.

Illustrations are the mistakes made in introducing the gipsy moth and the English sparrow into the United States, and wholly unconscious errors like that of transporting the phylloxera to France and the Hessian fly to Great Britain.

Losses of this class may take many special individual specific forms, such as excessive directive (as in meddling, domineering, etc.); or too small an amount of directive; or in directive of a low

order, where a high order is possible, as in the use of war instead of arbitration.

(2) The second important form of evils connected with directive are conscious and intentional misuse and misdirection, as in swindling, cheating, corrupting, and similar acts and processes.

Often classes (1) and (2) are combined in cases which are difficult precisely to analyze, as in many forms of sin and crime.

The Hand of Efficiency

However after full allowance for all limitations and drawbacks has been made, we still find a high surplusage of value in directive acts and processes. This we may sum up in the statement that just as symbolism is the tongue of efficiency, so directive is the hand of efficiency.

EFFICIENCY ANALYSIS

From the preceding discussion it follows that directive is essentially a group-making process, but that it is this process considered from a peculiar and limited point of view.

Directive as a Group-making Process

Thus as compared with reuse (1) in directive special attention is paid to the beginning or incipiency of group formation, the process of use and the matter of results being often left in the background for the time being:

(2) Directive also, to a greater extent than reuse, implies or is exerted in opposition to resisting forces or conditions;

(3) Hence directive lays emphasis on personality and especially on the will exerted in overcoming difficulties. Hence courage, perseverance, knowledge of human nature and similar qualities are specifically prominent in executive work as compared with other group work.

In other words directive is the group-making process considered from the incipient, concrete, subjective side.

In this separation of the directive part from the rest of the group-making process, the principle of diversity is utilized. This has the advantage of making it possible to group these new incipiencies and treat them in various efficient ways as by applying externalities to

them. Hence in the end it adds greatly to the efficiency of the group principle in its various forms.

Other Elements in Directive

Certain other special sources of efficiency in directive should also be mentioned.

For instance, motion implies space, and a knowledge of the properties of space, especially of its efficiency properties, is often a source of power in carrying on directive processes.

Similarly it is important to recognize that directive implies time in the general way in which all motion implies it, and also in the special way indicated by the word incipiency.

Limitation is also present in the form of opposing forces or conditions. Hence a knowledge of force and limitation may be a source of added efficiency in directive processes.

Unique Element

Also there is present in directive a unique element, not statable in terms of the other Efficients or categories. This is present particularly in the ethelistic element involved in directive. We glimpse this element in the power which living matter has to initiate motion, to move itself, and in what we call freedom of the will, all difficult of analysis, yet all important possible sources of new efficiency at any time.

Definitions

It will be well also to define as closely as we can certain more or less familiar species of directive to which we shall have occasion to refer at times.

Initiative is a species of directive in which the ideas of incipiency and usually of opposition are emphasized, all other elements being left vague and indefinite.

Substitution is the dual directive act of removing one object and putting another in its place.

Utilization is the act of transforming that which has been unused into something directly useful.

METHODS OF APPLICATION

The most fundamental way in which to utilize directive to the utmost as a source of efficiency is to realize and apply the fact that directive is essentially groupish and multiplicative in nature. Many details of this method suggest themselves at once as a result of the preceding discussion, but particular mention may be made of the following.

The Multiplicative Principle an Aid in Many Ways

The multiplicative aspect of the matter is helpful in realizing the great variety of objects or entities which may be directed, the many different routes and ways in which they may be moved, and also in holding all these data in mind in a panoramic form in readiness for use as opportunity may arise. This principle also enables us to realize and anticipate the extreme power and extraordinary results of directive action in its developed forms. It should also cause us to keep in mind the possibly great destructiveness of unwise directive. Any artificial method of work, even that of the cultivation of plants, particularly on a large scale, is exposed to great dangers since as some one has said such a process upsets the balance of nature. In other words, in all directive work the universe is to be regarded as in an almost even balance between gigantic opposed multiplicative systems, and hence as composed of enormous opportunities and of equally great dangers.

Adaptation to Special Situations

In certain situations careful study needs to be made in order to determine the most efficient available species of directive action. In some cases only vague directive such as merely breaking up or shaking up a situation is possible, more definite action being postponed till the immediate results of such incipient action become apparent. Another limited form of action is that which is implied in the injunction to cast one's bread upon the waters. Among the most important but most difficult species of directive are those involved in the management of other workers, in the full utilization of the power peculiar to each individual, in eugenics, and in the equitable distribution of the profits of modern methods of work. Knowledge not only of the general principles of efficiency but of a great variety of technical facts is called for in dealing with such problems.

Certain cases of definite, stubborn, and wide-spread limitation, such as the fact that sunshine reaches the earth in a scattered diffused form, must often be dealt with in a number of different directive processes. The investigation of methods of meeting such cases takes the form of a study of sciences like physics, chemistry, and agriculture. Similarly a knowledge of the special limitations and possibilities of reciprocal directive, as in exchange and commerce, requires a knowledge of the technical facts and laws of psychology and sociology.

Relations of Action and Inaction

In this connection attention should be called to the importance of studying the relation of negative directive (or inaction) to positive directive processes. Efficiency is often gained by knowing laws of limitation and the peculiarities of special situations well enough to be able to determine when to apply active directive and when to leave matters to the dialectic processes of nature. Often it is best to use these two methods in alternate succession. As an illustration we have the fact that it is often a source of efficiency not to try at first to make one's work too finished and complete, but to leave something to the self-developing power of circumstances, and later to apply methods of correction and adjustment.

A special form of combined positive and negative directive work is that in which the directive process has in large part become automatic and self-regulative. As has been pointed out elsewhere such forms are of a high order of efficiency.

Superiority of Constructive Work

Particular attention should also be called to the fact that in directive processes constructive work should always if possible be dominant, and destructive work such as punishment and surgery, incidental and secondary. This principle constitutes a directive of directive in that it is a directive of the highest species of this efficiency instrument to the first place.

Externality as an Aid in Directive

All kinds of externality are particularly useful in directive processes but especial mention may be made of the freedom and scope given to directive methods by using abstract externality in the form of a broad view of things.

In particular many efficiency processes should be organized so as to produce not only the result immediately called for but also a surplus, often a continuous stream of results, and arrangements made to direct this surplus to the apices of other efficiency processes.

Ideal Species

In general the entire body of the Efficients when organized together form the most powerful and comprehensive of all aids in directive work. Or to state the matter in a still more general way the ideal form of the instrument under consideration is the directive of efficiental data in efficiental ways for efficiental ends.

Recapitulation of Chapter

To sum up the chapter, one of the most obvious methods of obtaining better results in any situation is that of directive action or management. Simple as this method often seems, it nevertheless needs careful analysis and study in order to realize even its main efficiency possibilities.

Species of management may be classified according to the various kinds of material moved, or according to the various agencies which move objects, or with respect to the ways in which objects may be moved, or according to the various results aimed at. Individual mention may be made of such special forms as inaction, initiative, and indirect management.

Directive action not only has such palpable uses as the utilization of otherwise unused materials and the prevention of various kinds of waste, but it also has the more fundamental value of leading to the formation of groups, systems, and various other Efficients, and hence to the efficiency results characteristic of each of these. In a peculiar sense it thus leads to self-developing results and often to transcendent fruitage. Each special kind of directive produces its own peculiar efficiency results. Hence also directive finds a special field of application in each department of life.

The same general principles which have been laid down in previous chapters as to the best methods of using other Efficients, apply also to directive. In addition, special attention is here called to the importance of a careful study of action and inaction in any field of operations, and also to the desirability of making constructive processes as dominant as possible over those which are destructive.

EXERCISE 12

State the class or classes of directive action to which each of the following instances belongs:

1. Winding up a watch
2. Pruning a tree
3. Vaccination
4. The manufacture of shoes
5. The choice of one of two objects
6. Physical exercise
7. Use of lieutenants.
8. Exchange of goods with a foreign country.
9. Changing the direction of a moving object.
10. Directing sheep by directing their leader.
11. The use of a furnace with a thermostat attached.
12. Introduction of the gipsy moth into the United States.
13. Making a woman's hat out of parts of old hats.
14. Damming of the Mississippi River and generation of electricity by the water power thus made available.

State as many as you can of the different kinds of directive involved in the work of a

15. Farmer
16. Teacher
17. Editor of a paper
18. Sculptor
19. Manager of a factory
20. President of the United States

Give two examples of each of the following species of directive action:

21. Imitation
22. Prophetic
23. Indefinite
24. Reciprocal
25. Homogeneous
26. Heterogeneous
27. Negative
28. Grouped
29. Multiplicatively grouped
30. Of low order
31. Of high order
32. Indirect
33. Substitution
34. Utilization
35. Exchange
36. Complex
37. Give a multiplicative series of methods of starting a fire.
38. State in what ways a mountain may, in effect, be removed by man.
39. Point out the elements of directive in the use of a fireless cooker and the advantages which result.
40. Give an example of a process of directive which has been cumulatively complex in its development.
41. State, as far as possible in efficiental terms, the advantageous results of each of the cases of directive mentioned in Exs. 1-14.

Give an instance in which an important result of the following kind has been obtained by the use of directive:

42. An economy of force
43. An economy of time
44. A concentration of force
45. Formation of a group
46. Of a uniformity
47. Of a diversity
48. The saving of life
49. An externality
50. A loss
51. Self-developing results
52. Point out the advantages and the disadvantages in carrying a number of keys on one ring.
53. Give an example where intense heat is obtained by a directive process.
54. Also one where intense light is thus obtained.

55. Discuss the principles of directive involved in a use of a portable saw mill.

56. Also in making and using an artesian well.

57. State the different kinds of directive action involved in transforming an object.

58. Give an example in which the efficiency of a directive process is increased by the use of diversity.

59. By use of uniformity.

Give two examples in which directive action is restricted in itself or in its results by limitations in:

60. Objects 61. Directors

62. Give some of the advantages which result from the use of artificial light.

63. Give two examples where a directive process has been aided (or might be improved) by the use of an improvised symbolism.

64. Give an example of a directive process which has been improved by being made at least partly automatic.

65. State the advantages of having the reserve of supplies in a factory kept up automatically by the use of a predetermined maximum and minimum.

66. Give an illustration of the value of partial or entire inaction with reference to a given process.

67. Give an instance where action and inaction are combined advantageously in alternate succession.

68. Discuss the advantages which result from the fact that man is now able to transport live fish from any one part of the inhabited world to any other part.

69. State the ways in which a wide knowledge of the state of the market for any given article aids in the distribution and sale of the article.

70. Why is it often more advantageous to move a threshing machine to wheat stacks rather than to move the stacks to the machine? Give two other illustrations of efficiency obtained by the directive method here involved.

71. Discuss will power as an element of success.

CHAPTER XII

KINEMATIC AND DYNAMIC

Illustrations

At the conference on Scientific Management held at Dartmouth College in the year 1911, an efficiency engineer stated that in a certain factory which he had been called on to inspect, he had found one machine whose speed he was able to increase forty-four fold. Increasing the speed of a machine in this way evidently meant that the capital invested in the machine and the wages of the workman operating it were both made 44 times as productive as before and meant also an increase in efficiency in the whole establishment in other ways.

On another occasion an analytic examination of the work of a bricklayer showed that the workman in laying a brick was making 18 distinct motions such as stooping down to pick up a brick, lifting it, turning it over to examine it on all sides, etc.; that this number of motions might be reduced to 5; and that in this way the number of bricks laid by the workman in one hour could thus be increased from 120 to 350. In this case the motions made by a workman in a given operation constituted a convenient symbol for the energy and time expended by him, and a convenient means by which to study and increase the degree of efficiency in his work. These illustrations show how the study of motions as such may be an important source of efficiency.

Again man in propelling a canoe by a paddle necessarily expends considerable muscular energy. When for this muscular power some prehistoric savage substituted wind energy applied by means of a sail, he adopted an efficiency process of the first importance, viz.: the utilization by man of the forces and motions in the external world. For such utilization not only means an economy of human energy and the employment in higher fields of the power thus saved, but also greater velocity, larger and more concentrated masses of energy, and power in more manageable form. The instance just given illustrates the fact that the study of force as such, particularly forces in the outside world, may be an important source of efficiency to man.

General Statement

Hence it will be a prime source of efficiency temporarily to isolate motion, speed, and force as a related group, to objectify them, and to concentrate attention upon them with a view to developing them to the utmost as factors in obtaining results.

At the outset it will be convenient to observe that motion for our purpose means necessarily a regrouping in several ways; that speed means a concentration of motion with respect to time; and that force is a useful externality or handle by which to control motion and its results. It is difficult to discriminate a motion in all cases from the force or forces connected with it. But in many instances it will be an aid in obtaining the utmost efficiency from the categories in hand, to treat as more or less pure or independent, those motions where the forces involved are small or obscure, as well as those which are logically separable from the forces producing them.

CLASSIFICATION OF MOTIONS

Classification of Objects which move

Motions may be classified by grouping them according to the objects which move. Thus we may have motions of organic or inorganic objects; of large, small, light, or heavy objects; of material things; or of ideal entities. Illustrations of the last class named are the growth of a reputation, spread of an idea, or a motion like that recognized in the phrase, quick as thought.

In this connection it is to be noted that all directed objects (see the preceding chapter) are in a state of motion but that many other motions are, for the present at least and in certain ways, best treated as independent of any directing power. As examples of the latter class we have the motions of the atoms in a molecule and of the solar system through space, and a host of motions intermediate between these two in some respect.

It should also be observed that every object moves in some way or ways but not in all ways. Hence we have the general statement, useful at least as a first approximation, that motion is the normal state of the universe, and that any state of apparent rest is a case of motions which cancel each in some respect.

Classification of Forms of Motion

Motions may also be classified according to their forms or other abstract individualizing qualities. In this respect the different species of directive given in the preceding chapter are also different cases of motion. But owing to the peculiarities of motion and force as sources of efficiency when considered independently of human originating power, certain of these species become of such peculiar importance as to be worthy of special mention. To this list some new species should also be added.

For instance particular mention may be made of *wave* or alternating motion. This species is of such importance that it will be treated at length in a separate chapter (see p. 230).

It is also important to note that spatially all motions may be regarded as compounded of two fundamental species, viz.: motions of *translation* and motions of *rotation.*

Attention should also be called to hidden or *obscure* motions. It is sometimes of service to group such motions according to the causes which make them invisible. Among these causes are smallness of the moving object, extreme slowness or quickness of the motion involved, great distance of the object, intervening opaque materials, and lack of color in, or the ideal nature of, the moving object.

Groups of Motions

Motions may be classified in different ways with reference to the group principle. Thus a resultant motion is a highly fused group of motions.

Of especial importance are certain cases of multiplicatively grouped motion. An instance is a motion obtained by gearing a set of wheels so that the cogs on the rim of each wheel act upon the cogs on the hub of the rest. Such a system may be used to produce great velocity as in the apparatus used to measure the velocity of light, or to produce slowness or uniformity as in the mechanism of a clock. Another form of multiplicatively grouped motion is illustrated in the swinging of a golf club, the body in this case being treated as jointed from the toes up, each part rotating upon the end of the part that precedes it in order. Still another species under this general class is the combination of motions embodied in the sun, planets, and satellites forming the solar system.

A case of multiplicatively grouped motion widely different from the preceding is that of a propagation of some sort through any genetic multiplicative group system as the transmission of a characteristic through several generations of a person's descendants, or that of the spread of a motion from the apex of any group system throughout all its suborders, as when a piece of information is conveyed from the commanding general throughout an army.

Motions of High Order

For our purpose it is also important to notice that certain motions are higher than others in what may be regarded as an order of materials scale. Thus a motion may be of high order owing to its position in a multiplicative group system of motion, an example being the motion of the end of a golf club. Another important case is motion of material of a high order even when the actual concrete velocity is small. Thus with respect to the progress of the world, the motion of Jesus as he toiled up Calvary was immeasurably more swift than that of a driver in a Roman chariot race.

Complex Motions

The number of ways in which motions may be combined and complexed is without limit. Instances are the motions involved in a widespread storm, in a war, or in a period of human progress.

CLASSIFICATION OF FORCES

For our purpose it is useful to regard forces as grouped in three different ways, viz.:

(1) According to the substances in which they are found or contained or by which they are conveyed;

(2) According to the species of motion which they produce;

(3) With respect to other results produced by them.

Sources of Forces

Viewed in the first of these ways, particular groups of forces are those obtained from wood, coal, oil, falling water, gas, winds, nitroglycerine, etc. In most of these cases the sun is the ultimate source of energy. Also certain chemical materials as radium, are found to be a prime source of energy. Viewing materials and substances

in other ways we get other groups of forces, such as atomic, molecular, and mass forces; or those termed physical, biological, mental, spiritual, and social. Some of the highest and most important forces reside in methods, especially in efficiency processes.

An important dual grouping of forces is that implied in the terms organic and inorganic, organic forces being those which appear only in connection with organic or so-called living matter, and inorganic those which may also appear in connection with dead matter. These classes have also been termed incarnate and uncarnate.

In this connection it is also well to draw a distinction between forces which act apparently by direct contact of bodies and those which, in appearance at least, act across empty space, examples of the latter being gravitation, electricity, and magnetism.

Forces Characterized by Motions produced by them

On taking up the second method of characterizing forces, we find that forms of motion produced by forces have already been described. But often when forces and motions are considered together in this way, individual species of force of especial importance arise. An instance is the classification of winds as local, continental, or planetary.

In this connection particular attention may also be called to the multiplicative grouping of forces and to forces of various orders in a multiplicative scale.

Forces Characterized by other Effects

In classifying forces primarily according to the results produced by them (other than motions pure and simple) we have for example forces of decay, repair, or growth; attractive forces like gravitation, chemical cohesion, and magnetism, and repulsive and radiating forces such as light, heat, and magnetism.

Mention at this point should be made of negative and resistant forces; also of mere continuance forces such as inertia and momentum. Closely connected with these ideas is a classification of forces according to the degree of human control of which they are capable.

Forces Characterized by a Combination of Methods

For some purposes certain forces are best characterized by a combination of the above principles of classification. Thus some forces, as we usually discriminate them, are determined partly by the

material in which they are found incorporated, partly by the kinds of motion to which they give rise, and in part by the more ultimate results which they produce. Examples are sound, heat, light, electricity and magnetism, elasticity, and chemical affinity. More abstract cases are the force of public opinion, of publicity, fashion, supply and demand, etc. Of higher order still are the forces which produce groups and the other Efficients, and also those which these agents exert. The latter class we may term efficiental forces. The force inherent in superior efficiency we shall find to be the most powerful and inclusive of all.

It is also necessary to allow for subtle and even transcendental forces as possible sources either of added efficiency or of limitation and opposition.

Looked at in their historical development, especially if we include abstract types, forces have been cumulatively complex from age to age.

EFFICIENCY FUNCTIONS OF MOTIONS AND SPEED

In investigating the efficiency meanings and values of motion, we consider three different cases or points of view. These are:

(1) The efficiency meaning of motion as such (i.e. when considered independently of forces):

(2) The uses of motion in cases where the forces involved are small, obscure, or beyond control;

(3) The efficiency function of motion when the related force is considerable in amount, more or less evident, and under control.

Later the efficiency value of superior speed will be considered.

Efficiency Value of Motion in Itself

First, then, considering motion by itself, it is observed that a moving point or object generates a group the elements of which are the moving point or object in its various positions. The group thus generated may take an extreme or infinite form as when a point generates a line, or a line a surface, and so on.

It is to be noted that not only does a moving object generate a group in the way mentioned but that it also, if its relations to objects outside of its path be considered, gives rise to a continuity of new multiplicative groupings and efficientizings. For instance, by tracing progressive changes in organisms we get the various unities which characterize the processes of evolution.

Uses of Motions Related to Slight Forces

Where the related forces are inconsiderable, the following illustrations will serve to make clear some of the efficiency values of motion. A single set of books kept in rapid circulation will take the place of several sets each kept in a distinct library. By taking advantage of the migration of birds a person is saved the labor of travelling to different places in order to observe various species of birds. A flower by utilizing the motion of the bee is saved the labor of producing a large amount of pollen as a means to cross fertilization. An astronomer in determining the distance of the sun, by taking his station near the earth's equator and utilizing the rotation of the earth on its axis, avoids the expense of measuring a large arc of the earth's circumference and taking observations from each end of the arc. A merchant who builds a store on a city corner where many people are passing and repassing, is saved a large amount of work and expense in looking up trade.

An important class of instances under this head is the motion or growth of ideal or mental objects. An example is the method of proof called superposition as used in geometry. A more general and important case is that of the useful results obtained by the more or less spontaneous motion of ideas among themselves as in the processes of the imagination.

Motion in several of the above instances has the effect of giving efficiency a new dimension. The new dimension and also the dialectic of advantages which sometimes results from it are illustrated in the new powers conferred by aerial flight, particularly in war.

Uses of Motions Related to Considerable Forces

The uses which may be made of motions where the related forces are considerable and more or less under control are illustrated in the case of the bricklayer given on p. 210. In such cases not only should the number of motions be considered, but also the duration, amplitude, and often form of each. In general in such cases waste motion is a symbol for waste energy. On the other hand graceful motion means highly efficient action. A somewhat similar utilization of motion as a symbol, is the recognition of a person by some peculiarity of gesture or gait. A more important case is that of reading the signs of the times, that is, grasping the future by means of slight changes in progress at any given time.

Transcendent Values

The benefits conferred by motion are often unique and not otherwise attainable. Thus the rotation of the planet Mars on its axis enables us to observe the planet's entire surface, a fact which is more fully appreciated when we recall that the moon by keeping the same face turned toward the earth, prevents us from ever seeing more than one of its hemispheres. A more important case is that where motion saves life.

Another class of the extreme or transcendental results of motion is that whereby a motion or change beyond what is termed a critical point produces a radical transformation in an object. An illustration is the fact that water becomes ice when its temperature passes through the freezing point, or that positive quantity becomes negative by passing through the zero point. Other important cases are the change of a solid into a liquid, or of a liquid into a gas.

Advantages of Speed

The advantages which often spring from increased velocity were also illustrated in the opening paragraph of the chapter. In some cases this element of additional speed adds what is in effect a new dimension to efficiency. This is recognized in the pedagogic precept first to learn to do a thing, then to learn to do it well, then to learn to do it quickly.

This new dimension of efficiency may take certain special forms. Thus superior speed in reaching Round Top at the battle of Gettysburg enabled the Northern army to win the battle. So in competitive races of various kinds the man who wins by even an inch takes the whole prize. This efficiency property of speed is recognized in the proverb, the early bird gets the worm. The fortunes of the Rothschild family were greatly advanced by the fact that their London representative learned the result of the Battle of Waterloo several hours earlier than any one else in the city. In like manner the farmer who first brings his crop to maturity and the business man who first learns the news that will affect the market, will each profit thereby.

Superior speed at one time or in one respect may mean a whole group of similar advantages. For instance the man who begins work first on a given day may be able to begin every enterprise

ahead of his competitors throughout the day. The same principle applies to much larger periods of time.

An increase of velocity or a superiority in speed may also lead to results apparently otherwise unattainable, or to results in some respect immeasurable in value. Thus in time of flood in a valley, increased speed may be a means of saving life. Also the rapidity with which plant breeders attain results enables us to enjoy results which the slower processes of nature could not achieve within our lifetime.

Complex Fruitages

The above useful results of motion and speed are often complexed in various ways. An instance is the circulation of the blood with the many advantages to the organism which result, and the enhancement of these advantages in those higher organisms where the circulation is most rapid.

Increasing Value of Motion and Speed

A comprehensive grasp of the utilities in motion is obtained by observing that the progressive development of various forms of life and of human civilization as a whole have alike, as a rule, been marked by an increasing use of motion in various forms, and by an increasing speed in processes. The speed of the fish exceeds that of the amoeba; that of the bird exceeds that of the fish; and the telegraph message moves more swiftly than the bird. The present superior efficiency of certain processes of manufacture are due to the fact that in the last ten years the average speed of machine work has increased threefold in the United States and that of metal cutting sevenfold where high-speed steel is used. On the other hand lack of motion, or of velocity, or of utilization of motions, usually means lack of power. Hence also it results that times of motions and change are periods of exceptional opportunity for the capable personality.

Limitations in the Utility of Motion and Speed

At the same time attention should be called to the fact that certain dangers and losses are connected with the use of motion, or of increased speed, in certain cases. Thus the danger in too much motion is expressed in the proverb, a rolling stone gathers no moss.

The loss occasioned by an excessive dependence on objective motions is illustrated by the limitations of a clam or oyster as compared with the fish.

Similarly certain natural limitations often exist with respect to speed, and any energy expended in trying to overcome these limitations in direct ways will be wasted. Also often when it is possible to increase speed, the attempt to do this will not pay, either because of the large outlay of energy called for, or on account of the compensating dangers and losses which result. Illustrations are the loss of the steamship *Titanic* in April, 1912, or the evil physical and moral effects of too high a speeding up of workmen in a factory, or of too fast a pace in any life process.

Beyond the mere dangers of misuse of motions, certain inherent defects are characteristic of motion as an independent phenomenon, the chief of which is its frequent lack of tangible externality by which it may be controlled. Hence arises the importance of force as an auxiliary efficiency concept in connection with the use of motion.

EFFICIENCY FUNCTIONS OF FORCE

From the point of view which is fundamental in the present investigation, we find in force and in the objects in which forces are stored or through which they act, efficiency properties of two principal kinds, viz.:

(1) Those properties which are an aid in the use of motion;

(2) Those which belong more distinctively to force (or energy) itself.

Force as a Source of Motion

In the first place, in treating motions by means of the objects in which are stored the forces which produce motions, we utilize certain permanences which characterize these objects. Thus the force stored in wood, oil, and coal does not vary appreciably in quantity or form, however long these objects are stored. Hence arises the possibility of transferring potentialities of motion (and its results) widely in space and time.

This adds greatly to the scope of directive processes in space and time, to the possibility of substituting a cheap source of motion for a costly one, and to many other of the advantages of a large externality.

Also fuel and some of the other force-sources of motion are capable of fine and exact subdivisions, and lead to certain important delicacies and accuracies in motion.

These properties of the sources of motion also make it possible to group multiplicatively and to systematize these sources and the motions which they produce in extraordinary ways.

Values Peculiar to Forces

Force also has certain independent powers in producing the desirable ends of efficiency processes as specified in Chapter IX. Thus heat produces warmth, health, and hence, in a measure, life itself. Light gives knowledge and externality in many ways, and has other useful properties. Inertia often means certain permanences in time, and similarly other negative forms of force have their peculiar functions. The specific and more or less concrete uses of each kind of force are described in books treating distinctively of them.

The general utility of objective forces as substitutes for more costly forms of human energy was pointed out in the introductory paragraphs of this chapter.

Transcendent Values

We should not fail to observe also that certain of the above values in some circumstances may be transcendental or unique. Instances are the utilization of night hours by the use of artificial light, or the development of life by the aid of artificial heat in certain situations where life could not exist otherwise. Other instances are certain esthetic results, as when light in itself becomes a source of pleasure. So various forces and motions may give rise to what is termed the joy of living.

Values in terms of Human Progress

From another point of view we may realize the efficiency values of force in the aggregate by observing that human civilization as it has progressed has been marked by an increasing use of various forces. Every great man has been full of power of some sort, and usually mere physical fire and passion have been an element or aspect of this. So every great race or nation has had great potential powers of combat as well as of constructive power.

Limitations and Dangers

This part of the discussion should not be concluded without a consideration of the fact that certain limitations and dangers are connected with the use of force. Thus physical forces in general are subject to the limitation which has been termed the dissipation of energy. This means that forces when transformed tend to pass into lower forms. Each particular form of force also has its own limitations.

Certain dangers and losses are also connected with the use of forces. Thus forces when used in too large or too small a way, or when otherwise misdirected, may produce floods, droughts, conflagrations, blindness, or death.

As a whole, however, the mastery of forces in connection with matter and substance tend to give a mechanical trigger control of the universe, with a dialectic of resulting values.

EFFICIENCY ANALYSIS OF MOTION

In classifying motions and more especially in investigating the efficiency functions of motion, it has been necessary to analyze the idea of motion to a certain extent. Certain other remarks however should be made on this matter in order to complete as far as possible the determination of the fundamental sources of efficiency contained in this concept.

Multiplicative Nature of Motion

The motion of an object implies certain diversities in the position and relations of the object, along with a uniformity in the object itself. This diversity and this uniformity are the primary sources of the group nature of motion, and hence of most of its efficiencies.

From another point of view the multiplicative nature of motion may be realized by observing that motion is a resultant of the ideas of space and time. We shall find later that these concepts, from the efficiency point of view, are primarily multiplicative in essence. Hence it follows that motion must be essentially of the same character. The formulas which are used to express the laws of motion in terms of space and time are multiplicative in form and hence illustrate the principle under discussion. Examples of such formulas are $s = vt$ and $s = \frac{1}{2} gt^2$.

Unique Element in Motion

In motion as in other categories of efficiency, there is also present a unique and irreducible element. In other words, after we have analyzed the conception of motion as far as we can in terms of the other Efficients and of conventional categories, something is left which is incapable of further analysis. This element is the source of useful results in ways not fully understood, and may at any time become the source of still other results.

Definitions

It may be useful to sum up the preceding discussion of the nature of motion in the form of an approximate definition.

Motion is a change of position accompanied by a continuous and manifold re-multiplicative grouping of an object in relation to all other objects in time and space.

Each special form of motion is also capable of its individual analysis and efficiency description. Thus

Change is vague and formless motion; or motion determined mainly by a difference in an object as viewed originally and after an interval of time.

It is sometimes useful to regard a motion as a mere change, even when the motion is fully understood. In this case change is the prime group in the system of diversities constituting the motion.

Kinematic is a term by which it is convenient to denote the abstract efficiental essence of motion and which it is possible to clothe under given circumstances with the more concrete categories which give rise to motion as conventionally understood.

EFFICIENCY ANALYSIS OF FORCE

The efficiency analysis of force, like that of motion, has been made in large part in the course of the preceding discussion.

Multiplicative Nature and Uniqueness of Force

It may be added, however, that the multiplicative nature of force or energy may be made more clear by a detailed study of the laws which govern special forces. Thus the central forces, such as gravitation, sound, magnetism, and light, are governed by what is known as the law of inverse squares. According to this law, the intensity

of each of these forces varies inversely as the square of the distance from the center where the force originates.

Force like motion also contains a unique source of efficiency which is constantly to be reckoned with.

Definitions

As an approximate definition we have the following:

Force is the aggregate of externalities which produce or prevent motion, directly or indirectly.

Each specific form of force also contains its own specific sources of efficiency and hence is capable of being described in efficiency terms of its own. Thus

A *cause* is a force, or that which contains a force, considered as preceding in time the motion associated with it.

Potential as a form of force means latent force. The term is often used with reference to the possible generation, throughout a given area, of force uniform in kind and degree. It is in this sense that lines or surfaces of equal potential are spoken of.

Dynamic is a term by which it is convenient to denote the abstract efficiental essence of force and which it is possible to clothe under given circumstances with the more concrete categories which give rise to force and energy as conventially understood.

METHOD FOR MOTIONS

In order to utilize motion to the utmost, it is important usually not to regard it as mere vague or spasmodic change, but to realize it as multiplicative in nature.

Variety and Extreme Forms of Motion

As with the other Efficients, this conception of the category in hand aids us in realizing the great number of forms which it can assume by the multiplicative combination of different objects, paths, causes, and results. This conception also helps us to anticipate the extreme forms which motion may assume, and in particular the extraordinary velocities which may characterize it. For evidently there is no appreciable limit to speed which may be generated by even a finite number of levers each rotating upon the end of the preceding. Hence it should occasion no surprise to find that the

velocity of light is 186,000 miles per second, and we are even able to deal with the statement that the velocity at which gravitation acts must be at least 7,000,000 times that of light.

In all practical affairs it is well constantly to realize that things not only hang together in groups, but that they also travel and change in multiplicative ways, as by swings and rushes, which present lightning strokes of both danger and opportunity.

In cases of difficulty and limitation it is often better to use crude and vague motion rather than no motion at all. Certain instances where this principle applies have been given in the chapter on directive action. A similar case is that of keeping up certain motions in a merely formal way, while waiting for them to pass into more effective action.

Motion Study in Specific Cases

The importance of the specific study of each individual kind of motion has already been pointed out. This applies to abstract or ideal motions as well as concrete ones. Thus it is important to note that at the present time the use of a new mechanical invention spreads rapidly while a new ethical or political idea travels more slowly. Much of the success of Napoleon was due to the fact that he knew the velocity with which his armies could be made to travel and he also knew the limits to this velocity.

In each department of life the detailed study of the motions involved forms an important source of increasing efficiency. Mention has already been made of the increased efficiency obtained in bricklaying by diminishing the number of motions involved in the laying of one brick. A similar study of almost any operation such as husking an ear of corn or harnessing a horse may produce similar results. A like opportunity for increasing efficiency is found in combining and arranging buildings, rooms and parts of rooms so as to reduce the size and number of motions of persons living and working in these.

The relative importance of speed in different cases often merits careful study. Thus great speed in firing a battle ship is of prime importance, while it is of little or no value in calculating the date of a distant eclipse of the moon.

Auxiliaries to Motions

Mention should also be made of the importance of using the best possible auxiliaries in connection with motions, as in generating or detecting very rapid or very slow motions. Symbolical and mechanical means may also often be made effective aids in keeping a machine running at its maximum speed efficiency. Thus "Mr. F. W. Taylor has succeeded in establishing formulae sufficiently trustworthy for the production of slide rules by which it is possible to determine in a few minutes the best speed and feed to use in executing any given piece of work in any given lathe and with any given set of tools". With practice and experience similar semi-mechanical methods may become more or less instinctive in the individual life with respect to higher matters also.

METHOD FOR FORCES

Similarly, in order to utilize forces to the utmost, it is important to regard them not as mere vague thrusts and pulls, but to understand that they are multiplicative in their essential nature.

Variety and Extreme Forms of Forces

This conception enables us to realize their great number and variety, their extreme forms, to grasp them in a shape ready for use, and to apply them most effectively. Thus the steam railroad would have been a practical fact much sooner if the extent of the friction between a rail and the rim of a wheel had been understood. The efficiency of the steam engine comes largely from the extreme facility with which water vapor takes up large quantities of energy and gives off the same by condensation.

Opposed Systems of Forces

In this connection it is important to realize that each case of seeming rest or stability is merely a case of equilibrium between balanced forces, and also that these balanced forces are multiplicative in nature. Thus in the human body opposite sets of nerves hold each other in check and the same is true of muscles. If one set is rendered inactive, the other set will act often with great energy. In general, the universe is in a state of approximate manifold equilibrium between opposed multiplicative forces. Any change or disturbance in one of two opposed sets of forces may result in sudden

explosions or transformations. In other words all things are in a state of triggerish opportunity or danger.

Study of Specific Forces

For this and other reasons the importance of making a specific study of each individual kind of force is evident. In making chemical syntheses, for instance, it is found that argon has too small an affinity for other substances to be widely useful, that fluorine has too great an affinity, and that the medium powers of carbon in this respect cause it to be the most widely used of substances. Similar special qualities are found to characterize all forces.

Forces of High Order

Other things being equal it is best to use the force which is most highly multiplicatively grouped in its nature or properties. For it is evident that such a force can be adapted most readily to any given end, such as delicacy or intensity of action. In such forces, a less costly form of energy can most readily be substituted in part for a more expensive form.

The advantage of using forces which govern, unlock, or control other forces should constantly be kept in mind. It is also best wherever possible to use abstract forces like pressure or influence instead of brute, concrete forces. Competition is often an effective force in producing results, but, as a rule, it is costly and of a low order, processes of coöperation and direct grouping being in themselves much higher.

In conclusion it is important to keep in mind that the highest forces are those which spring from the use of the Efficients in some form, and the highest motions are those which accompany such forces. For instance the particular conception which a person has of the summum bonum governs the life of that person, and the world life is changed more profoundly by the views of society in this respect than in any other way.

Hence the generation or utilization of efficiental force is one of the highest sources of efficiency.

Summary of Chapter

To sum up, motion and increased speed are often the sources of efficiency in various ways. The same is true of the utilization of

forces, whether these be human or external to man. Hence in the science of obtaining results, both motion and force merit close study. Motions are classified both with respect to their forms, and also with respect to the objects which move. Similarly forces are classified according to the substances in which they are contained, according to the motions to which they give rise, or with respect to other specific results produced by them. Also some forces are characterized by a combination of the above methods, examples being light, heat, publicity, and highest of all, efficiental forces.

In studying the efficiency functions of motions, it is convenient, first to investigate those of pure motion, then those of motions where the force involved is slight, and finally those of motions which are taken as representative of energy expended (as in bricklaying). The values inherent in superior speed also merit particular study. The fundamentally useful property in motion is that of producing new groups, but this property often takes a multitude of valuable special forms, some of them of transcendent value. Force is useful as a source of motion and also in other special ways. Hence human progress has ever been marked by an increase in the amount and speed of motions, and in the utilization of forces.

In order to make the most effective use of motion, its essentially multiplicative nature must be realized. Such a realization leads to an appreciation of its many varieties and of the extreme forms which some of these assume. Specific study of individual cases is highly important. Every motion is associated with some object or objects, hence the study of auxiliaries in connection with motions is peculiarly fruitful in results. The same remarks apply to the use of forces. In addition it should here be noted that apparent rest is due to a balance of forces which, if understood, may often be happily manipulated in whole or in part. Stress should always be laid on efficiental motions and efficiental forces.

EXERCISE 13

1. Give an example of each of the principal kinds of motion.
2. Also of each of the principal kinds of force.
3. In a given factory a machine which is capable of making 200 revolutions per minute is actually making only 50 revolutions per minute. State the speed efficiency of the machine as a per cent.
4. A given factory contains four machines which are being run at ¾, ½, ⅔,

and $\frac{3}{8}$ of their possible speeds respectively. If these machines work in parallel fashion, find their average speed efficiency. If they run tandem, find their resultant speed efficiency.

5. Three hours of labor by one man were once required to raise one bushel of wheat. Within the past century this time has been reduced to 9 minutes. Express the degrees of relative efficiency involved. What are the main sources of this increased efficiency?

6. In a given case the cost of floating 50,000 tons of coal from Pittsburgh to New Orleans was $35,000. To send this coal by railroad would have cost $150,000. State the degree of efficiency in the first process as compared with the second. State the principle sources of this efficiency.

7. In moving a certain building the wages of the workmen employed amounted to $18 per day. By using a team of horses at a cost of $3 per day, the time occupied in moving the building could have been reduced from six days to two. How much more efficient would the second method have been than the first?

8. Point out the advantages with respect to speed and power in the use of an autotruck as compared with a dray drawn by horses.

9. Give an example in which the superior speed of the automobile (as compared with the horse and wagon) is a means of saving life.

Give an example of a unique or transcendent value

10. In motion 11. In superior speed 12. In force 13. In superior force

14. Give an example where a motion is a useful symbol of a force.

15. Give an example where a force (or the material object in which a force is stored) is a useful handle by which to control a motion or another force.

16. Some of the air currents of the upper atmosphere travel with a velocity of 100 miles per hour. If these could be utilized by an aëroplane in crossing the Atlantic Ocean, what advantages would result?

17. Give an illustration of the power of publicity as a social force.

18. Give an example in which graceful motion is a sign of efficiency.

19. A laborer in husking one ear of corn was observed to be making eight motions. If the same work could have been done with five motions, what was the degree of efficiency in his work?

20. Make up and work a similar example concerning the process of hitching a horse.

21. Harnessing a horse 22. Washing a dish

23. Name four other processes of labor where the work involved might be diminished by an analysis of the process into constituent motions.

24. Give an example in which momentum (or inertia) is made a source of efficiency.

25. Point out some of the uses of friction.

26. Show how the dimensions of a force are increased by the use of the hydraulic press.

27. Give an illustration of the "power of an idea".

28. Give an instance where it is more advantageous to use abstract than concrete force.

29. Give three examples of the saving of human labor by the substitution for it of the forces of nature.

30. Why is speed in firing a gun on a battleship more important than speed in calculating an eclipse in an observatory.

31. Give an example where increased speed results in loss.

32. Give three examples of the misuse of natural forces.

33. State the advantages to a government of keeping the money issued by it in rapid circulation. To a merchant, of turning over his capital rapidly.

34. State the advantages of being able, in certain cases, to send money by telegraph instead of by mail.

35. State some of the advantages in business conducted by competition. Also some of the disadvantages.

36. Write an account of the increased utilization of motion as civilization has progressed.

37. Also of the use of increasingly high velocities.

38. Also of the increased use of forces of all kinds.

CHAPTER XIII

RHYTHM

Illustrations

After studying the work of men engaged in loading pig iron on a car, F. W. Taylor found that in order to load the maximum amount in a day, a workman must be free from load 58 per cent of the time. Hence in the Bethlehem Steel Works he required men doing this kind of work to sit down and rest at intervals so as to bring the part of the time when they were free from load up to the 58 per cent. This was an essential part of the method by which he increased the amount of pig iron loaded per day by each man from 12½ tons to 47 tons.

Similarly a great artist often finds that he can produce results of the highest order of excellence only by alternating long periods of rest with relatively short periods of intense exertion.

Not only do men consciously use the rhythmic method of work in order to gain added efficiency, but nature has also utilized the same principle in various ways in building up efficiency organisms like plants and animals. An example is the economy and power in the assimilation of food gained by the rhythm which characterizes the digestive process in man and the higher animals. The oscillating method also is present in many forms in the inanimate world and when rightly understood and utilized is a frequent source of efficiency. Thus by noting the tidal rise and fall of the sea, the navigator is able to pass over bars and to enter harbors otherwise inaccessible.

General Statement

As a preliminary definition or description we may say that a rhythm is the motion of an object in opposite directions in alternate succession; or more generally it is the alternate appearance and disappearance of an object or quality. The forces which produce rhythms are frequently complex and more or less obscure. Hence it is best usually to treat rhythms by means of the motions which characterize them.

CLASSIFICATION

Objects Capable of Rhythmic Change

It is an aid in using rhythms effectively to classify them according to the objects which move back and forth in the vibrations involved. Thus we may have a rhythmic motion of a material object, of a force, or of an idea.

In this connection it is well to note that every object or entity is subject to rhythmic motion or change of some kind, but not of all kinds. Thus the apparently immobile stone by the roadside is composed of molecules each of which is in a state of constant vibration. These vibrations themselves are subject to other rhythmic changes such as those produced by the daily and annual changes in temperature. All sounds made in the neighborhood also cause the stone to pulsate in other ways.

An investigation of the matter has shown that a state of mental attention is not uniform and continuous; on the contrary it is composed of distinct pulses or beats. A similar alternation characterizes each species of nervous and mental action, as perception, memory, and emotion.

Extreme Cases

Likewise what appears at first sight to be a uniform motion or an increasing or decreasing one, on examination is found to be rhythmic in some way, as when it is part of a rhythmic motion of great amplitude. Thus when a body falls from an elevated point toward the earth, it seems to move in one direction only, and with a constantly increasing velocity; but if its motion were not interrupted, the falling body would pass through the earth and would continue to vibrate back and forth past the earth's center. The axis of the earth is continually shifting its position slightly in a complex rhythmic way.

It is also to be carefully noted that the most abstract and complex ideas and entities are subject to rhythmic change. Thus we have rhythms of intensity, availability, efficiency, and of certain properties, at least, of space and time.

Rhythms Classified according to Form

It is also useful to classify rhythms according to their various forms. Thus rhythmic motions may be one, two, or three dimensional. Examples of linear rhythms are the undulations in a rod,

or wire, or those made by a point vibrating in a line past another fixed point. Examples of areal rhythms are the wave motion in a sounding board or in any surface. Illustrations of solid rhythms are wave motions in any fluid as in the air, ocean, or ether. The lines in which rhythms of one dimension act may be straight, circular, or of any curved or irregular shape. An important case is that which results from the combination of a circular motion with a rectilinear motion, as in the motion of a point in the tire of a moving wagon wheel.

Other Spatial Forms

Linear waves are of two principal kinds, viz.: (1) longitudinal waves, or alternate condensations and rarefactions such as are made in the air by a vibrating tuning fork or any sound wave; (2) transverse waves, or vibrations at right angles to the line of progress of the wave, such as are made by a stretched vibrating string, or by light in the ether.

Various wave motions may also be grouped and classified according to the lengths of the wave involved, their amplitude (i.e., breadth), their geometric form, or velocity.

The motion of a solid object back and forth in a line like that of a piston in a cylinder is termed reciprocating action.

Spherical wave motion is one which radiates in all directions from a center in a solid space, as when sound travels in all directions from the object producing it.

In some rhythms the moving object is not changed internally. An example is the earth as it rotates on its axis, or revolves about the sun. In other cases the moving object does change internally. Thus in wave motion in the ocean, as the wave moves forward, the drops of water composing the wave change in their relative position.

It is also well to observe that certain rhythms may be characterized as increasing or decreasing in one or more respects as in the length, amplitude, or velocity of successive waves. An important special form under this general class is the infinite decreasing rhythm. In some of these the whole space occupied is finite owing to the fact that the undulations diminish in length; or the time occupied by the motion is finite owing to the fact that successive waves occupy less and less time; or a rhythm with an infinite number of terms may be finite in both of these respects.

Numerical Forms

Rhythms may be characterized according to the category of number in different ways. The simplest of these is by the number of oscillations involved in a given instance. A much more important standard is the number of objects which alternately become prominent in a rhythmic motion. Thus the vibrating motion of a single object like that of a piston in a cylinder constitutes a unitary rhythm. The dominance in turn of two political parties, the alternate prominence of expansion and contraction, or of intake and output, are cases of dual rhythm. The cyclic changes of moisture through the forms cloud, rain, river, ocean, cloud, constitute a case of quadruple or fourfold rhythm.

Similarly when two or more simple or elemental rhythms are combined to make a single resultant rhythm, this resultant may be characterized by the number of its components.

Other Important Forms

With respect to *time,* rhythms may not only be termed temporary or permanent, but may also be named according to the periods of time in which a single alternation or a cycle of alternations is completed. For example we have diurnal, weekly, monthly, or annual rhythms.

With respect to the category of *order* (or direction) in many important rhythms certain subordinate parts may be regarded as retrograde or back eddies. Examples are the method of leaping ahead from one prominent point to another in a domain and returning from time to time to fill in details, or the method of construction and repair, of using rules and exceptions, or of action and reaction.

In relation to motion, special attention should be called to what may be termed *static* rhythms. Examples are a series of land ridges and depressions, or bands of alternate colors like the stripes of a zebra. In some cases static rhythms are the result of some preceding active rhythm, examples being the rings of annual growth in a tree, a set of geological strata, or the series of moraines formed by a glacier as it alternately advances and retreats. Also static rhythms may give rise to active rhythms in various ways, an example being the rhythm in mental cognition as the parts of a static rhythm are viewed in succession.

A highly developed and very important rhythm is that obtained by arranging the chemical elements according to the periodic law. Similarly it is often the source of certain kinds of efficiency to arrange the elements of a multiplicative group system in a cyclic linear order.

As to uniformity and diversity rhythms may be characterized as *continuous* or *discontinuous*. Examples of the former species are the motion of a wave in the ocean, or the alternate succession of day and night. Examples of the latter are the periodic reappearances of 17-year locusts, or the recurrence of eclipses. Also a rhythm may be homogeneous (or heterogeneous), either as to the material which moves, or with respect to the forms of the oscillations involved.

Grouped Rhythms

Evidently also rhythms may be grouped multiplicatively in various ways. One of these ways is to combine a number of rhythms to form a resultant, then to make a resultant of resultants, and so on. Or a given rhythm may give rise to a number of others, each of which in turn produces still others, the process being repeated during a series of stages. For instance the rhythmic motion of the earth about the sun produces many derived rhythms such as the change of seasons; each of these secondary rhythms gives rise to others of the third order and so on. Similarly the alternation of day and night gives rise to a whole system of derived rhythms.

Rhythms of High Order

Hence it follows that rhythms may be of different orders in various ways, or according to different standards, the most important of these standards being that of degrees of general efficiency. Thus a rhythm may be of low order because it belongs to a low order in a multiplicative group system, or because it is feeble or slow, or owing to the fact that it is motion of a material or in a medium of low order. An instance of the latter is the alternation in the moods of an insane person. On the other hand illustrations of rhythms of a high order are the alternate use of the abstract and concrete in education; of individualism and collectivism in sociology; of liberalism and conservatism in politics; of appearance and reality, or of the relative and absolute in philosophy. Other important cases are those where certain Efficients, or parts of Efficients, are used

in turn recurrently. Thus it may be the source of great efficiency to use a unit and multiplier in alternate succession, the product of each step being the unit to which the next multiplier is applied. Other important cases are the alternate use of algebraic and graphic symbolisms, whether in mathematical work, in art, or in life in general; of externality and remultiplicative grouping, these taking the form, for instance, of reading and meditation, or of travel and study, or of expansion and reorganization of any kind; of analysis and synthesis, as in chemistry; of idealism and realism.

It will be found later that perhaps the most fundamental and comprehensive case of all is the use, in cyclic succession, of phenomena, categories, and the Efficients.

Natural and Artificial Rhythms

It is also important to discriminate between the two leading classes of rhythms due to the presence or absence of the directive principle. These are:

(1) Natural rhythms or those which exist independently of man's volition, as for instance the alternation of day and night, or the beating of the human heart;

(2) Artificial rhythms or those constructed by man more or less intentionally, as the alternate use of humor and pathos in a novel, or the rhythmic recurrence of the classical and romantic movements in a field of art.

It will be found that each of these two classes has its own strongly marked efficiency functions. This follows from the fact that the causes and sources of artificial rhythms are in large measure under man's control, while natural rhythms must in large measure be dealt with as we find them.

It is to be noted in this connection that certain rhythms may be regarded as semi-natural or semi-artificial. For instance we have rhythms like the swinging of a pendulum which, after being initiated by man, continue to act for a considerable period without his aid. Also the rhythmic acts of one individual often serve as objective or natural rhythms to another person.

Complex Rhythms

It is useful also to remember that the above species of rhythms may be compounded and complexed in various ways. Illustrations

are the network of alternations which constitute the work of a farm during a year, or the combination of rhythms involved in a musical symphony.

EFFICIENCY FUNCTIONS OF NATURAL RHYTHMS

Since a rhythm is a form of motion and also involves forces from certain points of view, it follows that the efficiency functions of motions and forces as presented in the preceding chapter hold also for rhythms to a certain extent. However the rhythm, owing to its individual characteristics, is also the source of efficiency in ways peculiar to itself.

In discussing the latter class of uses it will be an aid to observe the distinction between natural and artificial rhythms. We shall consider first the efficiency uses of natural rhythms including under this head the uses of artificial rhythms when the latter are viewed objectively or apart from their originators.

Illustration of Uses of a Natural Rhythm

Owing to the somewhat complex nature of a rhythm, the efficiency functions of any specific rhythm are usually manifold and more or less interwoven. Thus a tide which enables a navigator to carry his ship over a bar is a special accumulation of water which man utilizes by the means of the directive principle. The advantage in this case may also be immeasurable in that it might be impossible to get the ship over the bar in any other way when pursued by a war vessel. Also if the navigator instead of hiring a tugboat uses the tide to carry his ship up a river, he applies the force principle of efficiency in the shape of substitution of a cheap force for a costly one as explained in the preceding chapter. This also constitutes another application of the directive principle.

The interval of time between successive tidal crests also constitutes an important natural unit, which is not subject to man's control and is therefore not liable to capricious change. Owing to the permanency of the causes producing tides this unit has an enormous multiplier giving rise to a multiplicative group system, which comprehends all tides the world over and which is also unlimited in time. The approximate uniformity in the depth of tides at any given place, together with the slight rhythm of changes in this depth, gives another cross multiplicative group systematization of tides.

The result is a general system of water changes which constitutes a wide field for the employment of the directive principle in adding efficiency to the processes of commerce.

The units, uniformities, and multiplicative group systems involved in the daily rhythm of day and night and the change of seasons are even less subject to change and more comprehensive than those connected with the tides, and hence enable us to plan and direct our activities even more widely and vitally.

Reuse in connection with Rhythms

It is to be noted also that the uniformity which characterizes the wave elements in a given natural rhythm, opens the way to reuse of our knowledge of one of these oscillation periods in connection with all the rest in the given rhythm.

Also the similarity between the general wave structure in one rhythm with that in other rhythmic systems enables us to apply to the rest what we learn concerning any one of them. Thus the general principles of wave motion, as for instance the laws which govern reflection of waves or their concentration, when once learned can be reused in mastering the properties of each particular form of such motion, as sound, heat, light, and electro-magnetic waves. This method is particularly useful in learning the action of invisible waves by observing the properties of those which are visible. It was by this means that Herz arrived at the properties of electro-magnetic waves and opened the way for wireless telegraphy.

Additional Illustrations

Another domain of useful applications of natural rhythms is illustrated by the fact that counting the number of rings of annual growth (a static rhythm) on a tree stump enables us to determine the age of a tree, and get knowledge which perhaps we could not obtain in any other way. Similarly an examination of a series of terminal moraines, or of a series of geological strata, gives us important knowledge of the past history of the earth, perhaps of the present position of valuable deposits as of coal and iron, and thus enables us to shape our directive action more effectively.

A study of tidal action enabled George H. Darwin to determine the past history of the earth in relation to the moon.

The arrangement of the chemical elements in the periodic system made it possible for Mendeleveff to predict the existence of chemi-

cal elements as yet unknown and also to foretell certain of their properties. It has also aided men in searching for and finding these. Among the elements thus discovered were gallium, scandium, and germanium. So afterward in the year 1894, when Ramsay had discovered argon, a gas distinctively different from the chemical elements previously known, "the periodic law immediately suggested the existence of a number of elements" allied to it. A search was made for these and the result was the quick discovery of neon, krypton, and xenon.

By the observation of the long continued vibration of a pendulum we obtain our most accurate determination of the force of gravitation. The combination of two rhythms in such a way that the elevations in one correspond to the depressions in the other often forms the most convenient way of obtaining a stable or uniform result. An example is that of obtaining a uniform income by investing partly in the stock of a company which produces iron, and partly in another company which buys iron and uses it as raw material.

A knowledge of the law of action and reaction, on the one hand prevents excessive optimism in times of prosperity, and on the other hand prevents depression in times of disaster.

The various efficiency attributes of many rhythms as of sound, light, color, flight of birds, dancing, make them sources of pleasure and efficiency beyond the possibility of complete analysis at present.

USES OF ARTIFICIAL RHYTHMS

A Source of Groups and New Power

The fundamental reason which has led man to form and use rhythms is the fact that in such rhythms he obtains groups at less cost than in other available ways and also at times he overcomes limitations otherwise insuperable. This principle was illustrated in the opening paragraphs of the chapter.

The element of repetition which characterizes a rhythm brings with it an inevitable reuse of the groupage which is the efficiency essence of each wave or undulation in the rhythm. The result is not only the direct efficiencies which characterize reuse, but also other auxiliary efficiencies such as are usually included under such terms as increased ease and mastery. These involve certain physiological, nervous, and mental economies, externalities, motion, force, and various self-developing efficiencies.

Illustrations of Efficiental Values

Particular forms of artificial rhythmic actions are often accompanied by characteristic groups of efficiencies. In the motion of a piston in a cylinder, there is reuse of the walls of the cylinder. In all cases of reciprocating action, some similar form of reuse is present. If a number of persons alternate in the use of an object, as a pump, the efficiency of the pump receives a large multiplier, at times with the immeasurable advantage of saving human life. When Stonewall Jackson made long marches and won battles by causing his men to lie down and rest at the end of each hour of marching, he used repeatedly the prime groups in the physical powers of his soldiers. So when a man pursues different occupations alternately, he reuses his prime groups of body and mind (and thus obtains a more or less continuous use of them), his more sensitive and specialized faculties being enabled to rest and recuperate in turn.

The back eddy method of progress, that is, the method of leaping ahead and then returning to correct errors and to perfect details, has the advantages which arise from the use of externality and from the treatment of prime groups first in a large domain of material. To take a specific instance, the most efficient method of political progress is to proceed by certain definite steps and to pause after each step till the reorganization called for by that step has been completed.

The alternate use of two contrasted objects or methods often brings efficiency in the form of a repeated use of a reciprocal unit and multiplier. This form of efficiency is illustrated in the alternate use of the concrete and abstract in education, or of the object and symbol, of the local and universal; or in the use of humor and pathos in a speech or in any literary narrative.

Negative Values

Certain negative values often accompany the use of artificial rhythms. For instance by the alternate conference of power on different political parties, important governmental evils are frequently removed or kept from taking deep root, or are even prevented from appearing at all.

Transcendent Fruitages

The results obtained by the use of artificial rhythms are often transcendent and unique. For instance by the use of the rhythmic

method a person is frequently able to overcome an otherwise insuperable difficulty, or to achieve otherwise unattainable results. An illustration is the breaking down of an obstacle by rhythmically repeated blows. A more abstract case is the solution of a mathematical or other problem which would otherwise be insoluble, by the use of successive like steps. A more subtle case is that of keeping an approximately straight track, as in driving a bicycle, by slight deviations alternately to the right and left, that is, by the alternate use of what may be regarded as positive and negative externality.

Complex Values

The above efficiency values may also occur in various complex forms. A comparatively simple case is that of a preacher who finds that he obtains better results by preaching a highly developed sermon occasionally than by attempting to make all of his discourses masterpieces. He thus not only uses his own powers with rhythmic effectiveness, but he sets up certain useful rhythms of self-activity and interest in his auditors. A rotation of crops by an agriculturist gives greater uniformity in income, and in the number of laborers employed, prevents the accumulation of crop pests, opens the way to economies in fertilizers, and has many other advantages. More complex still are the benefits obtained from a rhythmic variety of activity in the higher thought and culture life.

Limitations and Drawbacks

It is to be noted however that certain limitations and compensations are often connected with the use of rhythms. Thus while one political party is in power, the members of the other party are frequently idle or not working to their full capacity. Also a change from one party to another may produce costly breaks and interruptions in efficiency processes.

Human Progress Marked by More and Higher Rhythms

In the aggregate, however, in most fields, the advantages in the use of rhythms vastly predominate over their drawbacks. Hence it is not surprising to find that human progress has been marked by an increasing use of both natural and artificial rhythms, and of rhythms which take ever more comprehensive and vitally effective forms. An example of a recently developed and highly effective

form of objective rhythm is the use of electro-magnetic waves in wireless telegraphy, and of the propeller in aerial navigation. Instances of more ideal rhythms whose use has developed with advancing civilization, are the alternate employment of liberal and conservative movements in various lines, and the method of leaping ahead and returning to complete each step.

EFFICIENCY ANALYSIS OF RHYTHM

As with the study of the other Efficients, so here it will be an advantage to make some investigation of the sources of efficiency in rhythms apart from the partial presentation of the matter necessarily given in stating the efficiency functions of rhythms.

Multiplicative and Limitation Elements

The group or multiplicative principle is present in rhythms not only in ways already indicated but also in certain other ways as with respect to the principle of limitation. Thus in the vibrations of a tuning fork, as a prong bends away from its natural position the resistance increases in a multiplicative way till a point is reached where the momentum of the prong is overcome and the motion is reversed; the velocity in the opposite direction then increases in a multiplicative way till it again encounters a multiplying resistance, and so alternately. In the action of a geyser, in the oscillations in the supply and demand of an article of trade, in successive waves of insect pests, and their enemies, the same general law is found at work.

Externalities present in Rhythms

The group in some of its developed efficiental forms is also present in rhythms. For instance owing to the limitations inherent in wave action, a space exists between two successive waves or their crests which forms a natural externality to these waves and also a place where other externalities may be applied. Another inherent source of efficiency in a rhythm is the fact that a rhythm by mere continued action produces an externality homogeneous with the original data, and one to which the multiplicative group and order of material principles may be readily applied.

Other noteworthy elements of efficiency in this respect are the space and time categories which are combined in a rhythm. The

space category for instance opens the way for a multiplicative group treatment by introducing the idea of the deviation from a line (or standard) alternately in opposite directions.

Imperfectly Understood Sources of Efficiency

Rhythms often contain certain dimly understood and perhaps transcendent sources of efficiency. For instance it has been found that light acting intermittently on vegetation will, under some circumstances, produce more rapid effects than light acting continuously. But the reasons for such a result are not fully known. In the application of sewage to filter beds, intermissions covering 18 hours out of every 24 are found to be necessary. By the use of the alternating electric current power may be transmitted for long distances where such conveyance would not be practicable by the use of the continuous current.

Different Aspects

It should be stated that in classifying and using rhythms an element of relativity is often prominent. Thus what is a rhythm from one point of view is a mere collection of uniformities and differences in another aspect. An example is a series of glacial terminal moraines. Similarly a cyclic repetition of a certain group of elements may often be regarded as a linear multiplicative group; also a federal multiplicative group when made over into a linear form becomes a rhythm. An element of relativity is also present among different species of rhythm. For what is one species from one standpoint, may be another species when viewed differently.

A preliminary definition of the rhythm was given on p. 230. In the light of the further analytical examination which has just been made a rhythm may be described as a special form of multiplicative group system resulting from the operation of the multiplicative principle in a field of great limitation.

METHODS OF APPLICATION

Multiplicative Principle an Aid in Many Ways

In order to make the most efficient possible use of rhythms, it is important to realize their many groupish or multiplicative elements and aspects and to apply these aggressively. Thus the multiplicative principle enables us to anticipate the large number of existent

or possible rhythms; to realize their propagative nature; to grasp them in systems; and to detect, or be prepared for, the appearance of obscure rhythms of extreme forms, such as a tidal wave, in any time or place.

Rhythms in Difficult Cases

It is important also to realize that in a situation of difficulty it is often better to use a crude rhythm rather than no rhythm or other more developed Efficient. Thus the atomic theory of matter represents (or includes) certain useful ways of regarding the constitution of matter; the ether hypothesis includes other of these useful methods. It would be highly advantageous to combine these two views into a single more fundamental one. As this cannot be done at present, men of science often use the two hypotheses in alternate succession, and obtain many useful results in this way.

Adaptation to Special Situations

Again it is often a matter of careful consideration to determine what particular form of rhythm to use in a given case, or whether indeed to use a rhythm or some other Efficient. For example at times it may be best to let a panic of a fashion run its course and to wait for a natural reaction, instead of trying to check the movement at once or trying to substitute some other process. In like manner it is important to know when it is best to submit to a natural rhythm, as that of day and night, and when to supersede it by an artificial rhythm consisting of periods of intense work alternating with periods of rest determined by special circumstances. Thus the rhythm followed by a person doing manual labor would usually be very different from that employed by a great artist. In this connection as well as with reference to other considerations it is important to study the more or less technical properties of certain natural rhythms, as for instance, those of some physical forces, or those which characterize certain processes of physiology, psychology, and sociology.

Preference to Rhythms of High Order

Where a choice among various rhythms is possible, it is of course desirable to use that one which is highest in the scale of efficiency. Thus in certain forms of the steam engine much has been gained by

substituting a rotary for a reciprocating motion. Similarly the rotary snow plow is more efficient than one which acts by direct thrusts and recessions. One of the most profound and therefore useful rhythms is the alternation of directive with laissez faire methods.

An important method of elevating a rhythm in the scale of efficiency is to develop or master it till it becomes as automatic as possible, alike in its main action, its adjustments in details, and in its ability to give way to other more efficient processes.

Standard Forms and Systems

With rhythms as with other Efficients, it is often an aid to select certain standard forms, and to arrange and master these few standard forms as a multiplicative group system. It will be an advantage wherever possible to give the preference to these standard forms and often to utilize them by variant reuse instead of inventing altogether new forms.

Externality often an Aid

Externality has important uses in aiding us to discover, create, and operate rhythms. One rhythm may be a help to others in this respect. An example is the well known value of song in the operation of both subjective and objective rhythms.

It is desirable in many cases to use calculus methods in treating rhythms; that is, methods that are largely symbolic and mechanical and yet give the results of rationality. An instance is the analysis of a rotary motion into two linear rhythms at right angles to each other, only one of the two components in many cases being effective. Another instance is the compounding of two linear rhythms to obtain a circular rhythm. A more general instance is the use of the instrument called the harmonic analyzer.

In the mastery of rhythms many special methods beside the direct use of rhythms are available. Thus one's leisure may often be profitably utilized by watching the rhythmic motions involved in weather changes, as in noting the daily motions of storm centers and anticyclones. Practice of this sort aids in grasping the equally large but slower and more subtle rhythmic alternations of taste and thought in the world life.

Ideal Species

It is an important aid in obtaining the largest results from the use of rhythms to conceive of an ideal form of this Efficient and to apply this form under all possible circumstances. This ideal in its general form consists of the rhythmic motion or alternate use of the Efficients in efficiental ways for efficiental ends. It is also important for each person to study his limitations and opportunities and to give this ideal a special form adapted to his circumstances. This individual species will usually be found to be some particular form of the processes of leaping ahead by use of the best material at hand, no matter how fragmentary and imperfect this may be, and returning occasionally; in other words, a making of all of life into a comprehensive rhythm of externality and re-multiplicative grouping.

Summary of Chapter

By way of summary for this chapter, we may say that the alternation of rest with work is essential to the most efficient labor. This is an illustration of the many forms of rhythm each of which has its own peculiar uses. Rhythms may be classified according to the objects in them which change in pulsating ways, or according to the special forms of these changes. An important distinction is that between natural and artificial rhythms. Special mention may also be made of active and static rhythms.

A rhythm means a repetition, and it also often means condensations in the forms of waves. Hence, in a double way it means the formation of groups, and of group systems, accompanied by the useful properties characteristic of these Efficients, such as reuse, and often the attainment of results otherwise inaccesible. It is useful to consider separately the valuable functions of natural and artificial rhythms. Special mention is to be made of the value of the back-eddy method of progress; that is, of the method of leaping ahead and then returning periodically to perfect one's work. Human progress has been marked by increasing use of rhythms, especially of those of the highest type.

The rhythmic method is particularly useful in difficult situations. Special study should be given to hidden and deceptive forms of rhythm, and care should be taken to use the highest possible form in each particular case. The highest form of all is an efficiental rhythm reduced to an automatic and self-developing form.

EXERCISE 14

State the class or classes to which each of the following rhythms belongs:

1. Breathing.
2. Democracy, aristocracy, and monarchy when dominant in a country in cyclic succession.
3. Motion of a bird's wing.
4. Running around a closed track repeatedly.
5. Daily change of temperature of a typhoid patient.
6. Variations in the length of a solar day.
7. Successive returns of the same comet.
8. A sound which gradually dies away and then disappears.
9. The daily and annual changes in the color of prawns.
10. The so-called line of beauty.
11. The vertebral column.
12. Alternations of good and bad times.
13. The alternate use of induction and deduction.

Give an example of rhythmic change of

14. Distance	17. Velocity	20. Price
15. Temperature	18. Fashion	21. Value
16. Density	19. Thought	

22. Give an example of each of the principal kinds of rhythms.
23. Give two examples to show what is meant by the amplitude of a rhythm.
24. State, as far as you can, the amplitudes involved in Exs. 1-10.
25. Mention the various rhythms involved in the process of walking. What additional rhythms are involved in dancing?
26. Give some of the rhythms which result from the beating of the human heart.
27. Give two examples of rhythmic motions which are so rapid as to be invisible.
28. Give two examples of rhythmic motions which are difficult to perceive for some other reason.
29. The average annual profits on a certain farm when wheat alone was raised were $4000. Later when rotation of crops was practiced the profits averaged $5500 per year. Compute the degrees of relative efficiency involved.
30. When sailors pull on a rope they usually heave rhythmically and in unison. Why is this?
31. State the uses or advantages connected with each of the cases mentioned in Exs. 1-4.
32. Give an example of a rhythmic change in some processes of trade.
33. Mention a natural unit derived from a rhythmic motion.
34. Give a case where a knowledge of the law of action and reaction is useful.
35. Give two examples where an equilibrium or other uniformity is obtained by the use of an alternation across a mean.

36. Give an example of a uniformity obtained by the combination of two rhythms.

37. Name some book in which humor and pathos are made to alternate effectively.

38. Also one in which humor alternates with some element of strength other than pathos.

39. Give a case where the use of a rhythmic action gives a result otherwise unattainable.

40. State the advantages of repeatedly forgetting and relearning some subject. Also state the disadvantages.

41. Show how a rhythm often opens the way to reuse. Give illustrations.

42. Discuss the advantages and disadvantages of party government.

43. Why is it that when clearing in the daytime, after a storm, the sky is alternately overcast and clear?

44. Mention a case where it is advantageous to use directive and *laissez faire* methods alternately.

EXERCISE 15

Review

1. A yellow wall paper of a given kind reflects 42 per cent of the light falling on it, and a red wall paper 16 per cent. Express the relative light efficiency of these two kinds of paper.

2. Newton stated that he had accomplished what he did because he stood on the shoulders of giants. What principles of efficiency are implied in this statement?

3. In a certain city the average cost of delivering a ton of coal by motor truck is 18 cents, and by horse and wagon is 40 cents. Compute the degrees of relative efficiency involved. What are the main sources of superior efficiency in the first of the two methods used?

4. What principles of efficiency are involved in nipping an evil in the bud?

5. Why was it appropriate for Homer to speak of articulate speech as "winged words"?

6. Name the various uses which man makes of the tides?

What is the prime Efficient involved here?

7. What efficiency principles are involved in the statement "The pen is mightier than the sword"?

8. It costs $314 to send 24,000 pounds of glass a given distance as freight at the 100 pound rate and only $168 by the carload rate although in the latter case the glass must be paid for as if it weighed 30,000 pounds. State the degree of efficiency involved and the source of this efficiency.

9. What Efficient is illustrated in the aphorism, "An ounce of prevention is worth a pound of cure"?

10. A man who was digging 6 tons of iron ore by hand, later by a steam shovel was able to dig 200 tons per day. State the degree of labor efficiency involved and the sources of the increased efficiency.

11. A physician before he used antitoxin lost 82 per cent of his patients sick of diphtheria. After he used antitoxin he lost only 8 per cent. Compute the degree of the relative efficiencies here involved. State the efficiental sources of the increased efficiency.

12. What Efficients are involved in the circulation of the blood in the human body? State some of the advantages which result from this circulation?

13. State the elements of difference between a rotary motion and a reciprocating action. Discuss the relative advantages of each of these in snow plows. Also in steam engines.

14. What Efficients are implied in the statement "Thou hast ordered all things in measure and number and weight".

15. Compute the numerical value of $8976 \times 579 - 8976 \times 577$ by performing the multiplications before the substraction involved. Also calculate the value of the expression by first substracting 577 from 579. Estimate the efficiency of the second process as compared with the first.

16. Discuss the ordering of objects or acts in time as a source of efficiency.

17. Also discuss the orderly arrangement of objects in space as a source of efficiency.

18. Discuss the combining of order in time and order in space as a source of efficiency.

CHAPTER XIV
DIALECTIC

Illustrations

In the year 1856 a young chemist named W. H. Perkin was at work in London on the problem of the artificial synthesis of quinine. He did not succeed in compounding quinine, but in the course of his manipulations he accidently produced mauve, the first of the aniline dyes. This discovery not only led to the whole artificial dye industry, but to many other important results of which no one had any conception at the time. As some one has said, "Who would have dreamed that the discovery of the first aniline dye by Perkin should be an essential link in the development of modern bacteriology and therefore in the crusade against tuberculosis and other infectious diseases?" Similarly when Sir William Crookes attempted to obtain a more accurate determination of the atomic weight of the chemical element thallium, he did not have the remotest idea that his work would result in the discovery of Roentgen rays, or radium, and in the realization in some particulars of the dream of the medieval alchemists as to the transmutation of metals. Columbus failed to find a short route to Asia, but he made the far more important and unexpected discovery of a new world whose beneficial influence on the old world is only beginning. Nothing is more blighting and narrowing than the love of gold for its own sake, yet the search for gold has been the leading factor in the exploration of the world and in the civilization of distant and barbarous regions.

In business the application of any one kind of efficiency usually brings some additional unexpected useful results. Thus increased order and system bring increased speed and many other benefits. Rural free delivery of the mail has compelled the improvement of country roads, which in many sections has led to the consolidation of country schools and to many other wholly unforeseen improvements in rural life.

In higher fields the greatest thinkers tell us that as a rule their best ideas come to them, not when deliberately and consciously worked for, but in some unforeseen, sudden, and isolated way.

General Statement

The principle of efficiency illustrated in the above instances is that of the unexpected appearance or augmentation of useful results, owing to some unrealized development of the group or multiplicative category. This principle of efficiency we term dialectic. This source of efficiency has been frequently alluded to in a more or less casual way in the preceding pages, but we now propose to concentrate attention upon it for a time and to investigate it systematically. In one sense this is a study of by-products, but it is to be remembered that the by-product results here under consideration may be much more important than the result directly aimed at, and may develop over long stretches of time.

Dialectic in Different Fields of Material

Various dialectic processes may be grouped and classified according to the material given at the start; or, to express the matter in another way, according to the fields in which the different processes operate.

One of these fields is the world of *inorganic* matter. Thus a chemical element may unexpectedly assume a new allotropic or crystalline form, or even, as indicated above, be transmuted into another element. Veins of gold, or deposits of other useful material, are often found in strange positions or develop in unforeseen ways. Inorganic matter is transformed into living matter, and physical forces into biological and mental ones. Also forces seem at times to take on the properties of matter, and vice versa.

Living organisms evolve in surprising ways, an example being the development of the bird from the reptile. So at any time, under the hand of the plant breeder a strikingly new and useful species of vegetable life is liable to appear. Similarly a dialectic transformation may take place in a part of an organism, an example being the development of the skin into such diverse forms as the teeth and the crystalline lens of the eye, or that of hair into the horns of cattle.

In the *mental world* we have the various peculiar instincts or highly specialized psychic processes in animals, and in man the strange and unforeseen ways in which perceptions or words often pass into actions, conscious and voluntary acts into unconscious and involuntary ones, and the sudden and unexplained appearance of ideas often of the first order of importance.

Social processes are so complex that they are especially fitted as a field for the operation of dialectic. Instances vary from the capricious moods and actions of a mob to the extraordinary results which have sprung from the labors of statesmen like the founders of the American nation, of whom it is said that "they builded better than they knew". Probably no one of the American revolutionary statesmen dreamed that their work would result in the conversion of China into a republic within 150 years.

In the *ethical world* the principle in hand is illustrated by the attainment of happiness while simply seeking to do one's duty; and in religion, by the extraordinary phenomena which often attend conversion.

In the domain of *abstract thought,* we have the development of an idea of space into something that is spaceless, or of time into the timeless, or, as illustrated by the progress of modern scientific thought, of materialism into a form of idealism. In like manner idealism may give rise to materialism. The sudden development of one Efficient into a special form of some other Efficient has been frequently illustrated in the preceding pages. The general dialectic power of symbolism for instance has been discussed on pp. 177-178.

In concluding this part of the presentation it may be remarked that almost every important invention or idea has occurred to the discoverer in some small or special form and has afterward not only been generalized, but has found many unanticipated fields of usefulness. The man who first made a wheel could not have dreamed of its use as a part of a dynamo, nor could Plato or Aristotle have anticipated many of the uses of the terms idea and category invented by them respectively.

Classification according to Form

In characterizing dialectic processes according to their forms, and more particularly according to *spatial* and quantitative categories, many of the terms used for classification purposes in preceding chapters may be applied. Thus self-developing processes may be narrow, comprehensive, or universal; one, two, or three dimensional. A dialectic may work from within outward, or vice versa.

As to *number,* such a process may comprise a single event such as the discovery of the Fraunhofer lines, or it may be composed of many events occurring simultaneously or unfolding in succession.

An illustration of the latter species is the application of the spectroscope to one unforeseen field after another. Also a dialectic process may be dual, triple, or n-fold in the sense that it is composed of two, three, or n strands of action. An important case of two-stranded dialectic is a process which consists of both a movement forward to new results, and one backward to a deeper grasp of sources and origins. For example, the Roentgen rays have both had many unexpected useful applications in the field of medicine and also, by working in the opposite direction, have thrown new light on the atomic structure of matter. In a similar manner a new idea as to business management when introduced into an establishment, may lead on the one hand to a reorganization of the business internally, and on the other to an extension of external trade.

An important case of *temporal* dialectic is that in which surprising results follow from being in a given place first. Thus the Puritans by arriving in New England first dominated its civilization to an extraordinary degree, and like results have followed from the fact that their descendants first settled many parts of the western United States.

A process of *reciprocal* dialectic is one in which two elements act and react on each other, often in rhythmic fashion, thus producing many unexpected results. An example is the way in which abstract science and practical engineering have each developed the other. A still broader instance is the reciprocal influence of the new and old worlds on each other in the development of civilization.

A somewhat similar form of dialectic is that which may be termed *polar*. In this the existence of one strong form of efficiency tends to bring into being or to develop its opposite or complement. Thus in business the fact that one member of a firm is sanguine and daring may make another member cautious and conservative. Similarly a materialistic movement is sure to beget some form of idealism, often a very extravagant one. Polar dialectics may also be triple or manifold in form. Thus strong personalities often appear in groups of three, an example being the group of statesmen consisting of Webster, Clay, and Calhoun. A larger group is that composed of the great men prominent at the time of the American Revolution. Often one strong personality develops several others, each of which is strikingly different from the others in some respect. Instances are Napoleon and his marshals, and Lincoln and many of the great men associated with him.

An important special case of dynamic dialectic is that which may be termed *inductive* influence. An illustration is the expansive and unifying effect upon one's ideas which often accompanies a walk under the open sky after a day's work at the desk. More comprehensive instances are the results of living and working near great men, or surrounded by great books, in particular those containing efficiental material of high order of any kind.

Efficiental Forms

It goes without saying that dialectic processes may occur in groups and also in multiplicative group systems. As an example of the latter we have the extensive system of results which followed from the assumption of the erect position by man in his early development. Among these, in one direction, we have the differentiation between hands and feet, and the evolution of the opposable thumb. In another direction we have the grasp of a larger visual externality with its many efficiental consequences.

Processes of dialectic may be of *different orders,* either because they operate in fields of material which are essentially of different orders, or because they operate with different degrees of efficiency. An important case of dialectic of higher order is dialectic of dialectic. For instance when one form of evolution develops into a higher form, as brute evolution into that which is rational, we have the evolution of evolution, or evolution of the second order. Similarly we may have dialectic of dialectic.

With respect to *externality,* dialectic processes may be characterized by the amount of indirectness in them. In this connection we also have what may be termed matrix dialectic. This arises by virtue of the fact that if any process goes on in a given domain, some corresponding process must proceed outside of the domain. Thus in tying up a bundle, or forming a group of any kind, we unconsciously make the rest of the universe into a corresponding group.

An important instance of *complex* dialectic is the cumulative development of extraordinary results which has accompanied the succession of fundamental inventions such as articulate speech, or fire.

EFFICIENCY FUNCTIONS OF DIALECTIC

Many of the efficiency properties and results of dialectic processes have been indicated in connection with the preceding discus-

sions and illustrations. Others of them have been mentioned more or less incidentally in earlier chapters. It will be an advantage however at this point to make a more full and systematic statement of the matter.

Economies produced by Dialectic

In the first place, as a dialectic process often gives an extra result without extra effort it is the source of noteworthy economies. For instance when the lens was invented and used as a burning glass and it was afterward found that it could be used to remedy defective vision, the latter useful result was obtained practically without cost. This element of economy is greatly emphasized by the fact that the result obtained is frequently so different in quality from the anticipated and customary result, that the amount of effort required to hit upon it would have been enormous. Hence dialectic often means force and rationality acting independently of man, and serving as substitutes for human work of all orders.

A Source of Power

A dialectic process frequently advances not only without our aid, but seemingly in spite of insuperable difficulties or opposing forces. In this and in other ways it may be regarded as producing direct results, rather than as a mere economy.

The gain involved often in effect constitutes the creation of a new type or object. An instance is the evolution of the eye from the skin, or the bird from the reptile. Hellriegel when studying the nutrition of leguminous plants in the year 1886, made the unexpected discovery that the roots of leguminous plants fixate nitrogen. The result essentially amounted to the creation of new processes of efficiency in agriculture. In the field of abstract work, as in mathematical investigations, this creative function is even more apparent.

An important special form of this result is the conversion of the harmful into the useful, or of defeat into victory. The Colorado potato beetle at first greatly diminished the potato crop in certain sections of the country, but in the end, by causing agriculturists to study improved methods of potato culture, led to crops two or three times as large as those obtained before the appearance of the beetle. Similarly the San José scale has proved a blessing in disguise to the fruit grower.

Efficiental Results

Dialectic processes also produce efficiency results by giving rise to new and unexpected diversities and uniformities, and consequently to the fruits of these instruments. As to diversities, the word dialectic means literally a laying apart, that is, the development of new forms of diversity. A comprehensive illustration is what is called the divergence of species in the process of evolution. As examples of uniformity results we have the unexpected likenesses in efficiental essence which often appear in radically different domains as in language, mathematics, and different branches of science. Another kind of a result is that obtained when a few glimpses frame themselves together in a continuous whole. The multiplicative principle often shows an unexpected power to check and control its own exuberance and thus to produce uniformity in the guise of relative unity and system. This result often means not only a corrective of extravagance and error, but often a production of the efficiency values of so-called reality, or the discovery, or creation out of the relative and incidental, of an efficiency absolute (that is, of that which has all of the efficiency properties of the so-called absolute). A specific instance is that in which the observation and practice of the relatively useful leads to a recognition of an absolute in morals (the categorical imperative).

When a systematic (or multiplicatively grouped) arrangement of objects is made in one way, the result is often found to contain other unexpected cross systems of multiplicative groups. Thus objects when arranged rectangularly like the squares in a checkerboard or hills in a cornfield may be regarded as grouped systematically in a vast number of ways beside the original rectangular one. So a problem in mathematics (or elsewhere) that can be solved in one way may usually be solved in many other ways. In fact in any multiplicatively grouped system any element may be taken as the primal apex, and the whole multiplicatively grouped with respect to this apex.

Also in practical affairs, where many difficulties are present, effective system and order are often possible only because we are able to leave many matters of secondary importance to take care of themselves. In this case dialectic adjustment takes the form primarily of cancellation and elimination.

In a process of evolution the elimination of types between certain standard forms may give rise to a multiplicative ordering of ma-

terials. This occurs also when the materials and processes in a business are standardized. An example is the differentiation of seagoing vessels into certain well defined types. The same development is evident in automobiles and railroad trains.

A dialectic process often produces a multiplicative series in a more aggressive way as by the development of a higher type than any given at the start, and afterward of other progressively higher types. The evolution of plant and animal forms of life in the past history of the world is an example of this.

A process of dialectic may give rise to various other Efficients beside those which have been mentioned. Thus a new and important externality may be obtained either in some unforeseen direct way, or by a working back to causes or origins, or by a generation of or linking up with parallel procedures. Similarly a dialectic process often begets an important rhythm, as that of alternating steps of analysis and synthesis. A dialectic also may unexpectedly pass into or produce another and often more efficient dialectic. An extreme case is the dialectic of dialectic mentioned on p. 253.

Negative and Complex Values

Dialectic also has certain negative values or uses. Its corrective power mentioned above is one of these. Others are its power to reveal weakness and limitation. A still more important property in this connection is illustrated by the fact that certain evils, perhaps all evils in the long run, are self-destructive.

The complex values of dialectic assume many forms. One of the most important of these is its function of making clear the best ways in which to use the Efficients in combination. These complexes of its uses and results also appear in striking form in many places in human progress. It is to be noted also that these complex functions become more prominent as the world advances. For the more the group and multiplicative principles are recognized and employed, the more numerous and important will be their unexpected manifestations and fruitages.

Limitations and Drawbacks

It is not to be overlooked that dialectic as an instrument of efficiency has certain drawbacks and limitations.

Some of these are inherent in its nature. Thus it is often slow

and acts in a blind and costly way. Usually it is not flexible and readily manageable.

Again it frequently acts in a destructive instead of constructive way. Thus a mistake or evil may multiply itself in an unexpected manner. A scratch may produce death. Even that which is good in itself, owing to the presence of other unsuspected evils or limitations, may become a source of harm. An example is the injurious results which have followed from the introduction of rabbits into Australia. In business and in life in general, one must be constantly on the lookout for some unexpected accident or other extraordinary unfavorable development, and if possible have abundant means ready to meet such emergencies.

EFFICIENCY ANALYSIS

Result of the Multiplicative Principle

The primary cause which gives rise to dialectic as an efficiency instrument is the unrealized prodigality of results in a multiplicative process. For example 2^{100} when multiplied out gives a number so large that counting at the rate of 100 per minute it would take a billion billion years to count all the units included in the number. In other words the ramifications of the multiplicative principle are so broad and comprehensive, that frequently no one at the outset of a given application of it can grasp them all in detail.

This holds often even with respect to a single group. The extent of the group may be so great as to be only in part realized, or the developments of other groups may mask or hide the given group in part or entirely, for the time being. An understanding of this makes it clear why a mere change of position or of point of view often produces such vast consequences. These unrealized clusterings and hangings together of objects, may take special group forms, as those of imperfectly grasped reuse, units and multipliers, externality, etc. Thus George Westinghouse when he invented high pressure gas mains did not realize that he had discovered a process whose essence he could use later in transmitting electricity at high pressure. When a certain differential equation was first formed and solved, its author did not anticipate that this solution when applied to the Atlantic cable would result in making this cable a practical success. Owing to the common efficiental essence in a number of seemingly different acts, or in a number of departments, in learning to do one thing we often unconsciously learn to

do a vast number of other things. The often unnoticed uniformity in the size of algebraic symbols is the source of many of their dialectic properties . The outcome of a dialectic process often takes a rhythmic and in part paradoxical form. When typesetting machinery was invented it at first diminished the number of compositors employed, but later had the unexpected result of increasing this number. If the matter had been viewed in a broad way this result might have been anticipated at the start. An economical use of one's powers often results in dialectic results owing to the fact that part of a person's energy is left free to be employed in any high field which he may choose.

If the manifoldnesses involved in a single group are difficult to realize, much more difficult is it to grasp all the meanings and uses involved in a whole system of groups. This was illustrated for rectangular arrays of elements on p. 255. And vastly more difficult will it be to comprehend all the ramifications and intersections of several systems of groups. Thus modern bacteriology is due to a number of factors each unfolding in multiplicative style. The first is Pasteur's study of ferments, including a use of diversity in various forms, of various externalities, of directive, and other Efficients. The second was Perkin's discovery of aniline dyes; the third was the improvements in the microscope itself, due to a number of causes such as the invention of Jena glass. Other more general factors might be specified.

Element of Spontaneous Force

Another primal element of efficiency in dialectic is a certain apparently spontaneous force or energy which characterizes such processes. This force may be more or less concrete like many of those involved in ordinary physical growth, or it may be abstract and efficiental in nature. As an example of the latter force or energy, we have the fact often noted that when a group has been once formed it tends to increase in size, the reason being that laws of increased efficiency in a measure compel such a result. The force in a dialectic process may also be due in part to the action of unconscious subjective powers in a person or persons associated with the process. These various forces may also have a highly multiplicative structure which also is imperfectly realized. Hence, for instance, the surprising results which often follow from merely making a start in any enterprise, as for example, the spontaneous

flow of words and ideas which follows the mere taking of one's pen in hand. Some kind of difficulty or obstacle often stimulates this subjective multiplicative apparatus into action. This perhaps is the cause of the fact that a haze or gauze when thrown on a beautiful object, often increases the charm of the object. In its most general and aggressive form this subjective factor in dialectic is a generalization of Kant's doctrine of subjective categories.

The above various factors are often combined in a process of dialectic. Thus when a business like that, for instance, of the manufacture of glass is once established in a given place, it will often grow surprisingly, even though the place has no special adaptation to the business.

In modern life the number of multiplicative processes simultaneously at work in the social, educational, and business worlds, and in private individual lives, is so great that surprising results are constantly appearing.

Element of Limitation

In order fully to account for processes of dialectic, an element of limitation must be taken into the account. This element appears in two principal forms, viz.: our inability to grasp all the ramifications of a multiplicative process and in the obstructions or difficulties which make a process more or less necessary, and hold it in check throughout its progress.

Unique Element

In dialectic, as in each other Efficient, there is also present a unique and characteristic source of power. After we have carried as far as we can our analysis of the instrument in hand, an irreducible something is left over.

It may be well also to call attention to the respects in which dialectic differs from efficiency processes in general. These are the element of unexpectedness in its results, the obscurity of many of its elements, the native force and energy which carry it forward, and its rationality apart from human interference.

Above all it is to be borne in mind that dialectic is not mere vague haphazard change in which some useful spot is occasionally observed, but that it is a multiplicative process imperfectly apprehended and hence, by careful study and some management, capable of producing increasingly important results.

Definitions

As an approximate definition we may take the following:

Dialectic is an efficiency process in which results come of themselves, in whole or part, and are therefore altogether, or in some respect, unanticipated.

It may be well also to define a few similar terms.

Development is a vague and general species of dialectic. It may produce results which are either higher or lower than the data, or which are mixed in character.

Progress means a somewhat more definite series of changes than development and leads to plainly useful results.

Evolution is a dialectic process in which limitation is prominent, the steps are small, the process slow, but the change is toward what is higher.

METHOD OF APPLICATION

Multiplicative Principle an Aid in Various Ways

A realization of the multiplicative nature of dialectic is an aid in many ways toward the full utilization of this instrument of efficiency. For instance it facilitates a grasp of the great number of results which may be obtained from this source, the extreme forms of some of them, and the explosive suddenness with which results may appear.

It also facilitates the arranging different forms of dialectic in an order of materials scale and thus aids in applying to a given domain of material that form of dialectic which is best fitted to this field. Thus in certain difficult cases only the crudest form of dialectic can be used. Instances are the use of mere motion, as in shaking a fluid or agitating a situation, and waiting to see what results will follow; or the use of the mere juxtaposition or separation of the two objects. An important general case is the Baconian method of grouping facts according to their likenesses or differences, and waiting for general laws to appear among the spatial groups thus formed. Bacon himself would be astounded at the results which have already been obtained by his method. Dialectic by induction (see p. 253) may also be useful as an initial method in a difficult case.

Uses of Crude Dialectic

A less crude form of dialectic is that of making a start toward a given end, and using light and power as these appear. In its

more advanced form this process consists in the rhythmic use of successive externalities and re-multiplicative groupings, and the utilization of the dialectic results which appear at each stage of the process.

In a difficult case it is often well to avail oneself of dialectic in the double form often called compromise. A compromise in effect frequently consists in allowing two or more methods or processes to develop till one of them becomes clearly more efficient than the other, or some new method more effective than either appears on the scene.

Preference to Dialectic of High Order

The aim should be in all cases to make the dialectic used of as high an order as possible. Beside the dialectic processes of high order of efficiency described on p. 253, special mention may here be made of that process which consists essentially of perfecting the prime groups of a given domain or piece of work to the utmost, and leaving the rest of the process largely to dialectic development. Another important species is that which emphasizes the self corrective power in multiplicative processes, and thus makes it possible to use aggressive methods in an imperfectly known field. Special emphasis should also be laid on the ideal form described on p. 263.

Standard Varieties

With dialectic as with the other Efficients it is important when a good unit form has been found to reuse that species to the utmost; or, to express the matter in another form, to give it as large a multiplier as possible. An illustration is the generalization of a principle or method by using the dialectic habit which leads us, when we have thought of the numbers one, two, three, also to think of any number however large. Thus having observed that antitoxin aids in the cure of diphtheria in a few cases, the query arises in our mind whether it may not be an aid in all cases. This process may often be repeated to advantage in many different fields. A similar use may be made of the letters of the alphabet taken in order. Other standard cases are the dialectic which come from the use of the comparative method (that is, manifold homogeneous externalities) in scientific and historical investigations, or from the method of transformations. An extreme form of the latter is that

of expressing any given process in terms of any given Efficient, or, expressing each Efficient in terms of the other. Having mastered a few standard forms of dialectic it is often an advantage to arrange these as a multiplicative group system, as an aid in using them either directly or by variant reuse.

Relation to Other Efficients

Added efficiency in the utilization of dialectic is often obtained by studying the relations of this agent to each of the Efficients. For instance, it has been observed that, as the result of dialectic, a sudden, explosive disaster or opportunity is liable to occur at any time. Hence the importance of having on hand some margin, reserve, or other externality is apparent. This externality should, if possible, be of such a kind that it can be utilized in other ways in case the accident allowed for does not occur. Thus it is well to allow a margin of time in starting for a train, and, in case no unforeseen contingency arises to consume this margin, to spend the unoccupied time in reading. The same principle applies to vastly wider and higher fields.

It is especially important to know where to use directive and dialectic in relation to each other. Often, as already indicated, efficiency is obtained by using the two instruments in alternative succession, or in some other mottled way. Frequently the directive part of an operation should consist of the mere selection and launching of the proper dialectic process.

In general, after any efficiental process has been worked out consciously, an added element of power and efficiency can often be conferred upon it by developing it till it acts in more or less dialectic fashion.

Special conventional forms of dialectic, such as growth, evolution, and development, are utilized most effectively when viewed explicitly as species of dialectic, that is, as essentially multiplicative in nature.

Ideal Species

From the point of view under consideration in this book the ideal form of dialectic is that which proceeds from efficiental material, and acts in efficiental ways to produce efficiental results. To look at the matter in more detail, we can conceive of the Efficients as so per-

fected that they will of themselves attack every point of need or opportunity with manifold power, speed, and adaptation. In other words the process would possess the highest mechanistic and rationalistic properties in combination. The fact that processes of dialectic in actual operation fall so far short of this ideal, shows that some principle of a limitation is at work, and it is this principle which we shall next investigate.

Recapitulation of Chapter

As a summary of this chapter we may say that when working for a given end, a person is often surprised by the appearance of some valuable result which had not been anticipated. Often this unforeseen fruitage exceeds in value the end consciously aimed at. The principle of efficiency here involved is termed dialectic. It acts in all fields, from those which are material and inorganic, to those which are most abstract and ideal. The species of dialectic may also be usefully classified according to the various forms in which they occur. Special mention may be made of the kinds termed reciprocal, polar, and inductive.

Since the process of dialectic acts in hidden, unforseen ways, the results obtained from it may often be regarded as obtained absolutely without cost. Hence dialectic is a peculiarly effective source of economy and new power. Frequently a process of this kind gives rise to one or more of the Efficients with their corresponding fruitages. Dialectic also in special ways and to a peculiar degree supplies an element of charm and even of romance to kinds of work which might otherwise be limited in interest.

Since dialectic is primarily due to the multiplicative principle acting in one or more unobserved ways, in order to be able to utilize this source of efficiency to the utmost, a person must obtain some grasp of the great profusion of results which may arise from the action of the multiplicative principle in even one form, and hence of the vastly greater number of results arising when two or more species of it act in combination. This will make plain that no one can anticipate all the forms which dialectic may assume. In cases of great difficulty, after initiating elementary efficiency processes, much may be left to dialectic self-development aided by occasional efficiental regulation. In general, also, it is highly important to study and utilize the relations of dialectic to the other Efficients.

EXERCISE 16

1. Give an example of each of the principal forms of dialectic.

2. Reinforced concrete was first invented in modern times by a French gardener in making reservoirs for water. Give some unexpected results.

3. Name some of the by-products now obtained in the manufacture of steel.

4. In the refinement of oil.

5. State the dialectic of uses of a telephone directory.

Give an example where the result of a dialectic process is

6. A multiplicative group system.

7. A uniformity. 8. A diversity 9. An externality

10. Name some of the unexpected evil results which have followed the invention of printing.

11. Give an instance of your own of loss produced by dialectic.

12. Give an example of a dialectic process resulting from a positional relation.

13. Name some of the dialectic results which have followed from Napoleon's wars upon Germany.

14. State some of the unforeseen advantages which may come to a business man from keeping a considerable reserve of cash.

15. To any person from having a reserve of physical vigor.

16. Give an example where the use of system or organization has brought unexpected efficiency results.

17. As a blacksmith hammers he often sings and as he sings he hammers with greater energy and precision. Discuss the dialectic involved.

18. Why is it that if one member of a family expends his energies largely in processes of thought, some other member is likely to become a person of action.

19. Discuss the dialectic results of a notebook kept by a business man.

20. By a literary man.

21. Methods of wrong doing are often self destructive in unexpected ways. Give an example of this species of negative dialectic.

22. If a primitive man in building a camp fire happened to place a lump of iron ore among the stones about the fire and thus discovered the metal iron, what principle of efficiency was involved?

23. Discuss the rhythmic dialectic relations of fire and iron in their historical development.

24. Mere formal imitation of a process often ends in an intelligent grasp of the same. What Efficient is here involved?

25. Give an instance where it is advantageous to use the directive and dialectic methods in alternate succession.

26. Give an example of a difficult case where only crude dialectic can be employed.

CHAPTER XV

LIMITATION

Illustrations

In the eighteenth century the invention in England of machine and factory methods of weaving cloth and manufacturing other articles caused a considerable part of the rural population to move to the towns and cities for the sake of the higher wages there obtainable. The result was a scarcity of agricultural laborers. This compelled the landowners to study means of meeting the deficiency, the outcome being certain improved methods of agriculture, the most important of which, perhaps was the systematic use of the horse with adapted implements. In this way not only was the deficiency in hand labor made up but additional results were obtained which have ever since given England the leadership in agricultural efficiency.

No living person can run 100 yards in less than 9⅗ seconds, and probably this record will never be excelled by more than a fraction of a second. A knowledge of facts like this may be made the source of important efficiencies. Thus in the loading and firing of a 12-inch gun in the United States navy, by careful study it has been found that the shortest times in which the various operations involved can be performed are in seconds, as follows: ⅕, 3⅖, 4⅕, ⅗, 2⅗. By organizing the work of gun practice in accordance with these facts the firing efficiency of the fleet has been increased several hundred per cent. The heat obtained by burning a pound of coal is limited to a certain amount. In whatever ways physical energy may be transformed, it can never be increased or decreased. These facts have enabled engineers to determine the amount of waste energies in steam engines and, by diminishing this waste, to increase the efficiency of such engines threefold.

General Statement

The above instances illustrate the fact that the efficiency of processes is subject to various limitations. Some of these limitations are insuperable in the sense that they cannot be directly overcome.

But a knowledge even of these insuperable restrictions is often an aid to efficiency in certain ways, as in the negative way of preventing a waste of energy in striving to overcome these deficiencies by direct methods. In other cases by devising means by which to overcome a given limitation or part of a limitation, we not only succeed in diminishing or removing the deficiency, but may obtain additional results or increases in efficiency. In other words one of the most important kinds of efficiency is the negative efficiency which consists of the removal of waste, loss, error, and limitations of certain kinds. This negative efficiency is often accompanied by a dialectic of positive efficiency. Casual references to these facts have been made in the preceding chapters. We shall now isolate this principle of limitation in a measure, and concentrate attention upon it, in order as far as possible to avoid the losses and increase the efficiencies connected with it.

CLASSIFICATION

Limitations in Various Objects

Considered with reference to the category of efficiency every object is limited in many ways. For example every given kind of matter is limited as to the amount of it which is accessible to man. Two bodies of matter (as matter is ordinarily understood) cannot occupy the same space at the same time. Matter cannot be created or destroyed. Each particular kind of matter also has its characteristic limitations with respect to density, combustibility, fusibility, and to magnetic, electric, luminous, and other physical properties. Hence each kind of matter is restricted in efficiency power, and in its efficiental functions as in the kinds of reuse, groupability, and directive it is capable of, or can aid in.

In like manner the *force of energy* of any kind that is accessible to us is limited in amount. Transformations of energy are hemmed in in various ways. For instance, when the energy in coal is converted into motion by the use of the steam engine, nine-tenths of this energy is lost. Innumerable attempts have been made to find a way of converting coal directly into electricity, but all have been failures. A more general limitation in this respect is what is termed dissipation of energy, or, to use a more popular expression, the impossibility of perpetual motion. The efficiency of many processes is greatly restricted by the fact that only a certain amount

of energy can be stored in a given bulk of fuel, or in the core of an electro-magnet, or in a storage battery. Also physical energy, like matter, can neither be created or destroyed.

Living organisms are subject both to the limitations of the matter and energy composing them, and to other limitations peculiar to themselves as living objects. Thus protoplasm is limited as to the speed with which it can assimilate sunshine and food, the amount of each which it can absorb, and the ways in which it can transform what it absorbs. Hence organisms are limited as to the size which they can attain and their various efficiency properties. These facts are recognized in such proverbs as "You can't get blood out of a turnip", and the query "Can a leopard change his spots?" The limitations in the efficiency properties of vegetation are illustrated by the fact that after the saccharine matter in the sugar beet reaches about 18 per cent of the entire beet, further increase is apparently impossible owing to the fact that the ordinary vegetative processes in the beet break down when more sugar is present. Man, the most efficient of organisms, is limited in the speed with which he can walk, run, or perform any act, in his ability to replace a lost member, to increase his bodily height or his length of life. In fact every specific quality of living organisms is hemmed in by, and in one aspect means restrictions of various sorts.

Similarly *psychic powers* are limited in man and to a much greater extent in other organisms. Thus in the human mind we find many limitations with respect to perception, memory, imagination, reason, volition, and emotion. For instance the number of distinct objects which one can hold in his field of vision at one time is limited to five or six. The limitations in the reasoning and imaginative powers of the ordinary man are made evident by comparing him with the genius. Perhaps the most important psychic limitations for our purposes are man's circumscribed powers of conceiving and using groups and other fundamental efficiency instruments. Probably most other psychic limitations may be in large measure reduced to or expressed in terms of these more primary limitations.

The kinds of limitations which have been mentioned thus far are necessarily present in some form in the social, political, religious, industrial, educational, artistic, and similar worlds. Each of these fields also has its specific limitations. For example each particular form of government has its own inherent defects. The same re-

mark applies to each individual kind of music, or musical instrument. The time required for a given educational process cannot be diminished beyond a certain point.

Limitations in Abstract Fields

Important cases of inhibition are also found in more abstract fields. Thus *space* as we ordinarily understand the term, is restricted in the number of its dimensions and in many of its properties. For example only one straight line can pass through two given points, and a triangle cannot have the sum of two of its sides less than the third side. The circle cannot be squared by the use of the straight edge and compasses.

Similarly *numbers* have their characteristic limitations. Prime numbers cannot be factored, and two plus two cannot equal five.

Time, in the conventional meaning of the term, is limited to a single dimension, and also with respect to certain forms of control. Thus we cannot recall the past so as to live it over again and change its content.

Closely related to this are certain restrictions of *order.* To quote a familiar illustration, we cannot unfry an egg. So the inverse of a process is often more difficult than the direct form. An example is the process of the division of numbers as compared with that of multiplication. In some cases the inverse is easier than the direct process; for instance, a house can be burned down more readily than it can be built.

Limitations in *speed* are often of primary importance in an efficiency process.

Every species of *quality* has its characteristic limitations or circumscriptions. A soil that is fitted to raise one kind of crop is by that very fact unfitted to raise certain other kinds of produce. Gases are unfitted to perform certain of the functions of solids and vice versa. In fact the expression "it has the defects of its qualities" is a common proverb. A man who is especially qualified for one kind of work is often by that very fact unfitted for another kind.

Similarly each of the primary instruments of efficiency has its own special limitations or defects, many of which correspond to characteristic useful properties. These have been discussed in more or less detail in the preceding chapters.

Forms of Limitation

Thus far we have grouped limitations according to the materials or objects in which they occur. It is also useful to classify them according to certain characteristic forms.

Thus, with respect to the categories of space and quantity, restrictions may be large or small; may be scattered or compact; and may occupy various positions, the most important being an apexal or commanding one.

As to the category of force, the most important distinction is that between passive and active limitations. Among the former are those which take the forms of resistance or inertia. Among the latter are those which manifest themselves aggressively and which often assume such individual forms as disease, wickedness, insanity, or, in general, what is termed evil.

Aggressive limitations are often closely associated either with some form of actual personality as with individuals who are ignorant or wicked, or with personality used in a more or less symbolic sense, as when we speak of imps, goblins, devils, the father of lies, the beast, or the evil one.

Also with respect to personality limitations may be classified as subjective or objective.

It may also be well to note at this point that limitations often occur in a reciprocal form. For example in certain states of mind feeling and perception (or reasoning) restrict each other. More generally, specificality and generality (i.e., intension and extension) have reciprocally inhibitive powers.

Grouped Limitations

Limitations also occur in certain combined or grouped forms. For example several limitations combine to prevent the perfecting of the telescope lens, among these being chromatic aberration, spherical aberration, variations in the density and the chemical properties of a piece of glass, and instability in temperature. More familiar instances are the groups of limitation found in any one person, or form of government.

A given limitation is often the resultant of a number of component or contributing restrictions. Thus the limitation in the rapidity with which a man can run a given distance is determined by the limitations in his height, in heart, muscle, and nerve action, and in his power to store and to give out energy rapidly.

Similarly limitations often occur as multiplicatively grouped aggregates. For examples the limitation in the intensity of sunlight gives rise to a whole system of other limitations, such as those in the size of crops, in the closeness of human associations, and the rate of human progress. Each of these in turn produces its own distinct set of restrictions. Or viewed by the method of analysis the limitation in the number of bushels of wheat which can be grown on one acre of land is due to the diffused nature of sunshine, the limited powers of matter, force, and protoplasm, and defects in human knowledge and directive action. Again the limitation in the power of sunshine is the resultant of a number of more elemental limitations such as those in the nearness of the sun, in the temperature of the sun's surface, and in the angle at which the sun's rays fall on the earth. Similarly each of these restrictions could be resolved into a group of constituents. A still larger multiplicative group system of limitations could be obtained by analyzing the slowness of human progress into its causes.

Scales of Limitation

As a result it follows that limitations may often be arranged in an order of materials scale. A limitation may be of a high order owing to the fact that it occupies an apexal position in a multiplicative group system of limitations, an example being a limitation in human knowledge; or owing to the fact that it is a limitation in some material or object of a high order. Thus a serious lack of some kind in the president of the United States may check the development of the whole nation.

Relative and Absolute Limitations

The most important distinction in this connection is that between relative and absolute limitations. Some restrictions seem to be ultimate and beyond human control, while others are only superficial, contingent, or apparent. Examples of the absolute species are human inability to raise the dead to life, or the logical limitation in the number of sides of a triangle. Examples of the relative are limitations in human efficiency due to certain preventable diseases, as yellow fever or malaria.

An important form of relative limitation is that which exists only with reference to certain other limitations. Thus in a factory it may be possible to run a given machine with much greater rapidity

than is actually used, but it may not pay to do so owing to the limitations in the speed of other processes in the establishment. Gold exists in vast quantities in sea water, but it does not pay to extract it at present owing to the greater efficiency of other methods of obtaining this metal, and also, perhaps, to the limitations in our knowledge of the best way to extract this gold. The interior heat of the earth cannot at present be profitably utilized by man owing to the fact that tunnels to reach this heat, and make it practically available would cost at least $25,000,000. But at some later stage in the world's history it may be profitable to make even this expenditure.

In allowing for limitations in connection with efficiency processes, it is important to remember that restrictions of various kinds may be compounded and complexed in many intricate ways. Among the most important cases of complex limitation are those which are cumulative in form. As an illustration we have the limitations which accumulate progressively in the successive fields of matter, force, life, mind, society, and education.

EFFICIENCY RELATIONS OF LIMITATION

It will be an aid to efficiency in important ways to observe certain facts with reference to the nature of limitation.

Limitations are Multiplicative

In the first place it is important to realize that various limitations, in the vast majority of cases, are not scattered, discordant, or accidental in their relations, but on the contrary are connected in multiplicative systems. For instance there exist certain vital and primary limitations, such as ignorance, each of which includes many secondary and derived cases.

It follows that limitation is not primarily a lumpish, blind, and inert something, but that it is normally active and aggressive, and that different forms of it often combine to act in sudden and overwhelming ways. An example is the loss of the steamship *Titanic* in the year 1912.

Sources of Efficiency in Limitations

In naming and treating a given case as primarily a case of limitation, a certain human element is often present. This often proves a serious hindrance in the efficient treatment of such cases, though in

some it is a help. The thorn on the rose having pricked us, we regard the thorn as a drawback or limitation to the beauty of the rose, without stopping to consider the utility of the thorn as a protection to the rose under certain circumstances, and its consequent value in the general scheme of things. So in general limitation from the broader point of view is often entirely, always perhaps in part, mere diversity, our unpleasant feeling concerning it coming largely from narrow personal associations. In certain other aspects limitation is that which helps form units and even the Efficients themselves. Friction and inertia are often disagreeable. but they enable us to walk, to weave materials together as cloth, and to make many other groups and multiplicative groups.

In connection with the fact that limitation is in some respects diversity viewed in a certain personal way, the nearness of these two sides of the matter should be noted. From this it follows that wherever a so-called limitation, or even evil appears, some utility must be very close at hand. The defects of qualities and qualities themselves are in immediate contact. Often also the greater the limitation the greater the potential efficiency associated with it. The fact that Faraday lacked an extensive knowledge of technical mathematics left his mind free to take certain large and fundamental steps and, in effect, invent a new theory of the matter and ether. and discover among other things the principle of the dynamo. Graham Bell knew just enough of electricity to enable him to take the bold step which an expert electrician would have hesitated to take and which resulted in the making of the telephone.

EFFICIENCY METHODS OF TREATING LIMITATION

Externality the Best General Method

It has already been stated that the best general method of treating cases of limitation is by the use of externality; that is, by taking a larger view of things, so as readily to perceive such efficiency aspects of co-called limitations as diversity, units and their multipliers, groups, or more generally the fundamentally multiplicative nature of limitation and its relation to the multiplicative scheme of things. However, just as there is a whole scale of degrees of limitation, so there is a scale of degrees and kinds of externality which to apply, with a variety of special forms of efficiency as a result.

Treatment of Difficult Cases

In the first case, some forms of limitation are so fundamental and stubborn that only remote and indirect externalities can be applied to them as a means of remedying and overcoming them, and perhaps obtaining other more positive efficiency results. A ton of coal once burned cannot be burned again. But if the energy obtained by burning the coal be stored in some permanently useful shape, as in a manufactured machine, or as ideas worked out while the thinker is sustained by heat given off during the combustion, the energy in the coal may be reused and the coal in a manner be said to be reconsumed. Life may be prolonged indefinitely in the sense that the mind can forecast and in a measure anticipate many of the experiences of the future. Often the externality applicable to the class of restrictions under consideration takes the form of mere patience.

A knowledge of limitations also has important efficiency uses even when we do not attempt to overcome the given restrictions. If the given prohibitions are fundamental and insuperable along certain channels, we are saved the losses of various kinds which must follow an attempt to overcome limitations in such ways. Thus a knowledge that the circle cannot be squared by the use of the ruler and compasses, saves us the loss of time and energy involved in attempting thus to square it. In this case the externality involved is that of a large view of the aims and ends of action and of the means of attaining these aims, and the use of a totally new channel in reaching a large measure of positive non-equivalence.

Some Limitations a Direct Source of Efficiency

The limitations surrounding an object or domain are often better known or easier to grasp than the object itself, and hence form a convenient externality by which to deal with the object or subject in hand. Thus it is often easier to understand and deal with a man by observing what he is not or by his mistakes than by his direct qualities. Of a similar efficient nature are the proofs by exclusion as used in logic or mathematics. Similarly it is a well known fact that in the human body a set of nerves of a given kind is held in check by another set, and that often the best way to stimulate a man is temporarily to paralyze the inhibiting nerves to some degree. Or in general the most advantageous way of making a person or domain efficient may be merely to sever a set of restraining bonds. It is

evident that this use of negative externality may be built up in multiplicative group fashion.

So in general all increases of efficiency, in one aspect, are obtained by a removal of waste, ignorance, or other limitation; that is, all positive efficiency has a negative aspect. This negative aspect is often the more accessible and manageable of the two.

A Cause of Dialectic

Of somewhat similar nature are those instances where a lack of one desirable quality leads to the development of other and often more desirable ones. An illustration is the case of a woman who, owing to a deficiency in physical beauty, is led to the development in high degree of mental powers and a spirit of service. Often the wide and deep externality thus called into play leads to an important dialectic of results. For example the fact that our organs of speech can make only a limited number of distinct sounds has compelled men to reuse these organs in various ways, as in various order groups, thus producing the group system of articulate speech. This smallness of the number of primal units of speech has led to other efficiencies such as the invention of the phonetic alphabet, the typewriting machine, the Morse and other like alphabets with their many applications. So the limitation in the amount of available gold in the world has resulted in the wide use of symbols such as paper money and checks to represent this gold and as a consequence to great economics and efficiencies in commercial processes. Similarly a difficulty or defect often calls into action latent efficiency powers either in an individual or a social organization. An example is the effect produced in Germany by the conquests of Napoleon.

We next consider cases of limitation which may be remedied in some indirect way and often thus, from one point of view, made the source of higher efficiencies. An example is limitation in human vision as overcome by the lens. It is to be observed that the first use of the lens in the telescope to overcome deficiencies due to distance has brought with it a dialectic of results illustrated by the use of the lens in the microscope, and of the latter in the investigation of malaria, tuberculosis, and like diseases.

Applications of Externality

The externalities applied in such cases frequently takes many important special forms. Often a wide survey of material reveals

certain spots where a difficulty disappears, and light or power which can be reused in important ways becomes prominent. Thus both Darwin and Wallace by travelling and observing widely chanced upon facts which clearly indicated the variation of species, and which when fully interpreted gave the theory of evolution. Similarly a wide grouping of facts by the Baconian method, although applied first to the removal of difficulties, has often ultimately led to scientific discoveries and inventions of direct value.

Another way in which it is often advantageous to use externality is to allow an evil or limitation to develop its own consequences, and thus bring its own cure in whole or part. This process is often efficient by virtue of the fact that it arouses the attention, and utilizes the hitherto inert powers of a large number of more or less unoccupied persons. As has already been pointed out, it is often a source of efficiency to allow this method to take the form of alternate dialectic and directive.

Personifying evils and limitations (that is, annexing the externality of personality to them), as when we speak of the works of imps, goblins, and devils, often forms an effective method of summing up and making vivid the tendency and results of a given evil, and of arousing powers which will overcome it. This method however has compensating drawbacks.

Overcoming Limitations in Groups

A much more important principle in this connection is that of taking such a comprehensive view of matters as to be able to remove limitations and evils in groups, as by overcoming some deep-seated limitation which is the source of many more superficial ones. An example is the removal of many forms of disease, much poverty, ignorance, vice, and unhappiness in a city by improving its sanitary condition and by the education of its children.

Often, in overcoming limitations in groups, many forms of externality are used together which, beside overcoming the given limitations, bring added positive efficiency results. The Krupp firm in making cannon have to overcome limitations in the properties of iron, in the physical powers of workmen, and in human knowledge. In doing this work they employ coal and various physical forces, manifold machinery, and the largest physical and chemical laboratory in the world. But by so doing they have not only achieved

their immediate purpose, but have also discovered many mechaincal principles and scientific facts widely useful in the arts and trades.

Adapted Treatment of Limitation

Some species of limitation may be regarded as relative, in the sense that it pays under some circumstances to submit to them for the time being, and under others to try to overcome them; that is, at some times to apply only a remote and indirect externality, and at others a direct one. For instance in some parts of the world and in some ways it is a source of efficiency to overcome the limitations of peat as a fuel, and in others it does not pay to use peat in this way. The English system of weights and measures in itself has great defects as compared with the metric system. But it has the important property that its very deficiencies make it homogeneous with the great majority of efficiency methods in use the world over. Hence its mastery furnishes a convenient unit to reuse in connection with other processes. The same remark applies to the systems of spelling and pronunciation which characterize the English language. Perhaps the great practical power of the English and American peoples may be due, in a measure, to their training in these and similar disciplines. At the same time the English speaking man of science in his private work does not hesitate to set aside the English system of weights and measures for the more convenient metric system.

Similarly the wise man recognizes and submits to his personal limitations in selecting a field of life work. At the same time, under many special circumstances he tries to overcome these limitations as far as possible. A wider illustration of this principle is the wholesome moderation which was characteristic of the Greeks in all their process of life, and by which they were able to attain such vast and permanent results. So, in general, sacrifice and restraint, in some respects, are necessary to broad and vital efficiency processes of any sort.

Value of Exact Knowledge

At this point becomes especially evident the value of the exact knowledge, quantitative as far as possible, of the various species and forms of limitations. Vast amounts of such knowledge are found in various technical treatises on the different sciences and arts. Each

man in his particular field can often determine other more special and detailed limitation facts as to his work. Thus Mr. F. W. Taylor in his investigations which led to the system of efficiency which he calls scientific management, has found that work in certain shops is most efficient when each planner or overseer has under him not more than three men on the average, and that a shovel load for a common laborer should not exceed 21 pounds. In general if the boundary facts are known in any line or department of human knowledge or action, it is possible to construct with confidence the largest available or practical multiplicative group or order of materials system in that domain, with a view to varying from this by marginal reuse from time to time according to circumstances.

In this connection it is also well to note that in every situation a residuum of work by somebody is always necessary, and that it often does not pay to try to remove all of removable limitation, but that better results are obtained by working partly in other lines. Similarly instead of working with great rapidity in a single line, it often pays better to work at a more moderate rate in several directions.

From the preceding discussion it is evident that, in discriminating between cases of limitation and also for general purposes, it is an important source of efficiency to keep in mind the multiplicative nature of limitation. This facilitates an appreciation of the many forms which limitation may assume, the extreme nature of some of these, the grasp of them as a tabulated whole, and the marginal reuse and more general group methods of treating them.

Two Opposed Systems

Hence the best way to treat limitation as a whole is to consider it, not in an isolated fashion, but in connection with the general system of efficiency and surplusage, and to regard the two principles (of gain and limitation) as two opposed multiplicative systems; not to neglect or ignore parts of the two systems which seem to cancel each other, but to regard the two opposed systems as always existent and in action. Thus in psychology the process of forgetting is not to be regarded as a mere erasure of mental data, but as due to the encroachment of one multiplicative mental grouping upon another. The same ultimate two-systemed conception of things is found in a crude form in the doctrine of total depravity along with that of divine

grace in certain theologies. In all cases however in dealing with two opposed systems the positive or useful system is to be regarded as dominant over the negative one.

This general combination concept often takes various special forms. Thus it is frequently an advantage to conceive of it in the form of a rhythm in which the main action is that of positive multiplication, while the negative part consists of mere back eddies. The advantage in this method of viewing the matter is that it facilitates the application of dialectic in taking care of the limitations involved.

Other Uses of the Two Opposed Systems

Similarly the more general conception of two opposed multiplicative systems brings important efficiental values. For it provides an endless store of diversities and varied reciprocal externalities to be utilized in efficiency processes. In Germany where, at the present time, in many respects, scientific efficiency has reached its highest development, it is estimated that only $\frac{1}{1500}$ part of the sun force which falls on a cultivated field is collected in a useful form by the crops produced. When we consider the great array of efficiency processes which are here applied, we realize the vast extent and power of the limitations at work. We get a more comprehensive view of these when we consider that the world as a whole is moving forward only at a snail's pace with back eddies some of them tens of centuries long. We may thus realize to some extent the enormous results which it is possible to attain by a mere negating of limitation as well as the extent of more positive opportunities. In this connection it may also be noted that the greatness of a difficulty, or even defeat, often is a measure of the greatness of the related opportunity. The double multiplicative view of limitation both prevents a raw and unlimited optimism and inculcates a sane, wholesome, and discriminating optimism.

The most important general result which follows from conceiving the world as two opposite multiplicative systems lies in the fact that the separation thus involved enables us to conceive of an ideal goal of human endeavor, or, to express the matter in a more general form, to conceive of a summum bonum of all processes. For when considered apart from limitation, the positive surplusage treated in

Chapter IX becomes unlimited in amount and quality for each person and entity; or in other words each person and conscious entity would have a near and instant and unlimited wealth and ecstasy.

Summary of Chapter

To sum up the chapter, a fundamental but somewhat indirect source of efficiency is that of studying the limitations of a given process, determining whether these limitations are fundamental or superficial, and devising means to overcome them as far as possible. It is advantageous to classify limitations both according to the objects or fields in which they are found, and also according to the forms which they assume. Special mention may be made of the species of limitation respectively termed active, passive, absolute, relative, grouped, or complex, since a knowledge of each of these is an important special source of power. It is also important to realize whether each given limitation is personal and subjective, or objective. It is also at times an aid to new results to understand that a limitation is essentially a diversity, and hence may be made the source of those fruitages which are characteristic of diversities.

The principle of externality supplies the best method both for the accurate determination of limitations, and also for the advantageous treatment of them. It is evident also that various adapted species of externality should be applied to difficult kinds of limitation. Hence the value of thorough and exact knowledge in every field of operation. The most general and useful conception in dealing with any large domain of material is that of efficiency and limitation acting in the domain as two comprehensive and opposed multiplicative systems.

EXERCISE 17

1. Give an example of each of the principal kinds of limitation.

2. The limitation of the speed of a railroad train is the resultant of what component limitations?

3. It is said that the clay which underlies the city of Philadelphia contains over $125,000,000 worth of gold. State some of the reasons why this gold cannot be utilized.

4. It is estimated that one ton of radium would drive a 15,000 ton ship for thirty years at a speed of 15 knots per hour. Why is not radium used for propelling steamships?

5. State three cases in which friction is a limitation to human activity, and also three in which it is an aid.

6. Name some noted man the limitations of whose childhood have helped make him great.

7. Show that the limited amount of food in the world has had the effect of making the forms of animal life on the earth more efficient.

8. Give an illustration of the difference between a real and an apparent limitation.

9. Show how the phrase "defects of its qualities" is illustrated by the properties of water.

10. Of iron 11. Of electricity 12. Of money

13. Give a case where the limitations of an object are easier to recognize and to use than the object itself.

14. If the efficiency of man's body when used as a machine is 21 per cent and that of a small steam engine is 3 per cent, compare the two efficiencies. What means are there of overcoming the limitations involved in each case?

15. If a large steam engine is 10 per cent efficient, while a small one is 1.5 per cent efficient, state the degrees of relative efficiency involved, and also the sources of greater efficiency in the large engine.

16. If a steam engine is 10 per cent efficient and a gas engine ⅓ efficient, compare the efficiencies involved, and state in efficiental terms some of the sources of the difference.

17. Compare the relative advantages and disadvantages in having a building erected by contract and by day's labor.

18. State some of the causes which limit the amount of income from a safe investment.

19. A soil that is suitable for the raising of sweet potatoes is not fitted for raising tobacco and wheat. Give a similar illustration of your own.

20. Every musical instrument is said to have some characteristic limitation. Name these as far as you can.

21. Heat expands metal. Give an example where this property is useful. Another where it is a disadvantage.

22. Why is it that the tides, as sources of mechanical power, are not generally available for human use? Answer the same for the waves of the ocean.

23. Give an example of an efficiency blessing appearing in the disguise of a limitation.

24. Give an example where the use of an externality not only overcomes a given limitation but is also a source of positive efficiency.

25. Give an illustration of the fact that a wide survey of material often reveals a place or places where a given limitation disappears.

26. Give an example where the exact knowledge of limitations has led to their more efficient treatment.

27. Give an example of a group of limitations or evils which have been overcome by destroying their common root.

CHAPTER XVI

ERROR AND PARADOX

Removal of Waste and Error as a Source of Efficiency

In the preceding chapter it was stated that every efficiency process from the ideal point of view is a mere removal or undoing of limitation or unnecessary loss; that is, every efficiency process has a negative as well as a positive aspect. It was also pointed out that, in obtaining increased efficiency in a given domain, it is often an advantage to pay primary attention to this negative aspect which consists of the removal of limitation. This principle is of particular importance with respect to certain special kinds of limitation. One of these is that species of limitation where some element of personality is involved, as, for instance, in cases of ignorance and error. The removal of these is often an especially valuable means of creating or improving efficiency. One reason for this is that a mistake is something below the general level of current social efficiency, and the manifold adjacent social externality supplies abundant aids of various kinds in remedying the given source of loss. This applies for instance to a person of intemperate habits who lives in a highly moral community, or to a glass manufacturer who is melting his glass in pots while his neighbors use large tanks for the purpose. Another reason why particular attention to the prevention and correction of mistakes is a fruitful source of efficiency, is the fact that direct efficiency often means merely added luxury and hence something which may be dispensed with, while a mistake may mean ruin and the loss of all essentials. Hence awareness of error and loss is frequently far more stimulating and productive of the use of all possible factors and aids in obtaining results, than is the mere conception of new and additional sources of profit.

Further Advantages

For certain special reasons also, measures taken to prevent or to correct errors are often the source of peculiarly powerful dialectical (i.e. extra and unexpected) efficiencies. For in case of error we are concerned with the double fact that, objectively, the multiplica-

tive principle is giving results so profuse that they cannot be fully grasped, and also that, subjectively, certain powers are at work in like manner, such as the directive faculty or the mental powers in general. Hence error is best regarded in general as the multiplicative principle acting in a special doubly errant way, or, in other words, as double misgrouping of some kind. Socrates was persecuted and put to death because he was falsely classed with the selfish destroyers of the national customs. Selfishness is mainly due to the lack of externality and hence is a form of misgrouping. Errors in the use of words and figures are simpler cases of misgrouping. Hence the fundamental means of preventing and correcting errors is the acquirement of a thorough knowledge of the group principle in all of its aspects, including, in particular, an aggressive use of various kinds of externality. It is also to be remembered that these methods often lead to a new and vital dialectic. As an illustration we have the fact that a man who overcomes the habit of drunkenness must usually acquire some ideal, or religious principles, or even a new and profound philosophy of life, giving many aggressive efficiencies over and above the mere correction of a bad habit. In order to obtain a quick detection and correction of error and waste, a business man often devises some special method of keeping accounts, one, for instance, by which any element of his business can be summarized and balanced at the end of each day. This method not only brings the quick correction of errors, often in groups, but the knowledge thus obtained often leads to the development of new and positive efficiencies. Similarly the syllogistic method of examining a course of reasoning in order to detect possible fallacies, frequently leads to a searching analysis of data which, in the end, begets a more fundamental and efficient multiplicative grouping of a given domain.

General Method of Investigating Error

As error always occurs in the process of striving after results, in illustrating the different species of error it will be useful frequently to point out the efficiency principles which are distorted or misused in a given case. For if a given principle is found to be misused in many different cases, a more correct use of this principle will lead to the detection, correction, and prevention of errors in groups, with a corresponding intensification of dialectic results.

In order to condense the treatment as far as possible special attention will be paid to those extreme and persistent sources of error, the mastery of which confers a grasp of many lesser and more superficial ones.

CLASSIFICATION

Different Fields of Error

The domain of material in which a given error has been made to some extent determines the source of the given error, and also the form which the error takes. Hence in classifying errors according to material or content, we shall also to a certain extent be classifying them according to source and form.

Important errors have been made with respect to *inorganic objects,* and the correction of these mistakes has been the source of far-reaching efficiencies. For instance for a long time it was regarded as impossible that the addition of a fraction of one per cent of carbon to a mass of iron would of itself convert the iron into steel. The correction of this error has led to vast improvements in the manufacture of steel. A similar mistake was that concerning the resistance which air makes to projectiles. It is to be noted that the root of these errors was a failure to grasp the extreme results which follow from the multiplicative properties of matter. Other like paradoxical truths which have been or may become the sources of efficiency are the fact that water is more dense than either of its constituent gases, oxygen and hydrogen; that solder has a lower melting point than either of its components, lead and tin; that the process of tempering hardens steel and softens copper; and that the diamond chemically is pure carbon.

Similarly we have paradoxical properties and corresponding errors concerning *forces.* Illustrations are the fact that water will wear away a stone; that river courses are often more permanent than mountains; and that gravitation is the feeblest of known forces. Surprise and error concerning these statements arise from a failure to realize that the action of water in the above instances is a repeated one, and hence has a dimension not possessed by the cohesive force which holds a solid object together; and gravitation, as we are familiar with it, also has its source in enormously larger bodies than the other forces with which we are familiar. Lime binds sandy soils and loosens compact clay soils. It is an important

source of agricultural efficiency to take advantage of these two apparently contradictory properties.

In the domain of *living organisms,* a hybrid plant is sometimes more immune to disease than either parent species. A scratch at one time may be barely felt; at another it may produce death, the action in the first being additive, in the latter multiplicative. The extract from the thyroid gland in the human body is necessary to life, yet when injected directly into the veins it kills like lightning. The pancreatic fluid forms sugar in the intestine and destroys it in the blood. In an army it has often been observed that some frail looking clerk from a city store often can outmarch a husky young giant from a farm, the difference being due to the fact that the nervous energy characteristic of the former type of physique is of a higher order than the muscular energy prominent in the latter.

As a transition from the biological to the mental field we may take the case of two children born of the same parents but differing as much in mental capacity as, for instance, did Joseph and Napoleon Bonaparte. A possible explanation is that in the one child a set of factors may act together in an additive manner, or even cancel each other in a measure; while in the other child but slightly different factors act upon each other in multiplicative and progressive ways. Cases more detailed in nature are the fact that a nerve impulse will travel in less time from the tip of the finger than from the elbow to the brain; or that one minute's advice from an expert at a cost of $100 is cheaper than an hour's free advice from an ignoramus.

Error in More Abstract Fields

One of the most important forms of cognitive error is that of regarding the outside appearance as corresponding to or more important than, the inner reality. This error may be regarded as essentially a mistake with reference to the relation of the upper and lower parts of an order of materials scale, or as a misuse of symbolisms with respect to the object represented. A similar case is a mistake as to the relations of words and the actual intention of a speaker, or any distortion of the meaning of any act, word, or appearance. Memory, imagination, and reasoning are forms of reuse and multiplicative grouping, hence errors in them are essentially cases of misgrouping of some kind. Owing to their abstract nature

and to that of the groups involved, efficiencies of high order are possible in their treatment. Mention may also be here made of the paradox that the more one knows, the less one knows. The study of the illusions produced by a sleight-of-hand performer or so-called magician, is often instructive because the solution of a trick of this kind frequently reveals in explicit form the proximate efficiental cause of the error involved. Thus some of these illusions are due to the unrealized fact that the hand of the performer can move more rapidly than the eye can follow, or to a failure to realize the infinite number of surfaces that go to make up a solid (as in the flag trick), or to appreciate the vast number of externalities which may be employed in such performances (as externalities above, below, behind the stage, or in other persons), or the limitations in the scope of one's attentive consciousness (misgrouping). So, in general, various illusions may be shown to be due, at least proximately, to mis-efficientizing of some sort, and this knowledge may lead to the efficient avoidance or correction of error, often with many dialectic results.

Social and Religious Errors

By a cumulative combination of the preceding sources of mistakes, as well as for independent reasons, history, politics, and sociology are full of errors and paradoxes. Thus the greatest political crimes have been committed in the name of liberty. Civil and religious wars have been the most bloody and cruel.

Similarly the field of ethics is full of errors of the most vital sort, the dominant cause being the same extreme lack of externality. Examples are the error of living for the immediate present, and hence in prodigal and wasteful fashion, or in working consciously or unconsciously for narrow selfish ends. Other cases are the frequent inconsistency between knowledge and action. Still other instances are the errors connected with the paradoxical fact that intelligent selfishness leads to altruism.

In like manner the whole field of religion and theology is full of error due to misgrouping of the vast entities involved. Instances are the mistake of regarding some particular type of infinite externality as the only possible one; or of regarding some special method of approaching and utilizing infinite externality as the only one allowable. Also each form of conventional religion contains princi-

ples which, in certain respects, at least, are paradoxical and the occasion of many errors. For error as a rule is proportional to power, and "unexpected sources of error lurk in every complicated process".

When errors occur primarily with reference to certain categories, it is often advantageous to characterize and group them with respect to these categories, even though the various instances might also be classified according to the more concrete materials in which they occur.

Misconceptions as to Dimensions

Thus among the groups formed with reference to the various categories of space, are errors as to size, form, or position. Among the most subtle, and also the most important as sources of waste and hence of corrective efficiency, are those with respect to dimensions. Thus the fact that it is harder to keep money than to make it is explained by the fact that a higher dimension of time is involved in the former case than in the latter. A machine or a bridge which is a success in the model, when built on the large scale demanded by actual practice often fails, and indeed may not sustain its own weight, the error involved being due to the fact that the weight of an object increases as the cube of its linear dimensions, while its strength increases only as the square. Lightning strikes at a single point while the thunder which accompanies it is heard over a wide area. The unconscious assumption that the danger from lightning extends over the area throughout which the sound is heard is the source of the extreme terror often produced by a thunder storm. Similarly many other kinds of cowardice may be cured by a realization of the narrow and local nature of the evil which is feared. Likewise it is often of the highest importance to comprehend the fact that good is higher in the dimensions of its efficiency than evil; for while a method of doing good is indefinitely reusable and multiplicative, any form of robbery or wrongdoing can be repeated only a few times before it becomes self-destructive.

Some Famous Paradoxes and Errors Explained

To complete the discussion of the principle in hand, it may be well to note that in the dimensional principle which is under consideration, we find the explanation of Pascal's paradoxes concerning the

smallness and greatness of man. For man is small in his additive relations but great in his multiplicative essence and powers. A similar explanation holds for Kant's antinomies; for the universe, for instance, is small (or finite) when the multiplicative principle is used to condense it, and large (or infinite) when the same principle is used to expand it. A special application of the fact that the world is small owing to its multiplicative nature is found in the fact that a traveller in any given place on the earth, usually finds himself near some acquaintance, or at least near some acquaintance of an acquaintance. A somewhat different application is the fact that when processes are fully understood a very few steps will carry us from the data of common life to their most abstruse and technical development. An example is the way in which gunpowder is made in Jules Verne's Mysterious Island, by a shipwrecked man deprived of the ordinary materials supplied by civilized life.

Of somewhat similar nature are errors with respect to the ratio between cause and effect, or in the various proportions of objects. Illustrations are the mistake that the length of a man's jump will be in proportion to the length of his running start, or that growth and strength will correspond to the amount of food consumed. Another important error of this class is a disproportion between the means employed by a person and his power to use those means.

The fallacy of the undistributed middle, in which one word is unconsciously used in two or different senses, is an example of error with regard to number and symbols. Illustrations of the numerical paradoxes which are frequently connected with errors are the statements that the greatest things are both simple and complex, and that the universe is both unitary and multitudinous.

Temporal Errors

Besides ordinary errors as to time, we have its paradoxical properties such as are implied in the statements that life is short yet long; that all things are old yet new. Similarly errors occur with respect to order. These often take extreme resistant forms, while an understanding of them gives a deeper grouping of the facts of life with new and higher efficiencies in dealing with these facts. Thus in the history of the human race we find that contrary to the common supposition the use of ornaments such as bracelets, necklaces, and rouge, preceded that of clothing, and that animals were

first tamed to serve as pets rather than to serve for sources of food. This is an illustration of the fact that ideal efficiency is more primal than concrete utility. What was originally the bottom of a valley has in many cases become a mountain top. Similarly we have a general paradox as to order expressed in the statement, the first shall be last and the last first.

An important error as to velocity is indicated by the proverb "More haste, less speed". With respect to quality, we have that which confuses size with quality, as bigness with greatness.

A still more vital and inclusive classification of cases of error is obtained by grouping these according to the various Efficients which are primarily misapprehended or misused in the different instances.

Errors as to Uniformities and Diversities

In particular since the principles of uniformity and diversity are of primary importance in making groups and hence in forming the other Efficients, the correction of errors in regard to uniformity and diversity constitutes a conspicuous method of obtaining negative and dialectic efficiency. For instance the failure to realize that the surface of an iron rail, though apparently smooth, is rough enough to have great frictional properties, hindered the early development of railroads. For in the absence of this conception engineers thought that the rail and the wheel running on it must be cogged and must operate in rack and pinion fashion.

Besides such common violations of uniformity as inaccuracy or want of certitude, misconception as to uniformities often take the costly form frequently denominated "hasty generalizations". This consists in jumping to the conclusion that what is true of certain members of a class is true of all members of the class. For instance since most black and peaty soils are fertile some farmers have concluded that all such soils are rich and productive, whereas such soils in places along the Mississippi River, for example, are so deficient in potash as to be practically barren.

More paradoxical properties of uniformities and diversities about which errors are often made are illustrated by the fact that frequently the greater the number or extent of a uniformity the greater the diversities connected with it, and vice versa. The latter of these cases is illustrated by the properties of a book, which is essen-

tially composed of separate leaves, as compared with the ancient manuscript roll. In this connection may also be mentioned Napoleon's maxim, that nothing is so certain to happen as the improbable.

Errors as to Groups

A hasty generalization may also be regarded as a group made unduly large. In like manner groups may be made too small or otherwise misused. The misuse of the unit and multiplier principle is illustrated by the case of a person who is swindled as a result of taking some bait that has been held out, or by the practice on the part of some of magnifying the errors of other people and using them as pretexts and handles by which to obtain unworthy advantages.

Errors in regard to multiplicative groups are of many kinds. Among the most important is that of neglecting or failing to see the importance of the general multiplicative structure of a domain. Examples are the failure to realize that a well organized team of moderately able players is often stronger than an all star team not so well organized; or that the physical texture of a soil is more important than its chemical constituents. Another instance is that of mistaking a case of tandem for one of parallel elements of efficiency, or vice versa.

Among the most costly errors are those with respect to the relative importance of factors of efficiency in a given multiplicative or order of materials scale. Among those factors which are often mistakenly inverted are brain and brawn, blessedness and happiness, quantity and quality. An error of this kind often takes the form of consuming or expending the framework of life or character, as if it were something temporary or incidental.

Errors as to Externality

Errors in regard to externality are of great importance. Instances are egotism and selfishness in their various forms, or the use of one method of obtaining results to the exclusion of other methods. In striving for results it is of especial importance to remember that what is efficiency from a narrow point of view may be the grossest waste when viewed in a larger way, and that on the other hand what is seeming waste may be real economy and efficiency. In particular efficiency for narrow or selfish ends in the long run is not

efficiency at all, but relative waste. Similarly as a rule the best and quickest route to extreme technical efficiency lies through a manifold grounding in fundamental theoretical principles. Expressed in more paradoxical form the above principle becomes proverbs like, "Luxury is economy", or "The longest way round is the shortest way home".

Errors as to Other Efficients

Errors in regard to symbols are numerous and often vitally important because of the wide gap which usually exists between an object and the symbol representing it. Such errors are also costly owing to the highly abstract and efficiental nature of most symbols. Both of these causes open the way to aggressive and wilful abuses such as falsehood, swindling, and forgery. The extreme forms which verbal errors may take are illustrated by the fact that an entire legal document may be invalidated by the omission of a comma. Whole classes of symbolic errors are indicated by such words as misspelled, misread, misname, miscalculate, misrepresent, and other words beginning with the syllable *mis*.

A great variety of errors also appear in connection with the Efficient which we have termed directive. Large classes of such errors are indicated by words like mistake, misplace, misdirect, misuse, misapply, misguide, mismanage, as well as by many words formed differently.

Similarly numerous errors occur in the application of each of the other Efficients. These may take general forms, such as the over use or under use of any one Efficient, or the use of a wrong species or of a faulty combination of Efficients, or they may take more individual and specific forms. Important errors which relate to all processes are those concerning the relative values of the results of different processes.

Forms of Errors

Errors may also be classified according to the various categories of form. Thus important cases occur which may be characterized by one or more of the following adjectives: large, small, pervasive, cumulative, conscious, unconscious, resultant, manifold, reciprocal, temporary, permanent, passive, aggressive, primary, secondary, simple, complex, rhythmic. An example of a resultant error is one

composed of mistakes as to the size, shape, color, and duration of any object, or the combination of errors which led to the loss of the steamship *Titanic* in April, 1912; of passive errors, are those due to inertia, or imitation; of active errors, are the spendthrift habit or aggressive wrongdoing of any kind.

Scales of Error

Errors high in a multiplicative scale are naturally of prime importance. An error may be of high order either because it occurs with respect to some material of high order, as with respect to some fundamental principle of efficiency, or with reference to any Efficient. Also an error which leads to or includes a number of other errors is of high order. An example is selfishness or wilful ignorance about the laws of health. A like place is held by an error with respect to the nature and methods of correcting errors.

It is to be noted in this connection that the correction of any fundamental mistake includes the correction of a host of subsidiary errors, with a dialectic of other useful results.

Transcendent Errors

Certain errors are so extreme as to be difficult even of proximate explanation and hence may be termed transcendent. The tendency which persons feel to throw themselves from the top of a precipice, and to which an individual occasionally yields even with the certainty of losing his life, has been explained as due to the fascination produced by the thought "of the momentous consequences that would result from such a comparatively simple act". Nevertheless it is difficult to realize how disproportion of this kind could work such a spell that a person would sacrifice his life in order to have this surplusage experience. Of even more extreme nature however are certain cases of moral wrongdoing when the doer is fully aware of the deadly nature of the results of evil and of the beneficial outcome of what is good.

Paradoxes

In this connection special mention may be made of the paradox. The paradox is essentially a contradiction between outer appearance and inner reality, but one of so stubborn a nature that it persists in the mind even after it has been explained. Illustrations are the fact

that in winter "as the days begin to lengthen, the cold begins to strengthen"; or the ethical principle that it is more blessed to give than to receive; or the spiritual fact that he would gain his life must lose it. These paradoxes are all capable of explanation in terms of the Efficients. Thus receiving is a single or unitary act, while the right kind of giving is continuous and multiplicative in nature, and hence is of a higher order of efficiency; hence the contrast in this case is between the unit and the multiple or product. Similarly the concrete life must be lost in order that the abstract and comprehensive life may be won, an application being thus made of principles of externality and orders of material.

Similarly the human will is measurably free in the field of the efficiency abstract, but is relatively restrained in the concrete world. Nevertheless owing to the momentum or dynamic of certain crude psychic habits the feeling of contradiction persists to some extent ofter the explanation has been made. The study of paradoxes however is often particularly useful in that their explanation often brings out into peculiarly clear light the essential nature of many fundamental errors and the most efficient methods of preventing, correcting, or even utilizing these. Such a study also by removing difficulties opens the way to more extensive multiplicative groupings of errors than would otherwise be possible, and hence to greatly enlarged efficiencies.

Makers of Errors Classified

It is often useful to classify errors according to the agencies which make given mistakes. The principal classes of errators are:

(1) Individual persons, or certain faculties or parts of a personality such as the memory or imagination.

(2) Small groups of persons, as families.

(3) Societies or nations, or even humanity as a whole.

It will be noticed that the number of agents (that is, the size of the acting group) involved in these three cases increases progressively.

Complex Errors

The different species of error and paradox which have been mentioned in the preceding pages may be combined in various ways. A number of illustrations of complex mistakes have already been

given, but a still more complex case is found in the various errors which have acted as a continuous and often cumulative stream in human history, and which together with other limitations have held human efficiency in so many respects within narrow limits.

An illustration of paradox combined with error is the fact that Faraday, although he discovered the principle of the dynamo, classed the electro-magnetic engine with mesmerism and perpetual motion, and called them all "ill weeds which cannot be extirpated". A similar instance is the fact that Charles Darwin was publicly reprimanded in the Shrewsbury school for setting up a laboratory for scientific experiment.

These instances illustrate both the great difficulty in detecting and preventing complex or compound errors, and also the large efficiency results which may often be obtained by so doing.

NATURE AND EFFICIENCY RELATIONS OF ERROR

The nature and sources of error, or at least the best way of regarding these matters for efficiency purposes, have of necessity been indicated in large measure in the part of the chapter which precedes. It will be useful however to state some of these properties more systematically and to call attention to certain additional ones.

Dual Nature of Error

Every case of error may be regarded as dual in nature; that is, as containing

(1) a more or less ultimate element, viz.: the fundamental principle of limitation acting through personality;

(2) a relative or approximate element, viz.: that of the misuse in some way of an Efficient or of a combination of Efficients. The useful properties connected with the first of these elements have been stated in the preceding chapter and need not here be repeated.

Multiplicative Element in Error

With respect to the second of these two elements, the realization of error as due to, or at least accompanied by, the multiplicative principle facilitates an appreciation of the enormous number of ways in which error may arise, and of the explosively destructive consequences of many mistakes; aids in the recognition and admission of one's own mistakes, and in the treatment of those of others, charitably and in other efficient ways.

Advantages in Efficiental View of Error

Thus, for example, the above conception makes clear in striking yet safe ways that what is often regarded as pure folly or malice may contain much that is good and usable. As has been remarked in another connection, the dominant motive in a bullying schoolboy is pleasure in the exercise of his directive faculty, the nature of the effect on others being entirely secondary. The use of these powers may frequently be freed from error and directed into useful channels. The same analysis applies to many of the tyrannies and other errors of older persons. Or, in general, the efficiental view of error and wrongdoing often separates and sets free certain Efficients and allows and stimulates them to act aggressively and effectively.

In this connection it is well to note that error is often in proportion to power; that it is the man of energy and constructive capacity who makes mistakes. In other words, where there is much waste there is much to save and utilize.

The above multiplicative view of error also makes clear the need and value of treating error by manifold checks and tests and by inclusive group methods. It also indicates the rich dialectic results which are likely to follow such methods of treatment, and stimulates one to apply such processes.

Advantageous Use of Error and Waste

As has been stated in another place, in certain circumstances, especially when hemmed in by certain limitations which for the present seem insuperable, the elimination of waste is the most available source of additional efficiency. Often also error and waste are the most conspicuous signs of possible new positive efficiencies, and the most manageable routes by which to attain these. Also in competitive work a large element of success consists in merely taking advantage of the mistakes of others. Napoleon Bonaparte, for example, obtained many of his greatest victories in this way. This method of procedure in obtaining efficiency is entirely legitimate when the mistakes of evildoers are utilized as a means of overthrowing such people, or the errors of any class of persons are employed as a means of benefiting them. The mistakes made by other people may often also without impropriety be regarded as experiments made for our benefit without cost to us.

Similarly if the multiplicative elements which are in conflict in a

paradox be realized, by this very means these elements are set free to act independently, yet in a certain measure with mutual helpfulness since they stimulate each other to develop in different fields while still maintaining useful relations. The efficiental explanation of paradoxes also, as has been remarked, clears the field of obstacles and thus opens the way to deeper unities and freer action.

Era of Progress often preceded by Period of Error

After the above study of the relation of error and paradox to efficiency, it is not surprising to find that almost every important epoch of human progress has been preceded by a stage of error and need, the more or less efficiental treatment of which has led to the subsequent period of progress. Thus we have the age of the Sophists before Socrates, and the period of decadent Greek and Roman morality before Christ. Similarly the destructive Hume preceded the constructive Kant. At various times eras of political oppression have been followed by periods of freedom and progress. The principal ways in which error and evil tend to produce reaction and new efficiencies have been indicated in the preceding pages.

METHODS OF EFFICIENT TREATMENT

Many principles and details with respect to the most efficient methods of treating error are contained in, or follow obviously from the preceding discussion. It will be useful however to make certain additional statements as to this matter and to present the whole topic in a more systematic form.

Multiplicative Principle an Aid in Various Ways

The multiplicative principle not only aids us in realizing the great number of ways in which errors may arise, but also the extreme forms which errors and their consequences may take. Hence it also suggests the importance of using wherever possible a variety of checks and cross multiplicative groupings in preventing, correcting, or utilizing errors in a given domain of material; and it also makes clear the numerous tests which are usually available. Examples of checks in the forms of cross multiplicative groups are the double entry in bookkeeping and the initiative and referendum in government.

One of the methods by which the proprietor of a large business often secures efficiency in its management, is that of dividing the

business into sections or departments, and requiring daily reports of the cost and profit of each department. Any waste or error in any one section is thus quickly revealed and remedied. This constitutes a highly important application of the group method in the treatment of error, and one which frequently gives rise to new, aggressive, and often dialectical efficiencies.

Application of Principle of Diversity

The principle of diversity is often a valuable auxiliary in this connection. Thus if a page of proof be corrected first entirely with respect to the spelling, then with respect to the punctuation, and so on for other individual elements, a more thorough correction and utilization of error will be obtained than if all the sources of error be treated simultaneously.

Manifold Treatment of Highest Material

This multiplicity of safeguards is of especial importance with respect to material which in any given case stands highest in the scale of importance. Thus the traveller should frequently check up his money and at all times should vigilantly protect it from loss or error. So the wide awake business man carefully notes the character of each document to which he affixes his signature. Every wise man in like manner will jealously guard certain first principles of life and action to see that they are not impaired or encroached upon.

Similarly, cases of especially difficult error, persistent fallacies, paradoxes, and mysteries call for the use of extreme and manifold externality and cross treatment. With sufficient externality a clue to each tangle usually somewhere comes to the surface.

Application of Unit and Multiplier

The unit and multiplier principle is often of great service in the treatment of error. For instance its use often prevents the exaggeration of a difficulty and the dismay, discouragement, and other evil results which would follow. When applied in the form of experiment it frequently is a means of avoiding loss of time and resources, and sometimes, either directly or indirectly, leads to the solution of a difficulty. In cases of doubt the erroneous nature of a principle may often be detected by developing the consequences of such a principle by the unit and multiplier method. When used in the step by step

form of the Greek logic, this method often prevents or corrects theoretic errors, or unravels abstract difficulties. When utilized as a means of realizing the consequences of an act or principle, it serves to arouse the attention and to stimulate to action in safeguarding against mistakes.

Application of Other Efficients

The principle of reuse may also be of service in this connection. An instance is the reuse which one makes of the experience of others in the matter of mistakes and difficulties. A person utilizes this principle when consulting a specialist in a given line.

It is often important to remember that it does not pay to try to prevent all error and waste from this source, but rather that it is frequently best to proceed by the most available efficiency methods and from time to time to correct as far as possible such errors, and provide for such wastes as may have arisen. Similarly it may be advisable at times to allow the results of the errors of others to accumulate till in time they, in a measure, check themselves. This in effect constituties a use of the rhythmic method in the backeddy and often dialectic form.

In another aspect this method consists of giving the leading place to the creation of aggressive efficiency, and a secondary place to the prevention of all errors. It pays better to try to have a large surplus on hand with which to pay for some errors, rather than to try to avoid all mistakes. This is especially true in the more inventive and creative elements of life and action.

Study of Special Pitfalls

It is well to remember also that each special field of work or life has its own particular pitfalls or characteristic sources of error and consequent waste. Hence it is important carefully to study the mistakes which are made, or are likely to be made, in any given domain; to ascertain their sources as far as possible; and to treat them in systematic ways. At this point is evident the value of as complete a knowledge as possible of the facts and principles of the various branches of science. Similarly it behooves each person to make a study of the mistakes which are characteristic of his personality, and which therefore he is most likely to make.

Other General Principles for Treatment of Error

In the treatment of paradoxes it is desirable to keep in mind the fact that some powerful source of efficiency is usually close to, or is contained in, each apparent contradiction of this kind; and that it is important to use as extensively as possible each source of efficiency involved in such contradictions, no matter how opposed to each other some of these sources may seem to be at times. Thus the fact that lime when applied to soils both binds a loose soil and loosens a compact one shows the wide and manifold value of this chemical element in agriculture.

In all cases the power of externality to convert a mistake into a source of ultimate advantage is to be noted. In fact it is not too extreme a statement to say that, with few exceptions, no matter what error one commits, if one has or acquires large enough externality one may convert such error into something advantageous. In the rhythmic series involved all depends on the degree of ultimate externality.

In general it is important to remember that the entire body of the Efficients is the most effective agent available, both in detecting errors and in treating them in the most useful way.

Synopsis of Chapter

An especially important form of limitation is the active, personal one termed error. The peculiar gravity of this species of limitation arises from the fundamental place and function of personality in all efficiency processes. Also the correction of error and removal of waste often form a peculiarly easy way of improving a process. Errors may be conveniently classified according to the objects to which they relate, according to the personalities making mistakes, or with respect to the forms which errors assume. Of prime importance are those in regard to the fundamental qualities of objects, to spatial dimensions, to uniformities and diversities, to groups or other Efficients. A paradox is an error which persistently recurs after it has been explained.

It is of the first importance to realize that every error contains both an ultimate and an approximate or contingent element, and that the latter of these consists essentially in some mistake with reference to the multiplicative principle, and more particularly with reference to one or more of the Efficients. Often by realizing errors

in this way they may be avoided or corrected in large groups. Hence, in general, the best way to avoid, correct, and, in some cases, to utilize errors is by a thorough study and careful application of the multiplicative principle in its various forms.

EXERCISE 18

1. State and explain some costly mistake due to a misunderstanding of the dimensions involved in the material considered. (See p. 286.)

2. Give two instances of hasty generalizations with disastrous consequences. Show how these errors might have been avoided by efficiental means.

Give an example of

3. A fallacy
4. An idol of the den
5. An idol of the tribe
6. An idol of the forum
7. Of the other principal kinds of error.

8. At an expenditure of $300,000 a city so diminished the leaks and wastes in its water system that an expenditure of $3,600,000 in order to increase its water supply was avoided. State the degree of efficiency involved. Describe possible dialectic efficiencies which might follow.

9. Give an illustration of your own of increased efficiency obtained by the elimination of waste instead of by an enlargement of plant.

10. If in a given country the annual loss from forest fires is $50,000,000 and an annual expenditure of $5,000,000 will prevent half of this loss, state the degree of efficiency of the expenditure.

11. Give an instance of your own where the correction (or prevention) of an error has had dialectic efficiency results.

Explain each of the following paradoxes:

12. Hot and cold applications to a part of the human body often produce the same effect.
13. Genius and insanity are closely related.
14. At the seashore, the hotter the sun's rays the cooler the sea breeze.
15. A cool sea breeze means warm surf bathing.
16. Give an error that is dual
17. Manifold
18. Of high order
19. State the species of error implied in the statement "all is not gold that glitters".
20. Give two examples of optical illusions with explanations of the same. Show how these explanations may be made the basis of aggressive efficiencies.
21. How is it possible that some rivers at certain seasons of the year flow up stream?
22. Why should the river Nile be smaller near its mouth?
23. Why is it that that which looks most easy to do, as the playing of a violin by an expert, is often most difficult?
24. Give two examples of paradoxes with explanations of the same in terms of the principles of this book.

How might these explanations be made sources of efficiency?

25. Why is it that, in some respects, the best way to see the country is to go to the city?

26. Give an example of an error arising from confusing quantity with quality.

27. Why is it that many persons think of Palestine as larger than Newfoundland or Peru?

28. Give an example of a mistake due to inertia or momentum.

29. State and explain the principles of efficiency involved in the statement that the serpent, though it has no limbs can "outclimb the monkey, outswim the fish, outleap the jerboa, and, suddenly loosening the coils of its crouching spiral, it can spring into the air and seize the bird on the wing".

30. Why is it that a person is often the most solitary when in a crowd?

31. Give an example of an error which consists essentially of regarding a multiplicative process or relation as merely additive?

32. Give an instance where, instead of trying to prevent all error and waste greater net efficiency is obtained by employing in aggressive ways the energy that would be thus spent. In this connection what is the meaning of the business man's profit and loss account?

Give an instance

33. Where the use of externality has been the means of detecting error or loss.

34. Where the use of externality has prevented error or waste.

35. Where the use of cross multiplicative groups detects or prevents error.

36. Where the use of the unit and multiplier principle has prevented waste.

37. Why are the simplest things often the hardest to understand?

38. What efficiental term would be used in describing an error concerning errors?

CHAPTER XVII

COMBINATIONS OF EFFICIENTS

SUMMARY

Illustrations of Combinations of Efficients

When man supports himself by hunting and fishing, he uses food which is of a high nutritive value, but one which is relatively scarce. Hence in the hunting and fishing stage of civilization man necessarily lived in a somewhat isolated manner. When men who have tried to support themselves in this way have multiplied in numbers, it has become necessary for them, in the end, to subsist more and more on vegetable products. Such food is of a relatively low order, but it has the property that by externality and by the coöperation of the producers it may be made very abundant. Hence its preparation in time came to be accompanied by an elaborate social organization with many unexpected resultant advantages such as a high degree of mental development and of the arts and sciences which accompany such development. The complex social organization of certain kinds of insects, as of some species of ants, and their great relative intelligence find a like explanation. Originally ants were meat eaters and like spiders lived approximately solitary lives. The stages may be traced by which, as their food changed, first to the mixed form and then to a vegetable basis, their social organization and general intelligence also developed. These cases are illustrations of the fact that in a world full of diversity, limitation, and active competition, more effective results are obtained by using a combination of efficiency agencies than by the use of any single one.

General Statement of Advantages

In general the employment of a combination of the primal instruments of efficiency, instead of one only, usually makes possible a closer contact with the actualities of the world of fact than is otherwise possible. Hence result certain intense, and often sudden and dynamic efficiency results which are of the highest importance in a world of competition. Various efficiency methods when com-

bined in a given process often interact also in ways so manifold and complex as to be impossible to trace in detail.

For instance, in the rotation of crops in agriculture we evidently have a use of the principles of diversity, directive, and rhythm. But these result in other efficiency principles such as the retention and reuse of farm laborers the year round, the greater reuse of farm machinery, utilization of by-products, the prevention of waste as by the destruction of insect pests, a uniformity of returns, and many other useful results and agencies.

Similarly in various forms of business, in education, in government, and in almost all actual, fruitful processes, we find a combination of efficiency agencies at work. Often by careful study such combinations may be developed to a far higher degree of efficiency.

Apart from their so-called business or practical uses, various bizarre combinations of efficiency agencies prevent work from degenerating into a mere formalism, and go far to give spice, variety, and charm to life.

Classes of Combinations

Evidently the possible forms and species of combinations of Efficients is without limit in form and number. In the preceding chapters the different forms which each Efficient may assume have been outlined, and plainly the number of different combinations that may be made of these various species vastly exceed the varieties of any one agent. A realization of some of the principal forms of combinations will be an aid in selecting the best combination for use in any given case, and will be useful in many other ways.

Loose Combinations of Efficients

In the first place we have those combinations of Efficients which are comparatively loose and incoherent. A situation may be so difficult that in it, especially at first, it is possible to use only what may be termed efficiental opportunism; that is, to employ such local and fragmentary efficiency methods as may be available from time to time and to leave the rest to future development. Simple instances of this process are that of looking at a subject from various points of view, or of working at a problem along various lines. So in business greater efficiency is obtained by applying adapted methods of obtaining results, to buying, manufacturing,

selling, and accounting than could be obtained by trying to develop all of these by one exclusive procrustean method.

The more abstract cases of this general type of combination are illustrated by the advantages in combining and using different views of electricity, no matter how contradictory these theories may appear from some points of view. For if electricity be regarded in certain cases as composed of small units called electrons; in other cases, as a combination of positive and negative fluids; and in still others, as stresses or undulations in the ether, it is found that better results are obtained in the aggregate, than if any one of these seemingly opposed views were used alone.

An important instance of somewhat different nature is that of organizing a business or a nation in such a way that it shall be characterized at the same time by the utmost system (or multiplicative group organization) and also by the greatest possible freedom (diversity) and initiative (directive) on the part of the individuals and other units composing it.

Often it is well (or even necessary) for a time to develop each strand or element of efficiency in a given situation, with little or no regard to the other sources or strands, and to wait or work for later, more definite combinations and crystallizations.

As a transition to the next general class of combinations mention may be made of the conscious formation of habits as a source of efficiency. This involves the combined use of the following efficients: reuse both objective and subjective, specialism (or diversity) and uniformity in kinds of activity, externality in the shape of extra educational practice, a resulting speed and force of action, and the substitution of lower for higher and more costly energy.

Close Combinations of Efficients

Another important form of combinations of efficiency agents is that in which the component agents are so closely knit together that their individuality is largely lost in that of the whole. Thus the efficiency of the human body arises to a great extent from the fact that the various agents and sources of effectiveness in it are in many places and respects so completely fused together. We here find, for example, various uniformities and diversities among the cells, tissues and organs; multiplicative groups of many species, both articulate and constitutive; orders of materials, as illustrated by the series, brain, nerve, flesh, bone; externality operating through

the eye and hand, and many subjective reciprocal externalities; varied reuse of each organ; and symbolisms, force, motion, directive, rhythms, and dialectics of various kinds. But the whole aggregate is so closely knit together as to form a solid contexture of efficientisms; in fact a unity is present which is perhaps beyond analysis. This fact is the source of efficiency results which are far beyond those which could be obtained from the agents specified in the above list if used in scattered fashion. The same principle holds in even higher degree of the human mind. The efficiency relations involved in a thoroughly united human family are of like nature.

Many of the sources of success, that is, methods of obtaining results, as popularly stated, are of this mixed and closely fused nature. Illustrations are the terms shrewdness, good health, magnetism, native ability, or even genius. Thus shrewdness includes power to perceive differences (or diversities) where other persons see only uniformity; to look beyond the immediate present (externality); to penetrate beneath the surface of things (use of orders of material); and the avoidance of error. Any highly developed or thoroughly mastered art of doing things, as contrasted with the science or knowledge of them is largely a thoroughly fused combination of efficiency processes of the general type under discussion. Instances in another field are the concepts of harmony, beauty, and sublimity. Thus many mechanical and formal rules are combined in a statue like the Venus of Milo, but they are so completely combined, or interact so subtly, that their complete analysis is impossible.

In an even more abstract world, it may be shown that concepts like those of space, time, and cause are instruments of the highest efficiency, and that this efficiency is greatly increased if, in certain respects, each of them be regarded as a closely knit aggregate of primal Efficients.

Combinations built up progressively

Systems of Efficients which have been built up progressively in time, or are the result of a species of growth, even though these systems contain elements which are in large part mere survivals by virtue of inertia, are often peculiarly rich and effective. An example is the English form of government. The human body and human mind as described above, and most of the general concepts in use are of the same general nature.

Mottled Combinations of Efficients

As a result of human directive action and of processes of natural development, many combinations are of a varied or mottled character with respect to the closeness with which their components are aggregated. Thus in some parts of any highly developed business the Efficients are loosely aggregated and in others they are highly fused. The same will be found to be true of almost all efficiency processes in actual operation in the concrete world if thoroughly examined. In many cases this diversity is the result of what seems caprice or even error, in others it means a closer adaptation to limitations and qualities beyond control, and hence is a source of important added efficiency.

The above considerations suggest certain remarks as to the best methods of using combinations of Efficients in actual practice. In the first place it may be noted that such combinations in some cases form a crude, initial instrument by which to attack a problem or treat a situation; but that in other cases they form a highly finished, final instrument; and that they may have every degree of development and value between these extremes.

Efficient Transitions

Again in order to obtain results of the highest order from the use of groups of Efficients, especially of the more developed combinations, it is often important to make the transition from one component in them to another with a certain deft and invisible swiftness. The changes in the moods and methods of an accomplished orator illustrate this. Frequently added efficiency is obtained by the concentration of some one or more of the involved Efficients into a momentary intensity. This principle also is illustrated by some single word, tone, or gesture of the orator. These intense master strokes of action are characteristic of workmanship of the highest order in all fields however diverse, as in trimming a woman's hat, or painting a picture; driving an automobile, or governing a nation.

Application of Directive and Dialectic

Similarly in employing combinations of the primal instruments of efficiency it is of great importance to know when to allow them to act of themselves in whole or in part. By practice their use in many ways may be made subconscious and instinctive, and it often

pays not to interfere with such methods of use when these have been acquired. In other words, often the most efficient way of using efficiental pluralisms is to treat them much as if they were living organisms.

Two Principal Forms of Combinations

It is important to note that, in this relation, highly developed combinations of Efficients take two principal forms, viz.: (1) those characterized by peculiarly intense adaptations to the local peculiarities of a given field, and (2) more general and abstract aggregations of the Efficients. It is well, if possible, always to use the latter of these two species in connection with the former. The close contact with so-called realities involved in the first species, together with the resultant momentum and dynamic, often leads to a certain narrowness when this kind of combination alone is used. The concurrent application of the general ideal species not only tends to prevent these drawbacks but to produce other more positive results by the externality thus supplied, and the interactions and dialectic which arise as a consequence.

Universal Efficiency Machine

The general observation may also be made that it is often a source of efficiency to regard the entire aggregate of the Efficients as a kind of universal machine, capable of variation by marginal reuse in different ways, and thus made applicable to special problems and situations.

General Summary

At this point it will be of advantage to sum up the results arrived at in the present and preceding chapters.

An efficient process is one in which the results exceed the expenditure. The primary instrument of efficiency is the group.

The group principle takes several different forms called the Efficients. Among the most important of these are reuse, the unit and multiplier, multiplicative groups, orders of material, uniformity and diversity, externality, symbolism, kinematic and dynamic, directive, dialectic, rhythm, combinations, with limitation and error as negative species.

The primary aim and end of efficiency is subpersonal surplusage

or positive non-equivalence. This is the common essence of value, happiness, blessedness, gain, and even of more special and concrete terms such as wealth, fame, food, and life.

In the excess of the result over the datum, the idea of the one to the many, or of the group, appears again. Hence many-to-oneness has two principle aspects (1) that of the means or method (2) that of end or result. In this double way, in the realm of efficiency, the fundamental principle is the group.

By multiplicative principle is meant the group with the aspect of force or dialectic which is inherent in it made prominent, and hence suggestive of the different forms which the group assumes and of the results which it produces.

In order to apply the above scheme of toolage, it is important explicitly to recognize that this scheme is built on a full recognition of the principle of limitation. Limitation is primarily subpersonal in nature, and multiplicative in its properties.

Hence for efficiency purposes the totality of things is to be regarded as composed of two giant opposed multiplicative systems, positive and negative, the one which produces positive results being, on the whole, dominant.

EXERCISE 19

1. Three men working independently in one year earned \$3000, \$3500, and \$4000. After combining as a corporation they together earned \$20,000 in one year. State the degree of efficiency involved, and some of the Efficients which might operate to produce this result.

2. A blacksmith and a wheelwright together can make a shovel for \$2. This can be made in a factory for 20 cents. State the degree of efficiency involved in the machine made tool and the sources of this efficiency.

3. State the principal Efficients which make the aëroplane possible.

4. Answer the same for the dirigible.

5. What Efficients are combined in the modern processes of spinning threads and yarn as compared with the former use of the distaff and spinning wheel.

6. What Efficients are combined in modern methods of weaving cloth as compared with more primitive methods.

7. State in both efficiental and more conventional terms the advantages in using garments made of cloth as compared with the use of bark or the skins of animals as clothing.

8. Multiply 5 bu. 2 pk. 5 qt. by 17. Also multiply 427 qt. by 17. Estimate the labor efficiency of the second process as compared with the first. Name the prime sources of this increased efficiency.

9. One steamer crossed the Atlantic Ocean in 9 days 10 hours, and another steamer crossed by the same route in 5 days 2 hours. Compute the time efficiency of the second as compared with the first steamer. State some of the possible efficiental sources of this efficiency.

10. A herd of cows when fed in a more or less haphazard fashion produced $3600 worth of milk in one year at a cost of $2200. Later, when fed at the same expense with rations compounded scientifically, they produced $4800 worth of milk in one year. State the degree and sources of the efficiency involved.

11. In a certain poultry establishment, the 15,000 hens which are kept are divided into groups of 100 each. A record is kept of the daily food and number of eggs laid by each group. When the number of eggs laid by any one group falls below a certain standard, the food of the group is changed. What Efficients are involved in this method of conducting the business?

12. Instead of paying $600 a year for a teacher for his children, $400 for a private watchman for his property, and $100 for upkeep of roads near his property, a man pays $108 taxes. The financial efficiency of the second method is how many times that of the first? State the fundamental sources of this efficiency.

13. Give an instance where a number of Efficients are in such close fusion as to have largely lost their individuality.

14. Express in terms of the Efficients the meaning of each of the following conventional categories of success: foresight, energy, prudence, reliability, popularity, punctuality, tact, sagacity, thrift, industry, health, coöperation.

CHAPTER XVIII

APPLICATIONS

Many more or less concrete applications of the principles arrived at have been given incidentally in the preceding pages. It will throw new light on these principles and be of service in other ways if a more systematic, though brief application of the efficiency methods advocated be made to specimen departments of human thought and endeavor. Since all applications to the world as it is must, in the ultimate analysis, be made by persons, we first consider the relation of the principles under discussion to the department of psychology.

PSYCHOLOGY

Memory as an Efficiency Agency

If a person can recall by act of memory the way in which a given word is spelled, he is often saved the labor of going to the dictionary and looking up the word. In this and similar ways one's memory is a means of obtaining economy and efficiency. As such it may have very great, even infinite or absolute values. For example if a person can remember, in case of need, the methods to be used in resuscitating a drowned person, memory may be the means of saving a life.

It should also be added in this connection that memory is groupish and efficiental in its organization. For instance facts are stored up by the mind in groups and recalled by means of this group relation (called the association of ideas). Hence a knowledge of the multiplicative principle in its various ramifications should be an aid in improving the memory subjectively. Objectively, also, a grasp of the general efficiental organization of the universe will be an aid in recalling and using any part of this whole.

Reason as an Instrument of Efficiency

If a savage man observes an animal floating down a stream on a log, he may infer that he himself can be transported in like manner, and that thus he may save the labor involved in walking on the land or in swimming. This one act of inference, when fully developed,

becomes the whole science and art of navigation, and this simple instances of the saving of labor becomes the vast and often transcendent fruitage flowing from world wide travel and commerce. Hence the reasoning faculty in man is an efficiency organ of great importance. Like memory it acts both subjectively and objectively by the group method in some form. Thus, after facts have been arranged in like groups, we may often infer that what is true of one member of a group is true of the other members also. Hence the reasoning faculty in man should be greatly improved in grasp and action by explicit knowledge of the efficiental group theory in its various aspects.

Efficiencies in the Various Senses

The faculty of perception in its elementary form is so primal that it is difficult of analysis. However it is evident that in the act of perceiving, a person is made aware of uniformities and diversities and hence of groups, both in himself and in the external world. To a certain extent also in an act of perception, the observer arranges objects and facts in groups. Another of the services of perception is to put the perceiver directly or potentially in useful relations with externality of various kinds. In this respect the senses may be arranged in an important order of materials scale. Thus at the lower end of the scale, the sense of touch gives notice, as a rule, of somewhat vague and small externality, while, at the other end, the sense of vision, by acquainting us with the fixed stars for instance, reveals a highly organized externality extending for untold millions of miles from the observer.

It is to be noted also that the organs of sense act by the aid of various auxiliaries which are multipliers or group organs in essence. Thus the sense of vision operates through a lens embedded in the eyeball, the sense of hearing is aided by bones which act as multiplying levers, and touch by hairs and papillae serving in like manner. Similarly the efficiency of perception is often increased by external objects used as auxiliaries and which concentrate or multiply the data apprehended, illustrations being the telescope, microscope, galvanometer, sounding board, and microphone.

Other Psychic Powers as Instruments of Efficiency

In like manner it might be shown that the imagination, will, and feelings are efficiental instruments of different kinds, and capable

of much improvement by efficiental methods. In this connection it may be remarked that what is often termed common sense is perception of, or judgment as to, primal groups in the efficiency organization of things, and may be cultivated in the ways indicated for the more conventional psychological powers which have been mentioned.

The Group Principle as Fundamental in Psychology

It is often desirable to be able to reduce mental processes, if only approximately, to some single primal principle. Various mental categories have been suggested for this central principle, the most prominent being feeling, cognition, or volition. But it is easy to show that each of these three mental categories contains or implies the other two. For example both action and feeling in some form are essential elements in cognition. From our point of view the central or essential element of mind is the multiplicative, or group principle operating within the domain which we term personality. Feeling, cognition, volition, and other mental processes are certain specific forms of this central principle. According to efficiency convenience they may be regarded either as somewhat distinct faculties, or as the modes in which a unitary mentality acts. A central mental principle like that suggested has important uses in that it enables us to apply our knowledge of groups to all mental processes and phenomena, and, for example, to use marginal reuse of various kinds in dealing with them.

Appreciation of Diverse Psychic Processes

This unitary category also has the advantage of aiding us to realize psychological processes widely different from our own, as for instance those of children, primitive peoples, and the lower animals. Since all these processes are multiplicative and efficiental in nature it is easy to comprehend that some of them in some particular, may be far more highly developed than our own corresponding powers, and also that animals, for example, may have some mental instruments or faculties very different from those of man.

This instrumental view of mind also gives us added insight into the processes of what we term genius. Thus we are enabled to realize, for instance, that every really great man is master in some form of part or all of the fundamental efficiental processes. Colum-

bus had the externality and directive faculties to an unusual degree, and Napoleon possessed great powers in the use of the group principle in its different forms. We are also able to understand the simplicity which characterizes many great men, since the mastery of even one item of the fundamental efficiency processes may mean vast power in the more concrete world. This view of genius also enables us to comprehend the fact that a great nature may be fragmentary, contradictory, and have other serious faults, and yet produce vastly useful results.

If mind is regarded as essentially a multiplicatively grouping entity, it is made easier to conceive that it, or the germs of it, may be present in the primordial atoms, monads, or substance which evolution presupposes; or, to state the matter in another form, we may conceive of both mind and matter as developing from some more primal form of the group principle by radial evolution.

Deductions from Efficiental View of Mind

Specific mention should be made of certain deductions from the above views concerning mind, which are capable of immediate and general application.

(1) The view presented makes obvious the importance of acquiring the power of rapidly and accurately reading the efficiental powers and habits of other persons, especially their externality scope, their multiplicative grouping faculties; and their surplusage tastes and preferences. In other words the analysis of the mind in terms of the Efficients should be made primary and essential and that into perception, memory, etc., secondary and auxiliary.

(2) The above view also shows in a general way how the psychological faculties, as conventionally regarded, may be greatly improved in grasp and efficiency. Thus if the mind realizes the general efficiency organization of things as a whole, a multitude of new means of perceiving uniformities and diversities will be supplied and the faculty of perception will also be stimulated to greater depth and acuteness of action by the realization that every uniformity or diversity may have enormous mutlipliers with corresponding values or dangers. So courage is greatly increased by noting that, owing to the groupish nature of the universe, danger comes only in spots, and that, because of the dominance of the positive multiplicative system, evil tends in the long run to cancel itself. So, also, we can conceive

of memory and reason, feelings and conscience as developed and improved by efficiental means.

(3) The above view also makes it readily conceivable that with practice, and often by the method of dialectic, new efficiental mental organs and powers may be developed. An example is the species of pleasure which may come to be associated with an efficiental process or fact. Similarly new and higher processes of memory, imagination, and moral judgment may spring from efficiental data. We may conceive also that powers of mental grouping and obtaining results may be generated which shall transcend such terms as perception and reason.

EDUCATION

Man's Activities Statable in Efficiental Terms

If the daily activities of a reasonably intelligent farmer, for example, be examined, it will be found that by far the greatest part of them consists of acts and elements such as have been described in the preceding pages of this book. Thus by him or under his supervision in a single day scores or hundreds of acts of reuse will be performed. These will include the reuse of tools, machines, soils, animals, the farmer's experience, and that of other persons. Similarly the day's activities will contain many uses of auxiliary objects; of groups of objects; of multiplicative groups either in the form of organization as directly applied to the work of the farm, or of external multiplicative groups such as the post-office system, railroads and various social and political organizations. The farmer will need to use symbolisms in keeping his accounts, in buying and selling, and in directing his subordinates. Various rhythms will be present in his work due to the alternation of day and night, the change of seasons, and the use of processes such as the rotation of crops. His work will also include many kinds of directive and of utilized motion and force, and will be held in check by various forms of limitation and error.

Also if his life is to be successful in the higher sense, the farmer must cultivate uniformity in various forms such as accuracy, truth and justice; externality in the shape of sympathy and breadth of view; and other Efficients in various social and moral forms.

The successful agriculturalist also needs a certain amount of technical knowledge. Thus a considerable and more or less minute

acquaintance with scientific facts concerning plant and animal life (biology) will evidently be of service to him. The efficiency of his work in dealing with fertilizers will be appreciably increased by a knowledge of chemistry, in dealing with machinery by a knowledge of physics, and with soils by a knowledge of both these sciences. An acquaintance with psychology or the principles of the mind will be useful to him in commercial operations and in dealing with his employees. But the larger part, if not all, of this more technical knowledge is found on examination to consist of the Efficients clothed or combined in special ways. Hence we may say by way of summary that 99 per cent of a farmer's work and life consists of efficiental processes such as have been described in this book.

A like examination would show that by far the larger parts of the activities of other callings, as, for instance, the legal profession or electrical engineering, are, in like manner, primarily and essentially common sense efficiency processes.

Woman's Activities in Efficiental Terms

It is equally important to realize that the domestic work of woman has a similar significance. Thus in the work of the kitchen, groups of many kinds are used in combining articles of food to make meals and dishes, in storing appliances and supplies, and in systematizing activities. Externalities are employed in the form of utensils, heat (to produce diversity and re-multiplicative-grouping), the experience of other persons, and the application of departments of knowledge like chemistry, physics, biology, agriculture, and ethics. Reuse, either entire or marginal, occurs in plain forms such as the repeated use of a recipe or a menu. This principle is also applicable in less obvious ways as when articles of diet of different rates of digestion are combined in a given meal, so that the stomach can, in a measure, digest them in succession and thus reuse its powers. Rhythms are to be observed in the form of daily, weekly, and seasonal changes of work, and of articles prepared for food. Directive occurs in many different forms and is capable of being made a source of efficiency in many new ways each day. Every good dish, meal, day's or week's diet is a pluralism or combination of Efficients and in the preparation of these combinations a knowledge of pluralisms may be utilized and developed constantly. Similar applications in kitchen

work of other Efficients such as units and multipliers, orders of material, and dialectic might be indicated.

An examination of other domestic activities would show that they are statable in terms like those just given for kitchen work and have a like wide significance. For instance in house furnishing and ornamentation, the externalities and other Efficients which may be employed are almost without limit. A similar statement applies to dress and personal adornment. The same essential artistic and efficiental principles may be used by a woman in trimming a hat that were applied by Milton in writing his Paradise Lost. Social usages are full of groupings, regroupings, symbolisms, and other Efficients. This fact is illustrated by various social assemblages, by the reciprocal reuses at these of the efficiencies in which each person present excels, by letters of introduction, calling cards, bowings, tipping of hats, shaking of hands, and by all that is included under such terms as tact, courtesy, and friendship.

Education Should be primarily Efficiental

Since life in general and work in different callings consist essentially of efficiental activities, it follows that education should consist primarily of the setting free and development, in the most economical and effective way, of the efficiental powers of a person. This view of the matter, when fully realized, should lead to improvements in the processes of education, and should increase both the technical and general efficiental value of study. For instance, it suggests as a basis of educational work, a careful examination of the child's mind with respect to its powers and habits of grouping, multiplicative grouping, reuse and the other primary principles of efficiency. It indicates the value of training and selecting teachers with reference to their ability in developing the above powers in children.

Value of Different Studies in terms of the Efficients

It also throws new light on the relative educational value of different subjects of study and on the methods to be followed in obtaining the maximum of results from each study at the minimum of cost. Thus in linguistic study it is of fundamental importance to realize the efficiental elements which enter into the subject. In language as a subject of study, besides specific principles of symbolism, are found striking examples of reuse, groups, multiplicative

groups, and all of the other Efficients. Thus as an example of economy and reuse we have the fact that by reading a book of travels which costs a dollar (and often in fact may be had free) we may obtain in a few hours many of the results which could otherwise be procured only by an actual journey costing hundreds of dollars and weeks of time. Marginal reuse is illustrated by the employment of prefixes and suffixes in connection with word stems. A book is a multiplicative group in many ways. Since a large number of concrete words may often be replaced by a single abstract word, the use of the latter illustrates the group and order of materials principles. The comparison of various grammatical forms in different languages is a valuable training in the use of diversities and externality. The fact that language has been built up in a complex and bizarre fashion makes it a useful representative of all efficiency pluralisms.

It is well to note in this connection the highly condensed form and small cost of the subject matter of linguistic study as compared with the materials employed in the study of branches of science like engineering and agriculture. As a further result of this difference, language study as a part of education has the advantage that pupils can be trained in it in large groups and hence at small cost.

Most of the Efficients are present in mathematical subjects also, and often in a much more explicit and detachable form than in languages as ordinarily studied. For a further development of this idea the reader is referred to the various textbooks and articles by the author on mathematical subjects.

As a subject of educational study, physical and biological science when compared with language, has the advantage of being in large part directly related with concrete objects and hence being free from the errors which arise from the abstractions and detachment of language. In science also, as compared with mathematics, the Efficients often appear in more individual, tangible, and striking forms. Instances are externalities in the form of various kinds of apparatus, the units and multiplier characteristic of the experimental method, and directive of various concrete and abstract kinds.

Increasing the Efficiency Value of Studies

The above analysis not only indicates methods of determining the relative values of different studies, but also suggests methods of giving greater usefulness to such studies as are selected and pur-

sued. For instance it is made clear that in the study of science, the Efficients involved in a process should be made as explicit as possible and that much may also often be gained by giving a leading place to the functions and results of a process. Frequently it will be best to bring out the advantageous results (or efficiental functions) of a process first in a qualitative or mass form, and later to take up quantitative methods as a means of making results more manageable and increasing them in various ways. Thus in order to interest pupils and enable them to get their efficiency bearings, it is an advantage in the first presentation of the matter, to develop the various kinds of useful results which can be obtained from water falling over a dam. Later it may be shown that the utility of these results will be greatly increased by learning the laws of falling bodies and of the transformation of energy, and by thus reducing the processes involved to an exact commercial basis.

Informal Efficiental Training

The above view of education greatly enlarges the means by which the most fundamental and important part of education, viz. the development of the child's efficiental powers, may be carried on. Charles Dudley Warner has said that a training in doing chores has been an essential part of the development of every great man. The truth in this statement is based on the fact that doing chores consists essentially in grouping common objects, in reusing and directing them, and in other primary efficiency processes. Play and games may also be analyzed into these elements and hence may be made to have fundamental educational values. The study of music, etiquette, or even of changes in the weather may be shown to have similar uses. In fact, every field or species of occupation, however informal, when properly viewed will prove to be a means of mastering efficiental methods. This explains the fact that many men have been great who have been without special technical training of any kind, but who have thoroughly grasped the primal efficiental basis of life. It also explains why, after this basis has been mastered, the technique of one department after another may often be rapidly assimilated with little or no outside help.

Efficiency Treatment of Current Educational Problems

The above principles of education are also an aid in solving important special problems in educational methods. Thus efficiental

ideas throw light on the matter of attention and interest, on methods of discipline, on the functions of memory, on the relations of books to things, on the place and nature of heuristic methods, and on the best ways of treating both defective and exceptionally able pupils. It tends to culturize vocational studies and vocationalizes culture subjects.

General System of Efficiental Education

Perhaps the view presented should also lead to a radical reconstruction of the general system of education. Thus we can conceive of a study which underlies and in a measure includes the various conventional branches and which essentially consists of the science, art, and philosophy of obtaining results; that is, of efficiency.

The germs of this discipline could be instilled in the young mind during the processes of elementary training; its main outlines would be a specific, formal branch of the high school course; in college, the thoroughly developed subject could be taught along with the leading facts and processes of every department of knowledge and action; in the university, the system of efficiency principles which had been thus mastered could be applied in learning every possible fact about some one specialty and in developing that specialty to a new degree of perfection. It would seem that such a study would greatly accelerate and deepen education processes and enhance their values.

SOCIOLOGY

Efficiental Solution of Sociological Problems

From various points of view it is evident that the principles presented in this book have important applications to the subject of sociology. Thus the adoption of the views under consideration apparently would contribute toward the solution of certain current social and political problems. For if it became evident that all callings are analyzable into the same efficiental elements and that consequently the occupations of cook, engineer, and lawyer are, for example, not unlike in certain essentials, many social distinctions which now hinder progress and limit man's happiness, would be largely modified and perhaps in a measure would disappear. Among these are the distinctions between higher and lower social classes, and between noble and menial occupations. Similarly the distinction

between work and play would to a considerable extent pass away. For the two are resolvable into an activity which is back of and includes both of them as details. These changes would make easier the distribution of that residuum of work necessary in any social system; or, to express the matter in another way, the solution of the difficult and fundamental problem of the distribution of the rewards of human activity would be facilitated. In this connection a full realization of the many costless or inexpensive sources of efficiental pleasure, power, and wealth which are open to all, is of prime importance.

The principles advanced in this book also have an application to more special problems. For instance, with respect to race suicide, it is made clear that, apart from other considerations, in children are found Efficients and efficiental values in their highest and best forms. It will be realized that these far transcend any values and pleasures obtainable through elaborate dress, expensive foods, constant travel, and other forms of personal luxury the fondness for which is now dwarfing so many families.

A grasp of the Efficients which constitute the essence of every form of work would tend to prevent the narrowing and blighting effects of the division of labor in its extreme forms as now practised. For every item of work however specific is seen to be resolvable into and related to elements of activity and life of the widest possible significance.

The relation of the general conceptions under consideration to the problem of forms of government has been considered in various places in the preceding chapters; other like applications are indicated by such terms as efficiental wealth, labor, equality, freedom, trade and competition.

An Efficiental Sociology

The efficiency principles which have been presented suggest not only improvements in existing social methods, but also more fundamental reorganizations. For these principles, when fully applied, mean, on the one hand, a new and more profound personal freedom and development and, on the other hand, point to equally new and strikingly effective methods of coöperation and social organization. For if each person realizes the fulness of efficiental wealth available to him, he will feel free and will be eager to appreciate and develop this, no matter what views others may hold, and this

course when adopted by individuals will lead to the formation of new methods of social coöperation. The efficiental forces tending to social reorganization will also act in more general ways.

It is estimated that, at the time of the highest development of Greek civilization, every Greek freeman had on the average five helots or slaves who worked his farms and mines and performed virtually all other manual labor for him. Hence the Greek freeman had leisure to think and discuss and to develop the more abstract efficiental side of his life, and thus to create an efficiency in art, literature, and philosophy which has never been surpassed. The Romans had equal leisure, but largely wasted their surplusages in senseless and debasing luxury. Owing to the use of coal and machinery some modern nations have twenty steam and electric slaves at work for each adult male citizen, that is they possess four times as much servile labor as the Greeks had. These modern forms of surplus energy and the leisure obtained through them, are being employed in part as the Greeks used theirs, in part as the Romans theirs, and partly in an earnest and organized search for still greater leisure and power. Hence the raw material is already at hand for a far higher and more efficient civilization than the Greeks enjoyed. When the efficiental activities already in sight shall have been fully coördinated and developed, the result will be a civilization far surpassing any the world has yet seen.

BUSINESS

Analysis of Scientific Management in terms of the Efficients

In recent years much attention has been given to the methods of conducting business which have been termed scientific management and which were largely originated by Mr. F. W. Taylor of Philadelphia. Briefly summarized the principles of scientific management are as follows:

1. The dissection of any process of work (as brick laying) into its elements or motions. (In terms of the Efficients as presented in this book, this means the application of diversity and of units.)

2. The study of the most economical and efficient way of performing each of the elements in a given piece of work. (This includes the application of groupism, multiplicative groupism, directive and most of the Efficients.)

3. Standardizing the work; that is, determining how much work

a good (or bogey) workman following the most approved methods may be expected to perform in a given time. (This means a study of limitations and specific quality, and the consequent formation of uniformities and multiplicative groups.)

4. Instruction of workmen in the best methods of work. (This means an externality and scale of teachers, and a reuse of knowledge.)

5. Selection of the workmen best adapted for each given kind of work. (This includes reuse of work done by nature, use of diversity and specific quality, and hence in one aspect of limitation.)

6. A planning, organizing, and instructing department of workers (i.e., externality, orders of material, multiplicative groups, reuse).

7. Charting and routing of each extensive and complicated piece of work (i.e., use of linear multiplicative groups, graphs, and other multiplicative groups and symbolisms).

8. Sharing the increase in profits (i.e., a recognition of positive surplusage, externality, and uniformity).

By use of the above methods, for example, the amount of pig iron handled by one man in one day has been increased from 12½ tons to 47 tons, and the number of bricks laid from 1000 to 2700.

Applications of the Efficients are Wider

From the above analysis it is evident that the principles presented in this book underlie, surround, and include, so to speak, the methods advocated in the above system of scientific management. Hence if the individual workman is familiar with the Efficients as such, each day numberless ways will occur to him in which he can, in addition to the above prescribed methods, use the groups, unit and multiplier, reuse, directive, and other primal means to efficiency. The same is true of the employer, superintendents, and foremen.

Furthermore the more general efficiental point of view also makes plain, for instance, that systems of accounting are but systems of multiplictive grouping according to the categories of time, place, material, personality, etc., and that a mastery of the principles of multiplicative grouping opens the way for the development of a special system of accounting particularly adapted to any given business. The same statement applies to all forms and kinds of system and organization included in any given business.

Hence the efficiental principles stated in this book relate not only to the processes of manufacture, but to those of buying and selling

as well. For example, the reuse by other agents of the experience and methods of the most successful selling agents is provided for, as well as the grouping, standardization, and codification of such experience and methods.

Uses of the Efficients are More Fundamental

It will be seen that the principles presented in this book will also necessarily modify the methods and processes of business in more fundamental ways, as, for example, by modifying the views which men hold of life in general. Thus a sane view of the aims and ends of life will diminish the passion for senseless luxury, and hence transform the processes of manufacture and business which cater to such luxury, and also simplify the problem of profit sharing.

The ideas here presented also aid in a grasp of the underlying principles of business in other ways. For instance every form of business is in some way a case of intake and output, that is of externality and re-multiplicative-grouping, with surplusage results. Hence a specific knowledge of these efficiental processes is of use throughout the technique of any species of business. Again if we view the universe as in a state of almost exact balance between gigantic positive and negative multiplicative systems, the world is realized as a solid domain of opportunities of high order for him who is able to perceive and utilize such opportunities. Yet the same point of view makes clear the wisdom of making it a rule to be content with large sales and small margins of profit, while at the same time ever being on the lookout for a sudden explosive opportunity due to some extreme action of the multiplicative principle. The law of multiplication also makes peculiarly evident the importance of holding reserves and in other ways being prepared for sudden explosions in the commercial world.

Primary Aim of Business

It is of equal or greater importance to remember that the aim of business is the making of positive surplusage in every possible way, with special emphasis on those ways which are broad and fundamental. This method of conducting business brings the largest and best returns to the individual business man as well as to society in general. Thus business is carried on most profitably and successfully when both parties to a bargain are the gainers thereby. Also an employer conducts his business most successfully when he considers not only his technical business relations with his employees but

also their general welfare and interests, including their food, recreation, health, education, morals, and religion. In like manner employees will receive the best pay and largest rewards of all sorts when they take an interest in promoting all the elements and relations of the business in which they are engaged.

Similarly an intelligent commercial house perceives that its returns are in proportion not merely to the direct value of the goods sold, but to the surplusage of all kinds conveyed to their customers, including, for example, the pleasure and instruction imparted through works of art exhibited in salesrooms, through the artistic presentation of goods, musical entertainment, courtesy of employees, and through well served food supplied in attractive lunch rooms.

A Universal Practical Instrument

Finally if we look at the so-called practical world as a whole the system of Efficients as here presented forms a kind of universal tool or instrument, applicable in some form to every problem or opportunity that may arise. Thus the use of diversity, units, and groups supplies at least an initial method by which to attack any situation. Later, by marginal variation and adaptation, the whole system of Efficients forms a completer instrumentalism for dealing with the developed problem.

ART

General Statement

Since art is a special and somewhat technical form of symbolism, the discussions in Chapter X show in general how the principles under consideration in this book have an application to various kinds of artistic work. If the matter be looked into in more detail, it is found that grasp of the Efficients helps to determine the subject matter most desirable in works of art, the principles and forms which should be applied to this subject matter, and perhaps will suggest new and more efficient species of art. The result on the subjective side should be greater ease in the mastery of art, added pleasure and profit from the technical appreciation of art, and still broader results owing to the grasp of the general efficiental meanings and values of any given artistic work.

Efficiental Subject-matter

As to the subject matter of art our point of view leads to an emphatic preference for constructive, optimistic themes. Prominence is to be given to that which is noble and uplifting; not only to that which is beautiful in the more sensuous aspect, but even more emphatically to those forms of beauty which are capable of the most efficiental treatment for the most efficiental ends. For instance the theory in hand makes clear that the dominant part of Dante's Divine Comedy is the Paradiso, and that the greater attention given to the Inferno is due to a morbid form of interest, and to a lack of a grasp of life in all of its relations and in right perspective.

Efficiental Elements in Highest Forms of Arts

As to the forms of art, the principles under consideration aid one in realizing, for instance, that mere profusion of ornamentation cannot produce art of a high order. For such profusion means only a set of units without multipliers; or even worse, an assemblage of units which largely cancel and destroy each other. In contrast with this method the unit and multiplier principle would suggest the use of a few units of a high order of artistic excellence placed on a massive and constrained background which serves in various ways as a multiplier of the given units. Such a combination recognizes the limitation that is in all things, but it also has other more positive values. It often means economy in material used and hence in cost, and in energy expended by the observer in learning to understand and appreciate a given work. More important still, large margins or backgrounds constitute an externality of space in which self activity can function, and at the same time challenges and stimulates this self activity to fill in details, and to propagate and develop the given units according to personal taste or need. Similar results are obtained by an atmosphere of twilight or by any "noble dimness", in connection with one or more well developed units of art symbolism.

In like manner other Efficients are often found to be fundamental sources of power in works of art of a high order. An important case of the use of groupism in art is the combination of several successive phases or elements of an act or process in a single picture or statue. This principle is strikingly illustrated in many of the works of the sculptor Rodin. So in literature the mere use of "words, words, words" is additive, while the use of "local habitation

and a name" is multiplicative. This principle is widely illustrated by the almost paradoxial importance, in any linguistic statement, of the small words, like since, thus, hence, for etc., which are employed to denote space, time, number, cause, and other prime group relations, and whose right use is essential to a clear presentation of the multiplicative group essence of a subject.

An examination of works of art of the highest order also seems to suggest that each of these contains some expression or indication of infinite externality. This characteristic is illustrated, for instance, by the Sistine Madonna as contrasted with the Madonna of the Chair. In the first of these paintings, the mother of Christ is gazing at something distant and inspiring, perhaps at the epoch-making work to be performed by her son, or at the source of this. Another illustration of this principle of a widely different type is the avenue of trees receding in the distance in Hobbema's Avenue of Middleharnais.

Efficiental Elements in Other Departments of Art

This suggestion of infinite vistas of expansion and development is also found in various forms in literature of high order. In fiction one of the most subtle and powerful of these forms is that superpoise of the author by which he is enabled to project his characters against the background of the general efficiental organization of things, to measure them thus in various ways, and to suggest their further development. In accordance with this principle also we find the highest type of literature, not in Shakspere, in spite of his marvelous profusion of local excellence, nor in Dante's Inferno, so vivid in its artistic qualities but so negative in its main content, but in Dante's Paradiso with its spiral unlimited development of the individual soul.

Similarly other of the Efficients are often present in great works of art in more striking form. When thus present, they not only govern the form, but also constitute the most important part of the subject matter. Hence in them form and content coalesce.

It is well to note in this connection that each of the fine arts contains certain characteristic efficiental elements. Thus orchestral music of a high order is a swiftly changing combination of systems of groups and other Efficients which is highly educative as well as the source of the noblest pleasure.

Schools of Art

It is evident that in such conceptions different schools of art find a common principle or essence; that in the beauty of a primal and perhaps ultimate efficiency, realism and idealism, truth and beauty as ordinarily understood, utility and art merge or melt together. Thus the highest art expresses a pure surplusage which is independent of time and of any eccentricity of personal or national mood. This form of art is approximated by an object like the Parthenon which gives the impression of being neither large or small, young or old. This pure efficiental art may be regarded as at times acting in special channels and thus, on occasion, giving rise to certain individual forms or schools of art, each having its own peculiar efficiencies, examples being the classical, romantic, and impressionist schools.

Advantages of Efficiental View of Art

This view of the matter leads to varied and often profound marginal reuse in the appreciation of art. It also greatly enlarges the scope of artistic expression and pleasure. Thus the furnishings of a home, while in themselves simple and inexpensive, may be made to express artistic data of the first importance, as, for instance, the various efficiental principles. The same statement applies to the architecture and the external surroundings of the home, to dress and personal adornment, and to speech and personal manners. So also Ruskin has shown that the sky and clouds form a changing picture gallery to the properly trained eye, and he and others have made clear that the same is true of the landscape both as a whole and in its details. The ideal aimed at in the whole field of relatively unconventional art is that the individual, the home, the town or city, and nature shall each radiate positive surplusage in a continuous esthetic form.

The principles arrived at in the preceding discussion also enable us to conceive of the possibility of new and higher forms of art than those in current use. Such forms may express efficiental themes to efficiental sensuous powers in efficiental ways for efficiental ends, more explicitly and effectively than do the current types of art.

ETHICS

Efficiency in Christian Ethical Principles

If we examine Christ's injunction to return good for evil we find that, apart from other considerations, the principle involved, in the long run, is a highly efficient one, and far more so than the principle of retaliation. For the fruits of retaliation are single or local and negative, while the mode of life implied by the forgiveness of injuries is positive and means a continuous radiation of positive surplusage in spite of all injustice or misunderstandings, and hence is superior to retaliation in a dual infinite way in its general efficiency properties. An examination of the other paradoxical ethical principles taught by Christ would show that they have similar efficiency properties.

Efficiency in Various Fundamental Principles in Ethics

In like manner if we consider the various ideas which have been proposed, from time to time, as a fundamental principle of a system of ethics, we shall find that each of the more meritorious of these possesses efficiency properties of a high order. This is true of such primal ethical concepts as knowledge, harmony, moderation, benevolence, sympathy, utility, altruism, complete life, and happiness. Almost or perhaps all of these it will be noted, directly or indirectly, mean fundamental groupism of some sort.

Efficiental Analysis of the Different Virtues

In like manner it may be shown that the different virtues, so-called, have efficiental values of the first importance. Thus for instance modesty is a realization of diversity, externality, and limitation, and has the uses connected with these principles. Truthfulness has the values which come from a correspondence (or uniformity) between fact and symbol; patience those of externality and dialectic; sympathy and generosity those of externality and groupism. In fact the virtues may be arranged in groups according to the Efficients prominent in them. Thus we have

(1) The *externality group,* containing altruism, love, self-denial, modesty, reverence, patience;

(2) The *directive group,* containing charity, generosity;

(3) The *order of materials group,* of good will or motive, character, intention;

(4) The *uniformity group,* of honesty, truthfulness, justice;

(5) The *diversity and limitation group,* of chastity, temperance.

The more conventional classification of virtues in groups relating (I) to the individual, (II) to other persons and (III) to God, may be regarded as essentially a grouping with respect to different degrees of externality.

Solution of Ethical Problems

The above analysis of ethical principles in terms of the Efficients, aids in the solution of certain ethical problems. One of these is the problem of the relation of self-interest to altruism. In cases where these two principles seem to be in conflict the proper course to follow would seem to be that which would produce the greatest and highest surplusage independently of personal considerations. Sometimes self-development, and in other cases a primary attention to the interests of others will do this. In order to decide between these courses in specific cases, much study of the peculiarities of the given situation is necessary, such study being conducted as far as possible in terms of the fundamental categories of efficiency. At times a point is reached where self-interest and altruism merge, or are lost, in the deeper principle of the subpersonal or nonjective making of positive non-equivalence in all possible ways at all times.

Advantages in the Efficiental View of Ethics

So also from the efficiental standpoint, certain new developments of the conventional virtues are possible, and also perhaps the formation of what may be termed new virtues, with new vices corresponding to them. Thus any perfected mastery of an Efficient might be an efficiental virtue; and any habitual or intentional violation of the same, an efficiental vice. We are also enabled to arrive in this way at the conception of a summun bonum or goal of all effort such as has been indicated in other places.

A farther advantage of the above efficiental treatment of virtues, vices, and ethical principles seems to be that it can be embodied in systems of education without running counter to personal or religious bias in such matters. It also aids in the mastery and application of these principles in other ways. Rightly understood it adds a new imperativeness to moral laws.

RELIGION

Efficiency Values of Religion

Whatever his views as to conventional religions and doctrines, every one must admit that abundant evidence exists of the efficacious results which frequently follow the use of that vast and unbounded externality implicit in every religion worthy of the name. The word religion means a relaying or regrouping, and this regrouping in connection with and under the influence of infinite externality often leads to extraordinary efficiency results. These have many times been illustrated in the phenomena of religious conversions.

The efficiency values in religion have however often been greatly obscured by the abuses which from time to time have grown up in connection with religious principles. These abuses have frequently, indeeed, been of a gravity which corresponded to the enormous power for good inherent in an efficiental use of infinite externality, and constitute a striking illustration of the law of rhythm and backeddyism.

An examination of the conventional doctrines and ceremonies which characterize various religions would show that each of these technical details and outward forms is usually the source of certain special efficiencies though often marred in practice by drawbacks and evils. But the detailed examination of these matters lies beyond the scope of this book.

Advantages of the Efficiental View

The above method of viewing religion possesses certain advantages, which should, at least, receive mention. For instance it opens the way to a free and aggressive development of the primary and fundamental values in religion apart from those questions as to details and outward forms which, in the past, have been the source of so much bitter conflict. Perhaps it makes possible the study of such values as a part of a general system of education.

Possibly also it should lead to the development of some religious ideas into higher and transcendent forms. Thus as an outcome of it we can conceive of new ways of clothing infinite externality with auxiliary efficiental toolages. These in turn may react on the principles which they aid, the result being fruitages or categories of value of which we cannot now form an adequate conception.

General Outlook

The above statement concerning the efficiency meaning of religion, concludes our illustrative application of the principles arrived at in the preceding chapters, to certain specimen departments of human knowledge and action. These prime agents of efficiency might, of course, in like manner be applied to other departments of life and thought. The principle of limitation will, apparently, be always at work, but if the universe be regarded as a combination of two multiplicative systems, one positive and the other negative, in proportion as the negative one is overcome, the positive system will predominate in accelerating multiplicative fashion, with corresponding fruitages which cannot be fully anticipated.

EXERCISE 20

General Review

1. On a given machine an efficiency engineer found that compared with what it should have been, the width of the cutting point of the tool was ¼, the speed was ⅓, and the amplitude of the stroke 4 times (i.e. the stroke overran threefold). Express the efficiency of the machine as a per cent.

2. A crop of 600 bushels of corn which might have been sold at 84 cents a bushel was fed to farm animals and was thus made worth $1120. If the labor of feeding cost $84, compare the efficiency of the two methods of disposing of the corn. What Efficients are involved in the second process?

3. When potatoes were raised without spraying, on a given field, it cost $4000 to raise a crop worth $6400. A crop produced in a later year was sprayed at an additional cost of $400 and was worth $8800. State as a per cent the efficiency of the first crop in terms of the second.

4. What Efficients are involved in the process of reducing fractions to a common denominator and adding them?

5. If a modern cotton gin and compress are operated by one half the number of men required by a former type and turn out twice as much baled cotton, express the labor efficiency involved.

6. A city factory was using 1500 tons of soft coal per year thirty per cent of which went up the chimney as smoke. Later, after installing a smoke consuming grate and by utilizing some of the gases which had gone up the chimney, the amount of coal was reduced to 1050 tons. State the degree of fuel efficiency involved. State also the Efficients employed and indicate the resulting dialectic.

7. What principles of efficiency are used by a housekeeper who buys eggs when they are cheap and keeps them in liquid glass till they are dear?

8. What principle of efficiency was involved when a primitive man in making a fire accidently melted copper and tin ore together and thus made bronze?

9. State in efficiental terms the different methods of making "a little go a long way".

Explain the efficiental meaning of each of the following:

10. There are more ways of killing a dog than hanging him.
11. Patience and shuffle the cards (Spanish proverb).
12. The early bird gets the worm.
13. Give him rope enough and he will hang himself.
14. He can't see farther than the end of his nose.
15. Fine feathers don't make fine birds.
16. By indirection find direction out.
17. All waste comes from isolation of some sort.
18. Sweet are the uses of adversity.
19. He has something up his sleeve.
20. Explain the advantage to a business man of keeping his capital as far as possible in a fluid state.
21. To a general of keeping his forces concentrated as far as possible.
22. Why is a street corner near the center of a city usually a superior location for a place of business.
23. What is the reason for the statement that it does not make so much difference what subject we study as how we study it.
24. In terms of the Efficients discuss the utilities in play. How could a knowledge of the Efficients be made a means of increasing these utilities?
25. State the advantages in having one man in a factory distribute the supplies to the machines before the work is begun, as compared with having the workmen who use the supplies go to the store room for them as they are needed.
26. Explain the efficiency significance of the motto "e pluribus unum". Also of "non multa, sed multum".

Explain in efficiental terms.

27. The advantage to a large firm in employing a general purchasing agent.
28. The meaning of the phrase, cross fertilization of the sciences.
29. The advantages and disadvantages of combining several small businesses as a corporation,

    (1) With reference to the owners of the business;
    (2) To the employees;
    (3) To the general public.

30. What Efficients make it possible to photograph a star which cannot be seen through the best telescope.
31. Explain the source or sources of efficiency in watching the eye of a boxer instead of his hands.

State the various Efficients prominent in the subject of

32. Arithmetic	33. Algebra	34. Geometry
35. Physics	36. Botany	37. Language study

38. Before the appearance of the Colorado beetle a farmer in raising potatoes spent $26 per acre and produced 87 bushels per acre. After the

appearance of the beetle, he spent $42 and raised 170 bushels per acre. Compare the efficiency of the two processes. State the sources of increased efficiency.

39. Napoleon estimated his own presence with one of his armies as equivalent to the addition of 100,000 common soldiers to its strength. Explain as far as you can the efficiency of Napoleon as compared with a common soldier.

40. Explain the sources of efficiency in the use of a bicycle as compared with walking.

41. Explain as far as you can the non-equivalence between the energy used in pulling the trigger of a gun and the energy in the action of the bullet.

42. It is estimated that the cost of human unskilled labor is 1000 times the cost of the same work when done by the best steam engine. Name the principal reasons for his difference.

43. State in efficiental terms the meaning of each of the following elements of success: honesty, intelligence, accuracy, self-reliance, loyalty, shrewdness, courage.

44. State in efficiental terms the advantages of knowing the cause or causes of an event. Also in knowing the results of a process before they happen.

45. Give the efficiental reasons for the following statement: At a certain stage in a process of thought a certain vagueness of thought is useful.

46. Discuss in terms of the Efficients the reservoirs of power unconsciously possessed by every person.

47. Discuss the meaning of the following statement: "Culture is that which is left after you have forgotten everything which you have learned."

48. Explain the advantage in not trying to remember everything but only where everything may be found.

49. Also in not trying to form once for all a system which shall include every detail and situation of life but rather in having the means of making a system in any given situation.

APPENDIX

A. THE CATEGORIES AND A GENERAL PHILOSOPHY OF LIFE

Preliminary Illustration

Beside the prime sources of efficiency which have been investigated in the preceding pages, certain other abstract and general instruments of thought and action have long been in use which it will be of advantage to examine, if only briefly. As an illustration we may take the category of space.

If a set of pigeonholes is used for any given purpose it is usually much more convenient to arrange the set of compartments in the form of a square or rectangle rather than in linear order. Among the advantages which result from the areal arrangement are the economy to eye and hand in using the system owing to the fact that the distance between any two pigeonholes is greatly diminished on the average. Also after long use of such an arrangement, other benefits arise such as the utilization of various positional symbolic relations and various other groupages.

Similarly other areas, as the wall of a room or the surface of a farm, in time are often instinctively realized as saturated with facile reciprocal externalities and highly condensed groupages of many kinds, from which various dialectics result which would not be possible with a merely linear object. Hence a surface is frequently, if not always, a special efficiency instrument of great power.

The above illustration may be extended to space of three dimensions. For if a set of drawers be constructed each of which consists of a rectangularly arranged set of pigeonholes (or checkerboard arrangement of some sort), we shall have a still higher condensation (or groupism) of parts, with a consequent higher manifold and more facile externality and a dialectic of cross multiplicative groupings. In this way we are brought to realize that space itself, from a certain point of view, is an efficiency instrument of great importance; that it, in effect is a multiplicative group system of three orders, manifold, continuous, and saturated with many resulting efficiency properties. This conception of space is an important

aid in mastering the properties of space as commonly understood, in conceiving of additional properties possessed by it, and in making the most effective use of all these properties as sources of efficiency.

General Statement

A like point of view holds with respect to other and perhaps all of the so-called categories, such as time, cause, reality, substance, and quality. These entities may often be usefully regarded as special instruments of efficiency. They may have been evolved in a somewhat haphazard way, but with respect to certain uses they have been developed to a high pitch of instrumental power, accompanied, however, by equally high degrees of danger and limitation. A study of the more important of these categories from this point of view will not only be of direct advantage but will be useful as an aid in removing certain fundamental and pervasive efficiency errors, such as the tendency on the one hand to overuse the category of personality, and on the other those of mechanism and equivalence reality. Hence such a study should be an aid to the appreciation and extended use of the broadest and most permanent kind of efficiency, as distinguished from that which is narrow and personal. It will also be an aid in relating the idea of efficiency to other approximately ultimate ideas, and realizing its place in a general scheme or philosophy of life.

The categories under consideration are so interwoven in meaning that it is necessary to discuss each of them more or less in terms of the others, it being noted that our main object is to obtain efficiency results, and that the development of a simple filiar system among the concepts studied is merely secondary and auxiliary to the main purpose. However the cancellations and other dialectic results which arise, will produce this filiar ordering of the categories to a certain extent.

Classification of the Categories

It will be an advantage first to make an approximate grouping of the categories to be examined. This classification will be made primarily with reference to certain leading Efficients. We thus obtain

I. The *Reality,* or *Uniformity Group,* consisting of the concepts, Reality, Substance, Matter.

II. The *Quantity,* or *Pure Group,* containing Quantity, Space, Number, Infinity, Zero.

III. The *Quality,* or *Diversity Group,* containing Quality, Time, Order, Negation.

IV. The *Dynamic,* or *Motion* and *Force Group,* consisting of Motion and Force, Change, Cause, Teleology.

(The category of Teleology also forms a transition link to the next group.)

V. The *Vital* or *Directive Group,* containing Life, Mind, Personality, Subject (vs. object).

VI. The *Absolute Group,* in which the principle of orders of material is carried to an extreme development. It contains such concepts as the Absolute, Fundamental, and Ultimate.

I. REALITY GROUP OF CATEGORIES

Matter

It has been said by a certain scientific worker that the carbon molecule and other chemical elemental ideas are mere concepts (i.e., primal groups) to enable us to deal more efficiently with phenomena, and that the whole idea of *matter* itself is merely a secondary and auxiliary phenomenon. It is important to realize more definitely from the efficiency standpoint, what is meant by these and like statements.

As an illustration of the utility inherent in the idea of materiality and substantiality and similar concepts, we may take Democritus' theory of vision and the various useful results which have followed from it. According to this theory any given object is constantly giving off effluvia in the shape of fine atoms; and when these atoms enter the eye they produce an impression (or image) which results in the given object being seen. With all its imperfections this theory has the merit of suggesting that something exists between the eye and the object seen, and that, by directing or working with this something, the process of vision may in a measure be controlled and made more efficient. For instance, it is but a step from this view to the thought that by interposing a lens between the eye and the seen object the atomic effluvia from objects may be collected and vision so powerfully reinforced that the very distant or the very minute be made visible as in the telescope and microscope; or to Newton's idea that the corpuscles given off, if they be of different

sizes, may be sorted into groups, the prismatic colors and spectrum analysis being the result.

If we enlarge this idea of atoms given off by objects to the conception of a universal fluid in which all objects are immersed and which objects can cause to undulate in various ways, we have, in general, the modern idea of the ether. Regarding the ether as having certain of the properties of materiality, Faraday formed the idea that a forward thrust of the ether in one direction would result in a backward thrust in some nearby adapted channel, or what is called the principle of electrical induction. The efficiency outcome of this was the invention of the dynamo, and of electrical machinery in general. It also resulted in Clerk Maxwell's electro-magnetic theory of light, and wireless telegraphy.

Hence we may take John Stuart Mill's conception of matter, viz.: as the permanent possibility of sensation, and extend it into the statement that matter is the permanent possibility of groupisms and efficiencies of certain specific sorts. From this point of view matter means the combined use of externality in both space and time, of multiplicative groups (see for instance the various methods of dividing and subdividing and combining parts of matter), of rhythms and other positive Efficients, along with limitations or special qualities such as are implied in the terms impenetrability, incompressibility, and inertia. The various scientific theories of matter, such as the ethereal stress and strain theory, and the molecular-atomic hypothesis, in like manner may be viewed as special combinations or aspects of the toolage properties of matter.

It is ever to be borne in mind that while the concept matter is thus a powerful efficiency agent in certain relations, it also has connected with it serious dangers and limitations. Thus the over use of it, or the crude use of it, may prevent the recognition and application of other important sources of efficiency in a given case.

However the dangers will be somewhat prevented if instead of the conventional somewhat crude and lumpish view of matter, we substitute the above view which makes matter a special combination of the Efficients and hence a special instrument of efficiency.

The view of the categories of matter and substance here advocated also opens the way to using, aggressively yet safely, these categories as special sources of efficiency under some circumstances. Thus when forms of abstract groupism have become familiar, it

is frequently a source of added efficiency to regard them as possessing certain of the attributes of materiality, or as more or less substantial, and thus to obtain the advantages of being able to manipulate such forms swiftly, readily, and by a low order of force, and also of conceiving them as always at hand and ready to act of themselves in certain ways.

By emphasizing or otherwise modifying one or more of the efficiental elements which go to make up the concept of matter, it is possible readily to conceive of different species of matter, and also of different degrees of materiality, and then to use these widely in the special efficiency channels for which they are adapted.

Substance and Reality

By *substance* is ordinarily meant a substratum underlying the special qualities of an object; or that which is left after an object has been stripped of its so-called qualities and attributes. Hence the idea of matter may be refined and generalized into that of substance; or substance may take the special form of matter. Thus substance is a kind of prime group, or efficiency abstract essence of matter and other objects. Hence much that has been said about the efficiental nature and uses of matter applies also with slight modifications to the category of substance.

The concept of *reality* differs from that of substance in that reality often has connected with it the idea of positive and aggressive action, while substance is mainly negative or resistant. In the reality concept the mechanical triggerlike properties which are prominent in the categories of matter and substance are less explicitly developed. The term reality, since it is applicable to ideal entities, as to thoughts and forces, is far broader in scope than are matter and substance in their conventional connections. Looked at more independently, reality from our point of view, is the possibility of permanent and certain reuse. Thus when we say that a specified man or entity is the real thing, we mean that he or it can be depended upon as a source of vital and fundamental efficiencies.

It follows that the efficiencies involved in the categories, matter, substance, and reality form an approximate order of materials succession, increasing in abstractness and comprehensiveness.

So valuable are the efficiencies contained in the idea of reality and so widely are these applicable, that in many cases this concept has come to be regarded as the most fundamental source of efficiency,

and has even at times been so overused as to exclude all other methods of obtaining results except as subordinate details. In this connection may be noted the tendency to regard the discovery of one fundamental reality as the aim of all philosophic study, even of life itself.

As an aid in preventing an exaggerated emphasis of the reality concept, it is well to keep in mind the defects of this concept as an agent of efficiency. It is to be remembered that the reality concept is characterized by a tendency to premature, crude, and exclusive finality, by a lack of diversity, and by a certain negative or resistant element. These limitations tend constantly, so to speak, to fossilize the concept in hand.

Viewing the category of reality as a special aggregate of efficiencies opens the way to the idea of different degrees and forms of reality, and hence to a wider use of the concept in its efficiency relations.

II. QUANTITY GROUP OF CATEGORIES

Space

In the preceding series of categories the concept of matter was taken and refined and generalized, primarily with reference to our sense of touch. In like manner we may refine and extend it primarily with respect to our sense of vision, and we shall thus arrive at the concepts of space and quantity. *Space* then is a special efficiency method of treating objects or entities of any kind, as was indicated in the opening paragraphs of this chapter.

Other considerations may be adduced which will be an aid in realizing as subordinate and auxiliary that view which regards space as an object or thing (that is, the substance or thinghood aspect of space). A misapprehension of the purpose with which the first treatises on the properties of space were written, is largely responsible for the exaggerated emphasis on the latter aspect of the above category, and for the desire for absolute certitude which springs from this view. The title of the first systematic treatise on space, viz.: the one written by Euclid, is Elements of Geometry, and the word here translated Elements is exactly the same word which the early Greek philosophers used when they described the universe as composed of the elements earth, air, fire and water. In other words, just as the Greek philosophers sought to reduce the physical universe to systematic form and thus by the use of uniformity

and diversity and by various kinds of groupisms, to economize man's labor in mastering and extend his power of manipulating it, so Euclid's primary purpose was to reduce the various properties of space to a few primal units, and thus give man an efficient mastery of the multitudinous variety of geometrical facts. Flawless certitude of deduction was secondary and auxiliary in accomplishing this primary end. Similarly his so-called axioms were named by him "common notions"; that is, by them he meant general and primal groups, or agents, in obtaining results, not primal elements of certitude. If two things are equal to the same thing, we are able to save the labor of a direct comparison of the two things. Similarly two points form a simple and highly efficient representative of a straight line; and three points, of a plane. In like manner it might be shown that all the different definitions and theorems of geometry are each an individual agent of efficiency, and that the combination of these into a system is made merely with a view to obtaining a higher order of efficiency. It will be found also that space may be regarded as a multiplicative system built up out of different elements in an unlimited variety of ways.

One advantage of the above view is that it facilitates the formation and recognition of a great number of different kinds of space, and also the groupings of any given set of objects so as to form what is in effect a space, and thus of reusing our knowledge and grasp of ordinary space in order to obtain increased efficiency in treating an aggregate of given objects. For example, if sensations be classified according to their differences in quality, intensity, and duration, they may be grouped and treated as a space. Similarly we are able to conceive of degrees, germs, and seeds of spatiality, and to make these ideas sources of efficiency under certain circumstances.

Number

In like manner from the efficiency point of view, numbering is more important than *number,* number itself being but a special existential tool aspect or element in numbering. In other words, the category of number is to be regarded primarily as a method or instrument for obtaining efficiency, rather than as a static object. Hence it is important to realize so far as possible the efficiental components of the category of number.

Evidently the unit and multiplier principle constitutes a special case or application of number. Also in its more general aspect

number is to be regarded as a form of linear multiplicative groups characterized by the uniformity among its elements, by their separateness or discreteness, and by the special and highly efficient form of symbolism used in representing them. This symbolism obtains its efficiency from the uniformity in the size of the unit symbols employed, from the extreme use of the principle of neighborhood symbolism, and from the fact that the symbolism for each number indicates not only a magnitude but also a position in a scale. Because of its relative simplicity and wide applicability the number toolage becomes highly assimilated in the mind and often acts with great inherent speed and force, and with dialectic power. An illustration is the way in which the reuse method termed generalization frequently acts of itself, when the number concept or a few number symbols are present with given material.

The efficiencies connected with number are so important that the presence of number in some form seems necessary to a high degree of efficiency in the treatment of any domain of material. Thus Plato says, "If arithmetic, mensuration, and weighing be taken away from any art, that which remains will not be much." In like manner Lord Kelvin says, "I often remark that when you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science." In another connection by way of illustration, Kelvin adds, "Arrhenius did not originate the doctrine of electrolytic dissociation or of free ions; that was enunciated in 1857 by Clausius, and remained relatively barren. What Arrhenius did was to introduce measured quantities into the doctrine, and show its simple quantitative applicability to aqueous solutions; immediately it became fertile."

Quantity

The category of *quantity* differs in general from space and number in that it is of a higher and more abstract order and for many efficiency purposes may be regarded as including the two categories just treated as special cases. Hence much that has been said of space and number is true also of quantity, but with the difference that the efficiental elements and properties of quantity are more general and comprehensive. In particular it should be noted that quantity

is characterized by an unlimited possibility of uniformities, diversities, groups, and multiplicative groups. Its pervasiveness also adds to its importance as an instrument of efficiency.

Zero and Infinity

The conventional meaning assigned to *zero* is that of absence of quantity or substantiality of any particular kind. But to obtain the full efficiency significance of the term, the context (or externality) associated with a zero in any particular case must be considered. For instance an absence of the kind described is often due to a concentration of material in a process of group making. Hence such absences and emptinesses are often closely associated with corresponding groupages, and these absences may become important instruments of efficiency in connection with the corresponding groupages. Thus a person may often express his intentions or attitude in a highly efficient way by doing nothing. A Fabian policy is an approximation to this method of obtaining results. A more abstract case is the use of x for $1x^1$.

Much more important is the fact that zero frequently means a balance between two opposites, these opposites often being two extensive and highly developed multiplicative systems. The zero involved may be made a source of efficiency in connection with these systems in various ways. For instance when we can write $0 = a - a$, if we know or can control one of the a's, we can often learn or control the other with little or no expenditure of effort. In other words, often, where only blankness apparently is present, two equal and opposed systems may really be at work, and the wise use of these two systems in relation to each other may be made the source of efficiency. Also, frequently a convenient and powerful way of applying externality is to substitute for zero or emptiness two equal and opposed aggregates of some kind.

For efficiency purposes, *infinity* is best regarded as the group principle acting without limit, either as to the extent of its action or in some other respect. Hence it is primarily a method rather than an object. Nevertheless, as with other categories, the thinghood aspect of infinity has certain special uses under special circumstances. Thus the idea of infinite externality as an object has a more restraining and stimulating effect for certain persons in some forms of religion than infinity as a principle or method. The objectivity form of the infinity concept however brings with it peculiar dangers

also, such as its tendency to crush and overwhelm, or at least to create a feeling of mere dependence. On the other hand infinity when viewed as unlimited groupism is easier to grasp, and can be applied more widely and freely. This view also opens the way to devise infinities of various sorts and to reuse the specific efficiency grasp which we already have of this category.

III. THE QUALITY AND TIME GROUP OF CATEGORIES

Quality

A *quality* of an object, for our purpose, is to be regarded primarily as an extremely close resultant of a number of efficiental elements. Thus hardness (as of a piece of glass for instance) means force, acting particularly as a limitation to other specific forces, and hence persistence in time, and hence certain group-making powers (as with respect to the transmission of light and heat). It is to be noted that the frequent difficulty of analysing a quality into various elements is often an advantage in that it enables us quickly and readily to manipulate qualities by the use of a low order of force. It may however be a disadvantage because of the lack of diversity which it entails.

A broader and often more useful view is that which regards qualities and objects, when considered together, as forming a reciprocal group system. According to this view an object is composed of a substratum (or underlying substance) in which a number of qualities inhere. Also any one quality may be present in many different objects, and therefore bind these objects together as a group. Thus glass has the qualities of hardness, transparency, a certain specific gravity, etc. But transparency is a property of other objects besides glass, as of water, air, alcohol, quartz crystals, etc. A danger in this view of the matter is that it brings the idea of substance into what is perhaps an unduly prominent position. At the same time other dual reciprocal group relations involved tend to diminish this danger.

A realization of quality as essentially groupistic in nature should increase its value as an instrument in obtaining efficiency. For example, in a soil the quality of fertility is composed of a group of component qualities such as the physical qualities of looseness, and cohesion, and other textural properties; also of certain chemical properties due to the presence, in right proportions, of nitrogen,

phosphorus, potash, and other elements; also of certain biological qualities due to the presence of the proper bacteria. A knowledge of these elements of fertility gives great efficiency in rendering a given soil more productive.

Time

For efficiency purposes *time* is to be regarded as a special method of grouping phenomena. This multiplicative grouping is usually regarded as predominantly linear in form, though there is a sense in which time may be regarded as of two or more dimensions, or even as a stratum or other detail in some more comprehensive multiplicative group system. Hence, as with space and number, the substantiality aspect of time is merely special, incidental, and even superficial. Since time is often used as a symbol for one or more forces, it forms a convenient transition to the next group of categories to be considered.

IV. THE DYNAMIC GROUP OF CATEGORIES

Force and Motion

The categories of *force* and *motion* were discussed at length in chapter XII (p. 210) and are now referred to again merely with a view to safeguarding their use from certain errors and to developing their more efficient use with reference to certain of the categories under discussion in this chapter. For instance, a motion takes place in space and time, and the remarks which have been made concerning space and time in the present chapter should render the utilization of motion more accurate and fruitful.

Mechanism and Cause

In the idea of *mechanism* the groupages which are characteristic of matter and force are combined to produce certain new groupages and efficiencies, often higher and more intense than the component ones. Thus when a domain of material has been reduced to the mechanical state, large parts of this domain can be operated in a trigger-like way at a slight cost of effort and by a low order of force. Hence, in certain relations, mechanism is a highly useful source of efficiency. It is probably universally applicable in some forms or aspects. Hence it has often been the tendency to overuse it, just as its components, matter and force, have been overused; at times to make it the primary, fundamental, and perhaps exclusive

method of obtaining results; or even to regard it as something final in itself and, by overuse to the point of contradiction, exclusive of the very idea of surplusage in results. As a matter of fact from what has already been said in connection with matter and force it follows that mechanism holds a subordinate and superficial place in the general scheme of groupage.

A few additional words may also be said in regard to the category of *cause*. As has already been remarked, for our purpose the cause of an event is best regarded as a precedent, dynamic externality to the event. Hence a grasp of the causes of a fact is an important means of obtaining efficiency results. For instance, such a grasp is often an important aid in systematizing knowledge, or in directing processes. In many cases a cause may be regarded as a handle by which to control or even create effects. But here again it is to be remembered that there is nothing final or ultimate in the idea of causation, and that it is merely an item or aspect in the general scheme of groupage.

Teleology

The category of *teleology* forms a convenient transition to the next group of categories to be considered. The aims and ends of efficiency processes have already been discussed at length in Chapter IX (p. 152) and this matter of teleology (or final cause) is now referred to again only with a view of so restating the matter in connection with certain other categories as to make it safe to use this principle more aggressively for efficiency purposes than is often done in some connections. From the point of view here prominent, an aim (or final cause) is an externality which subjectively precedes but which objectively follows an act. In the past undue prominence has often been given to immediate, personal aims and results. Personality is frequently very capricious and erratic in its operations. Hence teleology which consists of personal aims, uses, and values has often led men into the grossest errors. As a consequence numerous scientific workers and investigators have gone to the other extreme and sought to discard all teleology, and thus have lost the help, often powerful, which is inherent in this particular species of externality when properly applied. The kind of teleology which is always safe to apply and which is almost invariably productive of useful results, is subpersonal, efficiental teleology. This consists of aiming to obtain groups and other Efficients, and hence, ultimately,

surplus of some kind. Almost all important results in scientific investigations in the past have been obtained by the use, consciously or unconsciously, of some teleologic aim of this kind. In this class of purposes belongs the aim of discovering uniformities and diversities, and later of classifying facts in groups. A higher species is the aim to obtain a good working hypothesis and later to develop it into a final theory (or multiplicative group system). More specific illustrations are the studies which Harvey made of the uses of the valves in the blood vessels and which helped him to his discovery of the circulation of the blood, and Darwin's investigations of the purposes and uses of parts of organisms in connection with the survival of the fittest and the evolution of forms of life. From the point of view here advocated, it should be an aid in obtaining results in all fields of work to regard the discovery and development of groups, multiplicative groups, externality, and other Efficients, as a certain primal, subpersonal aim and end of processes. As has already been suggested, we can in this connection even conceive of one final and comprehensive aim or summum bonum, viz.: a thoroughly perfected and universal groupage yielding the utmost surplusage to every entity and mode of existence. If not itself final, this aim should form an aid, not a hindrance, in obtaining a more ultimate result.

V. THE DIRECTIVE GROUP OF CATEGORIES

We now come to a group of categories closely related to the Efficient termed directive. This group includes the categories conventionally known as *life, mentality, personality,* and *subject* (vs. object).

Life

For our purpose a living organism may be regarded as essentially a set or domain of groups combined and acting in certain specific ways. In it directive, externality, and rhythm, and resulting surplusages are prominent. As has already been indicated in various places such aggregates have connected with them certain characteristic limitations and aptitudes for error commensurate with the extraordinary powers inherent in them.

If life be thus viewed as consisting of certain essential and specific groupages it becomes easy to conceive of a great variety of forms of life many of them widely different from those usually included under this term. Thus looking at the matter from the chemical

standpoint, just as our present terrestrial life seems based on carbon groupages, so it seems possible to conceive of a system of life based on silicon groupages, and hence to conceive of life of this kind as present in the hottest stars. Similarly we may conceive of vital groupages different in many non-chemical ways from those familiar to us on the earth, as present, for instance, on the planet Mars or other planets. Similarly, by development of the groupages involved, we may conceive of far higher orders and degrees of life than any with which we are at present acquainted.

If the category of life be viewed essentially in the above way, it also becomes easy to reuse our present grasp of this source of efficiency in an aggresive and somewhat artificial way. Thus we gain a certain confidence in the use of some Efficients and groupage aggregates by regarding them as agents possessing powers of independent action, and to be relied upon to produce results of themselves. We may even regard them as endowed, in a sense, with some attributes of personality or even of hyper-personality.

Mind

In continuance of the above views mind may be regarded as life plus (or with an especial development of) a high order of externality and directive. Consciousness, as is indicated by the etymology of the word (*con,* meaning together, and *scio,* to know) is a knowing of things together, or an awareness of them in groups; and mind is a development and use of these groups with reference to a varied and increasing externality. Hence it is easy to see that mind must be an efficiency instrument of the highest order of importance. This view of mind enables us to conceive of a great variety of mentality, including types very different from those familiar to us in human beings; as, for instance, of methods of sense perceptions in the animals widely different from those in man, and also of very superior and transcendent mental processes perhaps attainable in time by us and perhaps glimpsed in the intellectual processes of so-called geniuses. It also enables us to make certain aggressive and, in a sense, artificial uses of mentality similar to those suggested for the category of life.

Personality is life and mind in connection with a single organism and usually developed in some striking and specific way.

By *subject,* in this connection, is meant a personality, as contrasted with the rest of the universe, or some part of it, taken as object.

VI. THE CATEGORY OF THE ULTIMATE OR ABSOLUTE

Efficiency Properties of an Absolute

If we could discover a primal (ultimate, fundamental, central, dominating) category or entity, in terms of which all other entities could be stated, or from which they could be derived, we should evidently have a highly important source of efficiency. For if this fundamental entity should prove to be a thing or object, by controlling or directing it we could reduce the universe to a mechanical or triggerish state. If it were more abstract in nature it would still furnish an all-comprehensive unity (or uniformity), with vital and comprehensive uses such as that of furnishing the material or basis for unlimited group formations, or that of supplying an omnipresent and manifold externality, or that of opening the way to universal marginal reuse. Because of this inviting vista a large part of the history of human thought has consisted of the search after some one fundamental reality with the (often subconscious) hope of obtaining a single final and complete instrument of efficiency or means of obtaining results. Various names have been suggested for this primal category among these being the fundamental reality, truth, or principle; the ultimate reality, or the *absolute*.

It may be suggested at this point that the present investigation thus far points to the conclusion that all that is useful in the proposed category of efficiency now under discussion is obtained by taking the group concept itself as the ultimate and inclusive principle for the present.

In this connection it may be noted that if we examine the various ultimate principles which have been suggested in the past, we find that each of these may be regarded as a modification or aspect of the group principle.

List of Proposed Absolutes

These suggested ultimates may be classified as follows:

I. The materialistic class: matter, water, air, fire.

II. The force or energy class: energy, force, light (and darkness), motion, change.

III. The biologic class: life.

IV. The psychological or personal class: mind, reason, consciousness, pleasure, love, faith, will, idea, spirit, soul, God.

V. The ethical and practical class: good, duty, value.

VI. The methodological class: becoming, evolution, the way, unity, number, the category of order, relation, interaction, or system.

VII. The reality class: absolute, substance, being.

The Multiplicative Principle as a Provisional Absolute

The analysis which has already been made in this chapter of the various categories, and also that made of other of the above concepts in the course of preceding chapters, shows in a general way in what sense each of the above proposed ultimate principles may be regarded as a special form or modification of the group principle. When the multiplicative principle is taken as ultimate, the narrowness or special individuality of many of these primal categories is remedied by the tendency to expansion which is characteristic of the group idea. For instance, the multiplicative principle by its power of self-correction and of developing unity, remedies the chaotic tendency inherent in a personality or an ethical principle when each one applies it to suit himself, as when each one forms his own system of values.

For this and other reasons, by regarding each of the above proposed central principles as a modification of the group principle, the way is opened to use each of them, and the system derived from it, aggressively as a special form of instrumentation.

It is also to be noted that if something more underlying and inclusive than the multiplicative principle is discoverable, the expansive and corrective properties of this principle would seem to lead on to and to compel the attainment of this something in the end, in case such attainment is possible. Hence we arrive at the conclusion that for the present the largest results are to be obtained by regarding the group or multiplicative principle as the most ultimate in sight, and by viewing personality (in the conventional sense of the term), reality, thinghood, and all forms of equivalence as relatively secondary toolages.

The Categories in General

With respect to the categories as a whole, it should be remarked that each of them probably contains something characteristic and unique, which may at any time become the source of hitherto unrealized efficiencies; and also that, by modification of the different categories and by various combinations of them, what are in effect

important new and more or less independent species of them may be obtained.

Phenomena

Before closing the discussion of the categories, some mention should be made of what are conventionally known as *phenomena;* that is, of sights, sounds, and other sensations and perceptions, and of the objects which are taken as directly represented by these. Since phenomena may be almost altogether analyzed into matter, force, mind, and the other categories which we have been studying, it follows that phenomena are often special instrumentalisms. Many of these apparently have been evolved in a crude and haphazard manner, but almost all of them possess some peculiar and highly finished efficiency property. As illustrations we may mention the efficiency functions of wood, stone, light, and the change of seasons.

Efficients, Categories, and Phenomena

As a result of our investigations we have therefore found three general classes of efficiency instruments, viz.: those which have been respectively termed Efficients, Categories, and Phenomena. No one of these classes can be determined independently of the other two, and each can be resolved largely into the others. But in this circuit of transformation, positive surplusage everywhere may arise, and a varied cancellation of assumptions takes place. In this manner, from a new and more comprehensive point of view, the group is made to stand forth in the first place both as method and result, and as leading on to other more ultimate ideas, if such are called for, or are possible. We thus arrive at a general conception of the relation of efficiency to other primal ideas and to a general philosophy of life, which, in order to obtain the advantages connected with a name, we may term efficientism.

B. HISTORICAL SURVEY

Value of History of Efficiency Principles

A brief survey of the historical development of the principles which have been presented in this book will be of service in different ways. For instance it will clarify and develop these principles by presenting them in certain new relations to each other. It will exhibit the uses of the Efficients in the special forms in which they have been applied in the past. It may suggest new individual forms which they may take. It should shed light on the best places at which and methods by which to apply these principles to the present-day world. It will also aid in realizing the place of efficiency in a general philosophy of life.

Certain of the primal efficiency instruments (as uniformities, diversities, groups, rhythms) appear, though often in a crude form, in inanimate nature, and in various plants and animals. In man also we find them developed to a certain point before the dawn of any distinctive efficiency consciousness.

Primitive Instruments of Efficiency

We begin our more detailed survey with a statement of the initial, semi-conscious efficiency efforts of man. We shall first present in tabular form (see p. 351) the early instruments devised by man for making his exertions more productive. In this table the conventional names for these aids to efficiency are used, but we shall also state the more important of them in terms of the Efficients.

Thus it will be observed that each item in the table, as the use of caves, clubs, metals, and animals, involves externality of some sort.

Each of these items also arises by directive action of some kind, and is made more productive by means of reuse.

Each of the tabulated devices and arts involves some species of groupism. Multiplicative groupism appears in many highly condensed forms, but also in some which are more explicit and elaborate, such as tribal organization and articulate language. The order of materials concept is illustrated in the series of materials, wood,

INORGANIC	BIOLOGIC	PSYCHOLOGIC	SOCIAL	RELIGIOUS
Caves (as dwellings)	*Trees* (as dwellings)	*Articulate language*	*Family*	
Stones (as weapons)	*Clubs*	*Pictorial arts*	*Tribalism*	*Fetishism*
	Opposable thumb			
	Right handedness			*Taboo*
	Wild animals (as food)			
Shaped stones (as arrow and axe heads, pestle and mortar)	*Skins and tendons*			
	Tomahawk (bow and arrow)	*Graphic symbols*		*Animism*
			Totemism	*Magic*
Sun-dried pottery				
Fire (baked pottery)		*Music*	*Exogamy*	
Copper	*Domesticated Animals*	*Written language*		
				Confucianism
Bronze	*Pastoral life* (human slavery)	*Astronomy* (beginnings of science)		
Iron	*Agriculture*			
Steel				

stone, copper, bronze, iron, steel; in right handedness; and in the succession of linguistic forms, and of the mental processes which accompanied these.

Primitive Combinations of Efficients

It should also be realized that usually several of the primal instruments of efficiency are found operating together in one of the items tabulated above.

Thus man's erect position by raising the eyes to a higher position gives a larger outlook and hence increased externality in human life processes. It also leaves the hands free for efficiency functions which

are higher as a whole than those of the feet, and has thus produced an order of materials development of man's limbs, which in time has led to the formation of other orders of material.

The use of fire constitutes an externality which is useful both directly and also indirectly by the formation of diversities and re-groupings. The resulting actions and reactions form a rhythm of increasing power.

Crude as it is at first sight fetishism contains an important externality. In fact in fetishism and like practices are found the germs of the use of unlimited externality, together with certain consequent groupings, forms of reuse, and other efficientizings.

It is important to note that in all this early efficiental life, surplusage or profit holds a dominant place as compared with certitude or reality. The primitive man lived and worked for definite and directly useful results. This view of life so far as it is a general view or theory, takes the form of what is called animism or spiritism (or personeity). Spiritism is essentially that view of life in which the principal method of obtaining results is that of treating all nature processes as the acts or under the control of persons (beings, spirits), who can be propitiated and converted into friends and helpers. Hence animism puts surplusage and the personal methods of obtaining it in the first place as compared with the mechanical equivalence idea of life later called natural law. The use of spirits suggesting as it does large immediate returns at little cost, appealed to men in several important ways and, for the time being, was developed in preference to pure groupism with its better results attainable after a larger period of effort.

Crude as they were in many respects, the essential efficiencies involved in man's first inventions and discoveries produced in the aggregate remarkable gains. Thus land used pastorally was one hundred times as efficient as land used for the hunting of wild game; and, when used agriculturally, was four times as productive as when employed pastorally.

Efficiencies in Early Empires

The large alluvial plains of Egypt, Babylonia, India, and China in which men first applied agriculture extensively, fostered the development of efficiency in certain ways, but also gave rise to some backward tendencies. Such plains are characterized by vast uniformities which led to the promotion of giant homogeneous political,

social, linguistic, religious, and intellectual groups. Often one of these vast groups in the end took the form of a multiplicative group system, the most explicit example being the empire of the Medes and Persians with its satrapies, provinces, and subprovinces. In each individual empire, and between empires, also naturally developed forms and vast amounts of reuse, reciprocal externality, directive, symbolisms, and permanencies in time. Hence also arose important efficiency arts and sciences, such as systematic irrigation in agriculture, the astronomy of the Babylonians, the land measurement (or geometry) of the Egyptians, and the arithmetic of the Hindoos. The large size of the groups involved together with the accompanying lack of diversity led however to some evils, chief among which were certain extreme forms of uniformity. Conspicuous illustrations are the caste system of the Hindoos and the oriental doctrine of fatalism.

Similarly the category of personality was developed in striking ways in these early civilizations. One of the more concrete forms of such development was Confucianism among the Chinese; more sublimated species were Brahmanism and Buddhism among the Hindoos and the worship of Jehovah among the Hebrews. Besides giving other advantages, these highly developed views of spiritism were useful in making infinite externality a near and useful fact. Associated with these ideas were positive non-equivalence, or surplusage, taken as dominant over equivalence in the form now termed natural law. Connected with personality and surplusage in the highly developed forms however were grave evils such as magic and necromancy, certain views as to miracles, and a failure to recognize pure groupism as in many respects back of and dominant over personality, and leading to and requiring important kinds of universal equivalence or law.

Greeks discover the Group Principle

It is the immortal glory of the Greek mind to have discovered, in essential respects, the group as an abstract, primal, and inclusive principle. Near enough to Orientalism to benefit by its results, and yet far enough away to be able to perceive and to attempt to remedy its defects, aided by the geographical diversity of the land in which he lived and the variety of its natural resources, familiar with the sea and its efficiency uses, and influenced by other factors, the Greek was led to recognize the value of uniformity

and of grouping. His attempts to form comprehensive efficiency systems along this line naturally at first took somewhat concrete forms.

As an example we have the views of Thales. In his idea of water as the primal material out of which all things are made or generated, Thales, in effect, conceived of a universal first principle, free from the caprices of personality (but annexing the elements of personality occasionally in a subordinate manner), and therefore capable of being grouped and multiplicatively grouped in reliable ways, and hence open to mechanical trigger control.

Water of course has radical imperfections as a first principle, and other early Greek philosophers tried to get rid of these by using instead of water some other element, as air, fire, mind, being, motion, rest, or matter. Of these immediate successors of Thales, Democritus performed the greatest service in that he first developed in explicit group form some of the properties latent in the substance which he regarded as primordial, viz.: matter. He obtained this result by regarding matter as divided into units termed atoms, and by combining atoms in what were essentially groups to explain phenomena. The advantages thus attained are illustrated for instance in Democritus' theory of vision and its consequences as presented elsewhere (pp. 335-336).

All of the early Greek thinkers erred however in paying primary attention to the category of reality and in leaving the efficiency functions of their ideas to take care, so to speak, of themselves.

The Sophists performed a negative service by showing that certitude and ultimate reality could not be attained along the route by which these were being pursued. They performed a more positive service by making gain or advantage the primary aim of life and by developing certain grammatical and rhetorical groups and multiplicative group systems, of which the moods and tenses of verbs are examples, as instruments in attaining their ends. They erred however in adopting narrow and superficial views as to the nature of gain and profit.

Socrates was the first to discover and apply the group in an abstract, general form. The group as used by him is what is now usually termed the concept. Socrates also developed this into the incipient multiplicative group form embodied in the use of the terms genus and species. He applied the group, as thus conceived,

in the special and somewhat personal fields of ethics and sociology, and not at all in the domain of natural science. His primary aim also in thus applying the group was certitude and not utility. He could not however use it even in this narrow way without to a certain extent developing its abstract and general properties. Hence the work of Socrates, in spite of his failure to give the group a pure subpersonal form and free development, constitutes a prime epoch in the history of efficiency.

Plato and Aristotle

Plato took the group instrument invented by Socrates and developed it in important ways. In his hands it took the form of the idea or, as we would perhaps describe it, the seen or graphic efficiental essence of an object or domain, though he often gave it much more general and untrammelled meanings. For Plato separated the special form of the group which he termed the idea, from the world of concrete limitation and quality, and developed it freely with respect to various categories. Thus owing to his work the group acquired a name, became subpersonal, objective, self-existent, and creative of phenomena. Plato not only developed his group idea in these ways, but he also applied it to all departments of life and knowledge as they then existed. In the more abstract of these departments, as in mathematics, logic, ethics, and sociology, he by this means obtained results of importance; but his undue emphasis on certitude and his lack of appreciation of the enormous profusion of highly developed diversities in the world, make his investigations in the field of more concrete science, crude and unsound. His separation, for the time being, of ideas from the world of concrete phenomena, and his consequent free and aggressive development of their properties, form a work of the first importance. It is to be noted that in this work of Plato's we find an application of diversity, externality, and orders of material in some of their highest forms.

With Aristotle the group took the specific and dominant form of the category. Aristotle recognized more fully than did Plato the fundamental richness of diversity and of individual quality in phenomena, and the consequent need of using special adapted primal forms of the group when applying it in different fields and relations, and hence also the need of careful progressive steps in such applications. Among the special forms in which the group and

multiplicative groups appear in Aristotle, are his categories, his various kinds of causes, his syllogisms, and his methods of classification and organization. In certain important ways he also realizes substantiality or thinghood as subordinate to or as a special cause of grouphood and value. Thus he regards things as their purposes, ends, and relations, rather than as the matter composing them. Motion he also explains as the result of the desire of things for form. Similarly matter according to him is a potency of form; soul is an actuality of this; and God is *purus actus* (or pure groupism).

Aristotle's treatment of the group lacks the freedom and sweep of Plato's but it fits the details of the naïve phenomenal world far more closely. Because of his careful, direct application of what is essentially the group method to the facts of life as they are immediately apprehended, Aristotle was able to initiate important new departments of study and efficiency attainment, such as physics, botany, zoology, and medicine, and to put logic in what is essentially its present form.

Other similar products of the Greek use of the group were sciences like psychology, ethics, and esthetics; or more special products such as Greek literature, architecture, and sculpture. To this fundamental source also are to be traced the principles of measure, proportion, moderation, and symmetry, which pervaded the Greek civilization and filled it with a manifold creative life.

Limitations in Greek Grasp of Efficiency Principles

From the ideal standpoint, however, the Greek systems of efficiency had serious defects. One of these was the failure to realize the self-corrective power in the multiplicative principle when freely used. Hence results the tendency in the Greek mind to invent and employ elaborate and costly methods of preventing error. This characteristic is illustrated by Aristotle's use of his system of syllogisms primarily as a means of detecting fallacies, rather than as a method of putting knowledge in a productive and readily usable form. Other deficiencies are the undue prominence given to truth and reality as aims and ends, and the lack of a grasp and free use of infinite externality.

Cramped in these ways, the Greek efficiency systems were not able quickly to grasp and handle the world life as a whole on coming into sudden contact with it after the conquests of Alexander the

Great. Hence the primal Greek instruments of thought and action came to be used in a narrow and personal way as in Stoicism and Epicureanism, or in a mystical and flighty way as in Neo-Platonism. Hence while the use of the group concept continued to produce certain useful results in mathematics, art, literature, and some departments of science, it was prevented from quickly reaping its full fruitage by the fact that its use was made tributary to the search for a final reality.

Efficiencies Characteristic of Christianity

The next great advance in the development of the principles of efficiency was made by Christianity and consisted in putting infinite externality in a near and usable form. Orientalism had made familiar the idea of the infinite group, but in the unlimited group as thus conceived the individual was lost and immersed, or with respect to it was regarded as insignificant, the result being a tendency to fixity and fatalism. On the other hand in Christianity the individual was conserved, and the infinite group had to him the relation of infinite externality with all the possible uses of the same. Thus in its essence Christianity is a combination of oriental and Greek efficiency processes. With this union the broad and complete foundation was laid for every ultimate advance.

The Middle Ages

During the middle ages there was steady progress in the major elements of efficiency. Thus there was a constantly increasing appreciation of the brotherhood of man and the fatherhood of God, and of the vast groupages which these ideas imply. The Greek system of groups was also used to advantage, in a supplementary way, in Christian theology, and the church was made more effective, in some respects, by the adoption of the Roman-oriental system of governmental organization. However, in certain important respects the efficiency life of the world retrograded during the medieval epoch. The tendency to seek in some easy and immediate way the fruits latent in the Christian infinities was prominent. Hence, following the method of Plato, the vast amount of concrete limitation in the world and the need for patient study of detailed facts, and for their grouping and regrouping, were overlooked, the detailed activities of many centuries being given up to theoretic speculation, and mystical rhapsody. Nevertheless the major elements of efficiency

which were at work were so powerful that important special new elements of efficiency were being discovered or invented from time to time. Among these advances were the invention of the horseshoe which made the age of chivalry possible, the discovery of the magnetic needle, the invention of gunpowder, the discovery of many chemical facts by the alchemists, and the introduction into Europe of the Arabic arithmetic. Perhaps we may fairly add to the list the Copernican astronomy with the concrete externalities, and the re-multiplicative-groupings which it implies, and the invention of printing by movable type with the various efficiental processes inherent in it and flowing from it.

Relation of Bacon to Efficiency Principles

Bacon, like Aristotle, realized the vast richness of diversity and the great variety of limitations which characterized the world of nature and life as we find it. Hence he understood the need of a careful, methodic study of nature at first hand, in order to obtain results of vital and permanent value. He advocated the patient collection of facts and the arrangement of these in groups, with subsequent constructions from them of hierarchies of inclusive principles or laws. In other words, in essence he urged the discovery and formation of elementary groups and the building up of these into multiplicative group systems. Bacon's use of groups differs from that of Aristotle in important particulars. It is superior in its advocacy of the investigation, in cases of difficulty, of mere uniformities and diversities; in the analysis given of the sources of error in the process of forming groups; in a realization of the value of formal written tabulations of facts; in the use of the objective world as the best starting place for such studies; and in the orderliness and comprehensiveness of his proposed study of the objective world.

Bacon however was inferior to Aristotle in that he failed to realize the value of an early general group schematism, obtained from an examination of categories like space, time, substance, force, as well as phenomena, as an aid in mastering facts, even though such a schematism be subject to much later modification.

Of like nature is Bacon's failure to recognize the value of the working hypothesis. Rarely, if ever, has an important discovery been made by the application of the Baconian method pure and simple. The effective method of arriving at multiplicative group

systems of facts, is not to try as a first process to collect all possible facts and then at one stroke to frame them into a final system. Rather the successful method is to collect facts till an intelligent working hypothesis, or provisional multiplicative group system can be formed; to test this provisional system by deduction (i.e., reuse), by experiment (i.e., by externality and the use of units and multipliers); to arrive thus at new facts in a far more efficient way than could otherwise be done; to reconstruct the working hypothesis from time to time by the aid of the new facts thus obtained; and to proceed thus by rhythmic progression till a highly developed or even final result is obtained. An example is the process by which the law of gravitation was discovered, extending as this process does through periods of alternate observation of the heavenly bodies and the framing of theories from the time of the Babylonians to Newton. A more concise illustration is supplied by the succession of steps by which Darwin developed the doctrine of the evolution of species.

Descartes and Leibnitz

The general provisional multiplicative group schematism, broad enough to include the categories as well as the then known or ascertainable concrete facts, which Bacon failed to make, was attempted by Descartes, Spinoza, and others. The most important of these comprehensive toolages for our purpose is that of Leibnitz. The monads employed by him from one point of view are units or elements; from another are groups; and from still another constitute strands in a pluralism. The principle of preëstablished harmony which in effect combines the monads, is the group principle taken as underlying and controlling. Its use in essence, makes subpersonal groupism primal and dominant, and hence more fundamental than either personality or material mechanism. The emphasis placed by Leibnitz on the principles of continuity and differentials also constitutes a recognition of the special value of uniformity and diversity. Hence in the system of Leibnitz as a whole we find our main instruments of efficiency, but in a peculiar and highly individualized form. The value of these principles even in this form is demonstrated by the important results obtained by Leibnitz through their use, the most valuable of these being the differential and integral calculus in mathematics.

Locke and His Successors

Locke applied the Baconian method to the domain of mind. His aim was to build up all mental phenomena out of a few primal elements, as all language is formed out of the alphabet, or as Euclid built up his geometry out of a few fundamental properties of space. Locke thus obtained efficiency results of the greatest importance. These were applied by him and his successors, particularly in France, not only to psychology, but also to ethics, education, and government.

Berkely and Hume ignored the economy and efficiency significance of Locke's work, and spent much time and effort in showing that Locke's method did not reveal an ultimate reality or give fundamental certitude. This drove Kant, like Socrates under similar circumstances, to a recognition of the group. For Kant regards time, space, and the other categories as group methods in which the mind acts in relation to the objective world, and his synthetic judgments indicate the surplusages or advantages which result. Notwithstanding its great apparent emphasis on mind and personality, this point of view is closely related to that which regards groupism as the primal principle with mind, matter, space, time, etc., and as special modes in which the group principle acts or manifests itself.

Kant, Hegel, and Herbart

Kant also made an important contribution to the efficiency schematism by obtaining a principle of certitude in his categorical imperative. This principle is arrived at in effect by a use of unlimited externality with a consequent order of materials and dynamic result. Hence it constitutes a special manifestation of the self-corrective and unity-giving power of the multiplicative principle.

However in Kant's system the lack of explicitness as to whether the group is to be taken as more fundamental than mind and matter, leaves the world of principles and the world of phenomena largely uncoördinated. Hegel tried to combine these two worlds by taking the "idea", which, in spite of the element of mentality seemingly implied in it, is fundamentally a group principle, as primal and constitutive of all things. By this method he obtained many efficiental results, but in its application he failed to take the principle of limitation and of specific quality adequately into the account.

Herbart avoided the element of mentality which warps and cramps Hegel's system from the efficiency point of view, by taking as a fundamental principle what is essentially the abstract and subpersonal

group discussed in Chapter IV. This is found in his idea of particulars forming a unity (his *realen* and *susammen*). However he retrogrades in that he uses this fundamental conception primarily as a means of obtaining certitude or reality rather than efficiency, and in that he fails to realize its power when developed as an extended multiplicative series and in various qualitative forms. He nevertheless obtained useful results by an application of his method in the field of education; the most important of these being his principle of apperception. The use of apperception in education means, essentially, the statement of what is new in the terms of what is already known. Hence in it are contained the use of uniformity and diversity, reuse, and often of the unit and multiplier.

Evolutionary as related to Efficiency Principles

The method of evolution as presented by Darwin, Herbert Spencer, and others, may be regarded as a special combination of certain efficiency processes.

Thus in Darwin's principle of the survival of the fittest we have a multiplicative expansion of objects by reproduction, a resulting homogeneous externality and a re-multiplicative-grouping into a more effective system. The continuity characteristic of such a process involves an almost continuous stream of marginal reuse, each organism reusing almost in an entirety the organism which preceded it in the stream of development.

Similarly Herbert Spencer's more comprehensive view of evolution as a series of alternate integrations and dissipations may be viewed as a rhythm of groupings, new externality, and re-groupings. In these processes the principles of limitation and motion and force are also fully recognized. In certain forms of evolution still other groups are suggested, as in the idea of Mendel and de Vries that the elements of an organism may vary and be adapted in groups. The use of the order of materials is also suggested in the thought, for instance, that the capacity of an organism for variation may vary.

It is to be observed, however, that, as ordinarily considered, evolution is a slow and costly system of obtaining results, as compared with certain other existent or conceivable efficiency systems. Thus, from the ideal efficiency standpoint evolution is deficient because of its slight use of externality, orders of material, directive, symbolisms, and other similar highly efficient instruments. Hence, as usually understood, it forms a special detail in the general efficiency system which we are investigating.

Concrete and Scientific Developments

As has been suggested and illustrated at various points, parallel with the development of the abstract principles of efficiency, there has proceeded a development of these principles in certain more concrete and special fields, as in agriculture, mechanics, government, and the various arts and sciences. These two streams of progress have often interacted helpfully. As an illustration of the development of the Efficients in one of these special concrete forms we may take the changes which have characterized the progress of agriculture.

Similarly the progress of science, as a whole, has been essentially, a progressive development of efficiency instruments and methods. Thus the axioms and laws of science are primarily fundamental instruments of efficiency. For instance the principle that action and reaction are always equal is a primary source of economy, since when either action or reaction is known, the other is known also, and the labor of an independent investigation of it is saved. So also progress in science is made by the extended use of various kinds of auxiliaries, by units and multipliers, and reuse, and by combining results in multiplicative group systems which are full of both economy and dialectic power.

Pragmatism and Its Services

The brief examination of the history of the principles of efficiency made thus far shows that the realization of these principles has been greatly hindered by the fact that throughout by far the greater part of definitely recorded history, the discovery of an ultimate reality has been taken as the primary goal of human endeavor. It would seem that the concept of a fundamental reality cannot be ultimate; that if an ultimate were discovered we would at once proceed to put it to certain uses; and that the uses of this reality would therefore be more fundamental than the reality itself. If however the human mind during this long record has been unconsciously searching for a fundamental reality not for its own sake, but for the sake of its uses, such a fact has usually been so far in the background that no systematic investigation of these uses, or of methods of their individual attainment, has been attempted. In other words up to within a few years any supremacy attributed to efficiency or a kindred concept has been unconscious, or unsystematic and undeveloped.

Recently however in the movement termed pragmatism, a conscious and somewhat organized attempt has been made to put utility in the dominant place in human thought and processes. According to this view of life the certitude of a principle is determined by the consequences of that principle. Reality depends on workability, or the useful is the real. This view of the matter seems to make value or surplusage more primal and determinative than reality. But by their elaborate *a priori* discussions of the nature of reality and by requiring utility to satisfy certain rationalistic tests laid down as determinative of reality, the pragmatic thinkers again in effect put reality in the first place. They also do this in a negative way by failing to make any thoroughgoing examination and development of efficiency methods and instruments, and by omitting to show that reality may be accounted for as one of these instruments. The additional failure to make personality (as ordinarily understood) subordinate to efficiency in general, makes possible the chaos which arises when each person is allowed to construct his own efficiency (and hence reality) world, unharmonized with those of others by any systematic comprehensive means. In other words instead of making the group primary and fundamental, at least for the time being, the pragmatic movement spends its force in rationalistic discussions of the relations between two special forms or results of the group, viz.: reality and personal utility, and alternately giving the dominance to each of these. Even this imperfect treatment of the utility concept however has produced a number of widely useful results.

The pragmatic movement, for instance, has, in a measure, inspired the recent direct study of efficiency in more or less concrete fields, as in business management and in civic and governmental affairs. In these movements many special results of importance have been arrived at. The more important of these results have, however, been referred to in the chapters which precede, and no repetition of them is called for at this point.

Summary of History of Efficiency

The preceding historical survey may be conveniently summarized as follows. In the early stages of human history we find the group utilized in the form of the family, tribe, or clan; in flocks and herds; in various mechanical devices; in different kinds of language; and in ideas as to various forms of spirits. All this use of the group

is dominated by a naïve assumption of gain or surplusage as primal, with slight development of the idea of reality.

Later the great alluvial plains fostered the development of enormously large groups both concrete and abstract. These larger groups mastered and often absorbed the smaller ones formed earlier, and also those systems of groups which came into the great river plains from outside sources, principally Aryan. The ultimate result often was the formation of a gigantic multiplicative group system, operated from the apex, and illustrated by the Persian and Roman empires. In contrast with this was developed what may be termed the federal, or Greek-Christian-Teutonic multiplicative group system. In this form of group organization action originates mainly in the individual units or elements and proceeds toward the apex. The system however is so full of freedom that multiplicative action also proceeds from the apex toward the elements, and in many cross and mixed directions. In general the multiplicative principle acting thus tends to give unity along with the maximum of freedom. The Greek-Christian-Teutonic method when developed in the most abstract and inclusive way gives rise to a federal system of Efficients, acting in connection with a similar but weaker system of limitations, the two systems intersecting to form many special agencies and results. It makes fundamental neither the One nor the Many, but the One-in-the-Many. Hence this general schematic conception may be regarded as including monism, dualism, pluralism, idealism, mechanism, and the other special systems touched upon in the preceding historical survey, as details of itself, especially when the systems just mentioned are regarded as methods of obtaining results.

INDEX